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STRENGTH OF MATERIALS

Strength of Materials

Strength of materials deals with the relations between the external forces applied to elastic bodies and the resulting deformations and stresses. In the design of structures and machines, the application of the principles of strength of materials is necessary if satisfactory materials are to be utilized and adequate proportions obtained to resist functional forces.

Forces are produced by the action of gravity, by accelerations and impacts of moving parts, by gasses and fluids under pressure, by the transmission of mechanical power, etc. In order to analyze the stresses and deflections of a body, the magnitudes, directions and points of application of forces acting on the body must be known. Information given in the Mechanics section provides the basis for evaluating force systems.

The time element in the application of a force on a body is an important consideration. Thus a force may be static or change so slowly that its maximum value can be treated as if it were static; it may be suddenly applied, as with an impact; or it may have a repetitive or cyclic behavior.

The environment in which forces act on a machine or part is also important. Such factors as high and low temperatures; the presence of corrosive gases, vapors and liquids; radiation, etc. may have a marked effect on how well parts are able to resist stresses.

Throughout the Strength of Materials section in this Handbook, both English and metric SI data and formulas are given to cover the requirements of working in either system of measurement. Formulas and text relating exclusively to SI units are given in bold-face type.

Mechanical Properties of Materials.—Many mechanical properties of materials are determined from tests, some of which give relationships between stresses and strains as shown by the curves in the accompanying figures.

Stress is force per unit area and is usually expressed in pounds per square inch. If the stress tends to stretch or lengthen the material, it is called *tensile* stress; if to compress or shorten the material, a *compressive* stress; and if to shear the material, a *shearing* stress. Tensile and compressive stresses always act at right-angles to (normal to) the area being considered; shearing stresses are always in the plane of the area (at right-angles to compressive or tensile stresses).

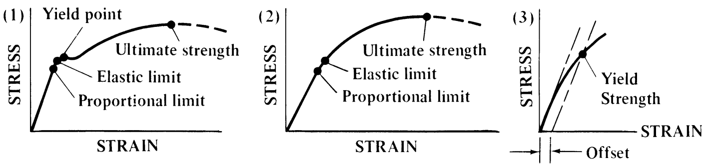


Fig. 1. Stress-strain curves

In the SI, the unit of stress is the pascal (Pa), the newton per meter squared (N/m^2). The megapascal (newtons per millimeter squared) is often an appropriate sub-multiple for use in practice.

Unit strain is the amount by which a dimension of a body changes when the body is subjected to a load, divided by the original value of the dimension. The simpler term *strain* is often used instead of unit strain.

Proportional limit is the point on a stress-strain curve at which it begins to deviate from the straight-line relationship between stress and strain.

Elastic limit is the maximum stress to which a test specimen may be subjected and still return to its original length upon release of the load. A material is said to be stressed within the *elastic region* when the working stress does not exceed the elastic limit, and to be stressed in the *plastic region* when the working stress does exceed the elastic limit. The elastic limit for steel is for all practical purposes the same as its proportional limit.

Yield point is a point on the stress-strain curve at which there is a sudden increase in strain without a corresponding increase in stress. Not all materials have a yield point. Some representative values of the yield point (in ksi) are as follows:

Aluminum, wrought, 2014-T6	60	Titanium, pure	55–70
Aluminum, wrought, 6061-T6	35	Titanium, alloy, 5Al, 2.5Sn	110
Beryllium copper	140	Steel for bridges and buildings, ASTM A7-61T, all shapes	33
Brass, naval	25–50		
Cast iron, malleable	32–45	Steel, castings, high strength, for structural purposes, ASTM A148.60 (seven grades)	40–145
Cast iron, nodular	45–65		
Magnesium, AZ80A-T5	38	Steel, stainless (0.08–0.2C, 17Cr, 7Ni) ¼	78

Yield strength, S_y, is the maximum stress that can be applied without permanent deformation of the test specimen. This is the value of the stress at the elastic limit for materials for which there is an elastic limit. Because of the difficulty in determining the elastic limit, and because many materials do not have an elastic region, yield strength is often determined by the offset method as illustrated by the accompanying figure at (3). Yield strength in such a case is the stress value on the stress-strain curve corresponding to a definite amount of permanent set or strain, usually 0.1 or 0.2 per cent of the original dimension.

Ultimate strength, S_u, (also called *tensile strength*) is the maximum stress value obtained on a stress-strain curve.

Modulus of elasticity, E, (also called *Young's modulus*) is the ratio of unit stress to unit strain within the proportional limit of a material in tension or compression. Some representative values of Young's modulus (in 10^6 psi) are as follows:

Aluminum, cast, pure	9	Magnesium, AZ80A-T5	6.5
Aluminum, wrought, 2014-T6	10.6	Titanium, pure	15.5
Beryllium copper	19	Titanium, alloy, 5 Al, 2.5 Sn	17
Brass, naval	15	Steel for bridges and buildings, ASTM A7-61T, all shapes	29
Bronze, phosphor, ASTM B159	15		
Cast iron, malleable	26	Steel, castings, high strength, for structural purposes, ASTM A148-60 (seven grades)	29
Cast iron, nodular	23.5		

Modulus of elasticity in shear, G, is the ratio of unit stress to unit strain within the proportional limit of a material in shear.

Poisson's ratio, μ, is the ratio of lateral strain to longitudinal strain for a given material subjected to uniform longitudinal stresses within the proportional limit. The term is found in certain equations associated with strength of materials. Values of Poisson's ratio for common materials are as follows:

Aluminum	0.334	Nickel silver	0.322
Beryllium copper	0.285	Phosphor bronze	0.349
Brass	0.340	Rubber	0.500
Cast iron, gray	0.211	Steel, cast	0.265
Copper	0.340	high carbon	0.295
Inconel	0.290	mild	0.303
Lead	0.431	nickel	0.291
Magnesium	0.350	Wrought iron	0.278
Monel metal	0.320	Zinc	0.331

Compressive Properties.—From compression tests, *compressive yield strength*, S_{cy}, and *compressive ultimate strength*, S_{cu}, are determined. Ductile materials under compression loading merely swell or buckle without fracture, hence do not have a compressive ultimate strength.

Shear Properties.—The properties of *shear yield strength*, S_{sy}, *shear ultimate strength*, S_{su}, and the *modulus of rigidity*, G, are determined by direct shear and torsional tests. The modulus of rigidity is also known as the modulus of elasticity in shear. It is the ratio of the shear stress, τ, to the shear strain, γ, in radians, within the proportional limit: $G = \tau/\gamma$.

Fatigue Properties.—When a material is subjected to many cycles of stress reversal or fluctuation (variation in magnitude without reversal), failure may occur, even though the maximum stress at any cycle is considerably less than the value at which failure would occur if the stress were constant. Fatigue properties are determined by subjecting test specimens to stress cycles and counting the number of cycles to failure. From a series of such tests in which maximum stress values are progressively reduced, S-N diagrams can be plotted as illustrated by the accompanying figures. The S-N diagram Fig. 2a shows the behavior of a material for which there is an *endurance limit*, S_{en}. Endurance limit is the stress value at which the number of cycles to failure is infinite. Steels have endurance limits that vary according to hardness, composition, and quality; but many non-ferrous metals do not. The S-N diagram Fig. 2b does not have an endurance limit. For a metal that does not have an endurance limit, it is standard practice to specify fatigue strength as the stress value corresponding to a specific number of stress reversals, usually 100,000,000 or 500,000,000.

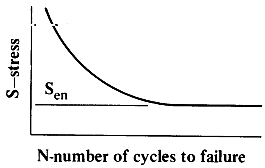


Fig. 2a. S-N endurance limit

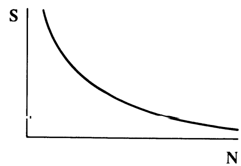


Fig. 2b. S-N no endurance limit

The Influence of Mean Stress on Fatigue.—Most published data on the fatigue properties of metals are for completely reversed alternating stresses, that is, the mean stress of the cycle is equal to zero. However, if a structure is subjected to stresses that fluctuate between different values of tension and compression, then the mean stress is not zero.

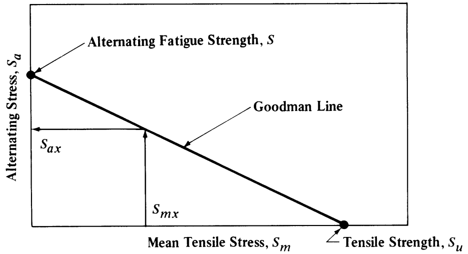
When fatigue data for a specified mean stress and design life are not available for a material, the influence of nonzero mean stress can be estimated from empirical relationships that relate failure at a given life, under zero mean stress, to failure at the same life under zero mean cyclic stress. One widely used formula is Goodman's linear relationship, which is

$$S_a = S(1 - S_m/S_u)$$

where S_a is the alternating stress associated with some nonzero mean stress, S_m. S is the alternating fatigue strength at zero mean stress. S_u is the ultimate tensile strength.

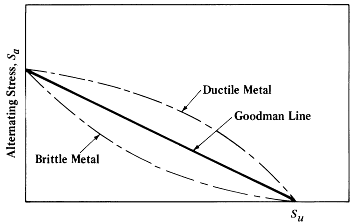
Goodman's linear relationship is usually represented graphically on a so-called *Goodman Diagram*, as shown below. The alternating fatigue strength or the alternating stress for a given number of endurance cycles is plotted on the ordinate (y-axis) and the static tensile strength is plotted on the abscissa (x-axis). The straight line joining the alternating fatigue strength, S, and the tensile strength, S_u, is the Goodman line.

The value of an alternating stress S_{ax} at a known value of mean stress S_{mx} is determined as shown by the dashed lines on the diagram.



Goodman Diagram

For ductile materials, the Goodman law is usually conservative, since approximately 90 per cent of actual test data for most ferrous and nonferrous alloys fall above the Goodman line, even at low endurance values where the yield strength is exceeded. For many brittle materials, however, actual test values can fall below the Goodman line, as illustrated below:



Mean Tensile Stress

As a rule of thumb, materials having an elongation of less than 5 per cent in a tensile test may be regarded as brittle. Those having an elongation of 5 per cent or more may be regarded as ductile.

Cumulative Fatigue Damage.—Most data are determined from tests at a constant stress amplitude. This is easy to do experimentally, and the data can be presented in a straightforward manner. In actual engineering applications, however, the alternating stress amplitude usually changes in some way during service operation. Such changes, referred to as "spectrum loading," make the direct use of standard S-N fatigue curves inappropriate. A problem exists, therefore, in predicting the fatigue life under varying stress amplitude from conventional, constant-amplitude S-N fatigue data.

The assumption in predicting spectrum loading effects is that operation at a given stress amplitude and number of cycles will produce a certain amount of permanent fatigue damage and that subsequent operation at different stress amplitude and number of cycles will produce additional fatigue damage and a sequential accumulation of total damage, which at a critical value will cause fatigue failure. Although the assumption appears simple, the amount of damage incurred at any stress amplitude and number of cycles has proven difficult to determine, and several "cumulative damage" theories have been advanced.

One of the first and simplest methods for evaluating cumulative damage is known as Miner's law or the linear damage rule, where it is assumed that n_1 cycles at a stress of S_1, for which the average number of cycles to failure is N_1, cause an amount of damage n_1/N_1. Failure is predicted to occur when

$$\Sigma n/N = 1$$

The term *n*/*N* is known as the "cycle ratio" or the damage fraction.

The greatest advantages of the Miner rule are its simplicity and prediction reliability, which approximates that of more complex theories. For these reasons the rule is widely used. It should be noted, however, that it does not account for all influences, and errors are to be expected in failure prediction ability.

Modes of Fatigue Failure.—Several modes of fatigue failure are:

Low/High-Cycle Fatigue: This fatigue process covers cyclic loading in two significantly different domains, with different physical mechanisms of failure. One domain is characterized by relatively low cyclic loads, strain cycles confined largely to the elastic range, and long lives or a high number of cycles to failure; traditionally, this has been called "high-cycle fatigue." The other domain has cyclic loads that are relatively high, significant amounts of plastic strain induced during each cycle, and short lives or a low number of cycles to failure. This domain has commonly been called "low-cycle fatigue" or cyclic strain-controlled fatigue.

The transition from low- to high-cycle fatigue behavior occurs in the range from approximately 10,000 to 100,000 cycles. Many define low-cycle fatigue as failure that occurs in 50,000 cycles or less.

Thermal Fatigue: Cyclic temperature changes in a machine part will produce cyclic stresses and strains if natural thermal expansions and contractions are either wholly or partially constrained. These cyclic strains produce fatigue failure just as though they were produced by external mechanical loading. When strain cycling is produced by a fluctuating temperature field, the failure process is termed "thermal fatigue."

While thermal fatigue and mechanical fatigue phenomena are very similar, and can be mathematically expressed by the same types of equations, the use of mechanical fatigue results to predict thermal fatigue performance must be done with care. For equal values of plastic strain range, the number of cycles to failure is usually up to 2.5 times lower for thermally cycled than for mechanically cycled samples.

Corrosion Fatigue: Corrosion fatigue is a failure mode where cyclic stresses and a corrosion-producing environment combine to initiate and propagate cracks in fewer stress cycles and at lower stress amplitudes than would be required in a more inert environment. The corrosion process forms pits and surface discontinuities that act as stress raisers to accelerate fatigue cracking. The cyclic loads may also cause cracking and flaking of the corrosion layer, baring fresh metal to the corrosive environment. Each process accelerates the other, making the cumulative result more serious.

Surface or Contact Fatigue: Surface fatigue failure is usually associated with rolling surfaces in contact, and results in pitting, cracking, and spalling of the contacting surfaces from cyclic Hertz contact stresses that cause the maximum values of cyclic shear stresses to be slightly below the surface. The cyclic subsurface shear stresses generate cracks that propagate to the contacting surface, dislodging particles in the process.

Combined Creep and Fatigue: In this failure mode, all of the conditions for both creep failure and fatigue failure exist simultaneously. Each process influences the other in producing failure, but this interaction is not well understood.

Factors of Safety.—There is always a risk that the working stress to which a member is subjected will exceed the strength of its material. The purpose of a factor of safety is to minimize this risk.

Factors of safety can be incorporated into design calculations in many ways. For most calculations the following equation is used:

$$s_w = S_m / f_s \qquad (1)$$

where f_s is the factor of safety, S_m is the strength of the material in pounds per square inch, and S_w is the allowable working stress, also in pounds per square inch. Since the factor of safety is greater than 1, the allowable working stress will be less than the strength of the material.

In general, S_m is based on yield strength for ductile materials, ultimate strength for brittle materials, and fatigue strength for parts subjected to cyclic stressing. Most strength values are obtained by testing standard specimens at 68°F. in normal atmospheres. If, however, the character of the stress or environment differs significantly from that used in obtaining standard strength data, then special data must be obtained. If special data are not available, standard data must be suitably modified.

General recommendations for values of factors of safety are given in the following list.

f_s	Application
1.3–1.5	For use with highly reliable materials where loading and environmental conditions are not severe, and where weight is an important consideration.
1.5–2	For applications using reliable materials where loading and environmental conditions are not severe.
2–2.5	For use with ordinary materials where loading and environmental conditions are not severe.
2.5–3	For less tried and for brittle materials where loading and environmental conditions are not severe.
3–4	For applications in which material properties are not reliable and where loading and environmental conditions are not severe, or where reliable materials are to be used under difficult loading and environmental conditions.

Working Stress.—Calculated working stresses are the products of calculated nominal stress values and stress concentration factors. Calculated nominal stress values are based on the assumption of idealized stress distributions. Such nominal stresses may be simple stresses, combined stresses, or cyclic stresses. Depending on the nature of the nominal stress, one of the following equations applies:

$s_w = K\sigma$ (2) $s_w = K\sigma'$ (4) $s_w = K\sigma_{cy}$ (6)

$s_w = K\tau$ (3) $s_w = K\tau'$ (5) $s_w = K\tau_{cy}$ (7)

where K is a stress concentration factor; σ and τ are, respectively, simple normal (tensile or compressive) and shear stresses; σ' and τ' are combined normal and shear stresses; σ_{cy} and τ_{cy} are cyclic normal and shear stresses.

Where there is uneven stress distribution, as illustrated in the table (on page 204) of simple stresses for Cases 3, 4 and 6, the maximum stress is the one to which the stress concentration factor is applied in computing working stresses. The location of the maximum stress in each case is discussed under the section *Simple Stresses* and the formulas for these maximum stresses are given in the *Table of Simple Stresses* on page 204.

Stress Concentration Factors.—Stress concentration is related to type of material, the nature of the stress, environmental conditions, and the geometry of parts. When stress concentration factors that specifically match all of the foregoing conditions are not available, the following equation may be used:

$$K = 1 + q(K_t - 1) \qquad (8)$$

K_t is a theoretical stress concentration factor that is a function only of the geometry of a part and the nature of the stress; q is the *index of sensitivity* of the material. If the geometry is such as to provide no theoretical stress concentration, $K_t = 1$.

Curves for evaluating K_t are on pages 201 through 204. For constant stresses in cast iron and in ductile materials, $q = 0$ (hence $K = 1$). For constant stresses in brittle materials such as hardened steel, q may be taken as 0.15; for very brittle materials such as steels that have been quenched but not drawn, q may be taken as 0.25. When stresses are suddenly applied (impact stresses) q ranges from 0.4 to 0.6 for ductile materials; for cast iron it is taken as 0.5; and, for brittle materials, 1.

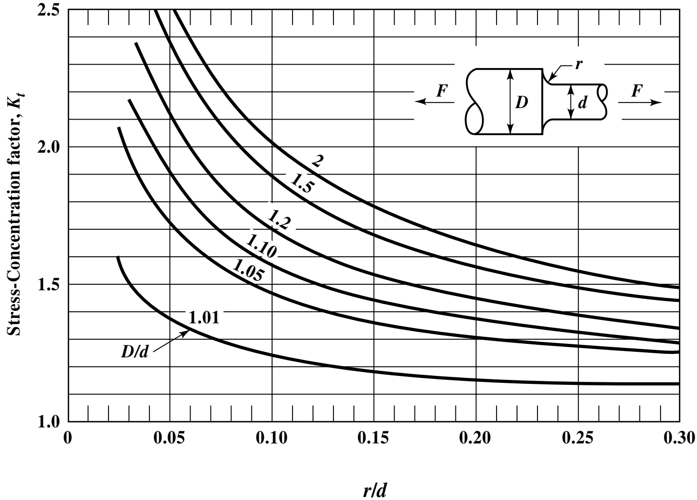


Fig. 3. Stress-concentration factor, K_t, for a filleted shaft in tension

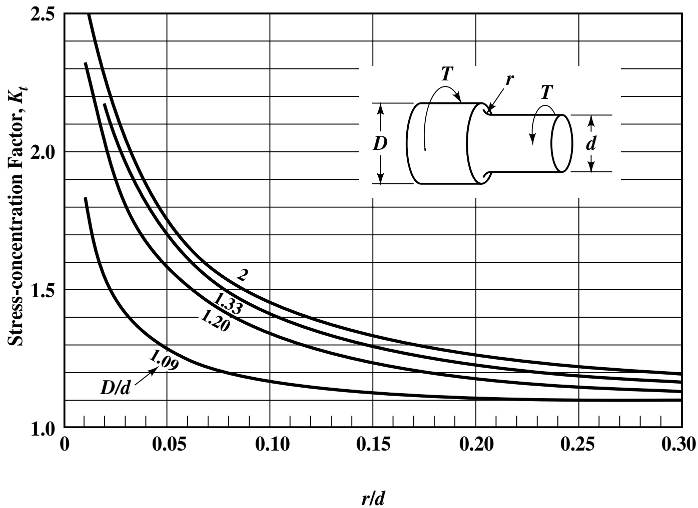


Fig. 4. Stress-concentration factor, K_t, for a filleted shaft in torsion[a]

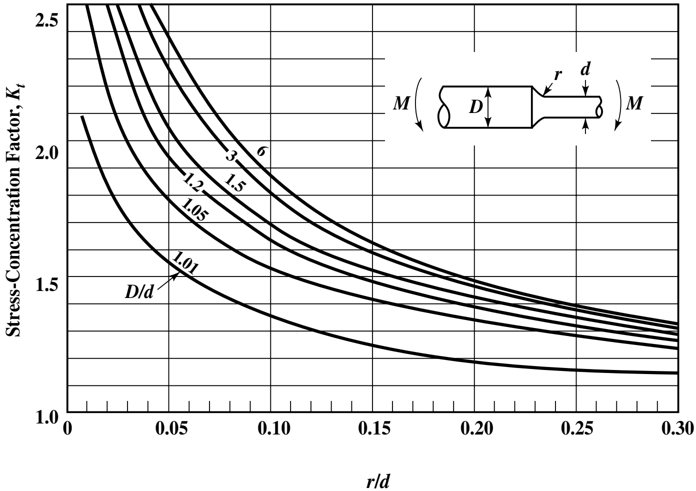


Fig. 5. Stress-concentration factor, K_t, for a shaft with shoulder fillet in bending[a]

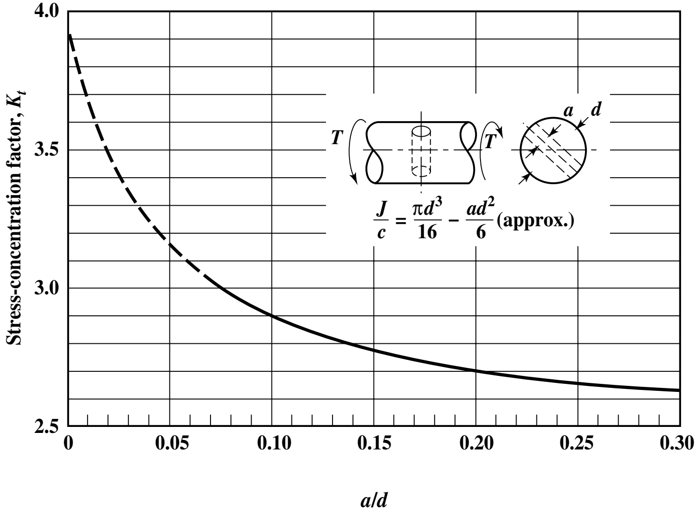


Fig. 6. Stress-concentration factor, K_t, for a shaft, with a transverse hole, in torsion[a]

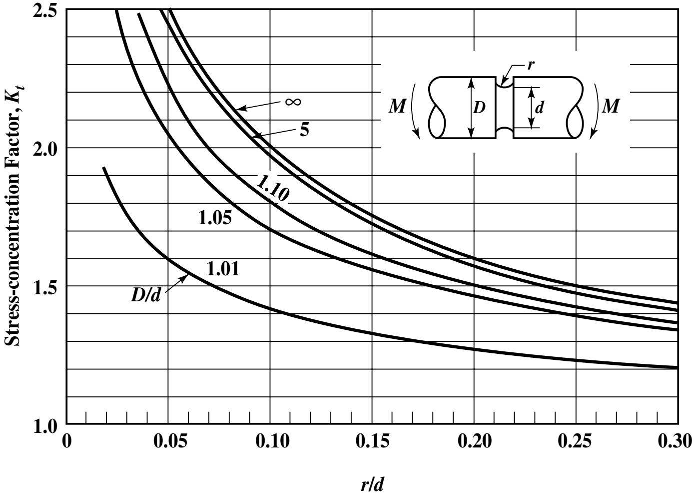


Fig. 7. Stress-concentration factor, K_t, for a grooved shaft in bending[a]

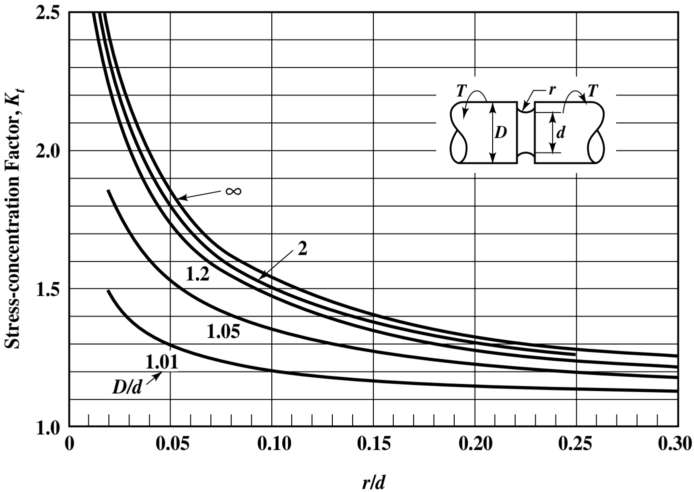


Fig. 8. Stress-concentration factor, K_t, for a grooved shaft in torsion[a]

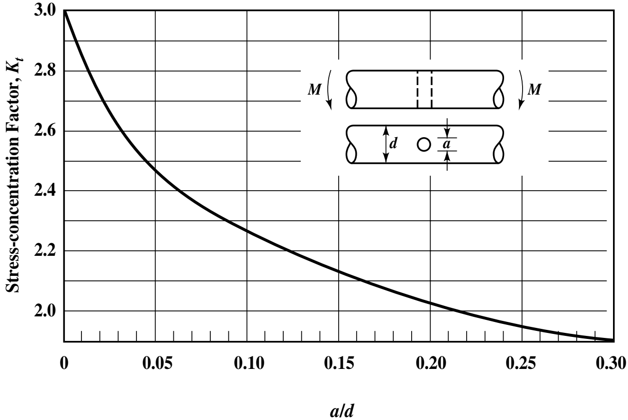


Fig. 9. Stress-concentration factor, K_t, for a shaft, with a transverse hole, in bending[a]

[a] Source: R. E. Peterson, Design Factors for Stress Concentration, *Machine Design*, vol. 23, 1951. For other stress concentration charts, see Lipson and Juvinall, *The Handbook of Stress and Strength*, The Macmillan Co., 1963.

Simple Stresses.—Simple stresses are produced by constant conditions of loading on elements that can be represented as beams, rods, or bars. The table on page 204 summarizes information pertaining to the calculation of simple stresses. Following is an explanation of the symbols used in simple stress formulae: σ = simple normal (tensile or compressive) stress in pounds per square inch; τ = simple shear stress in pounds per square inch; F = external force in pounds; V = shearing force in pounds; M = bending moment in inch-pounds; T = torsional moment in inch-pounds; A = cross-sectional area in square inches; Z = section modulus in inches^3; Z_p = polar section modulus in inches^3; I = moment of inertia in inches^4; J = polar moment of inertia in inches^4; a = area of the web of wide flange and I beams in square inches; y = perpendicular distance from axis through center of gravity of cross-sectional area to stressed fiber in inches; c = radial distance from center of gravity to stressed fiber in inches.

Table of Simple Stresses

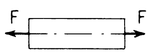
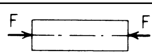
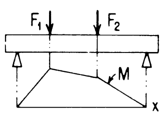
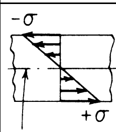
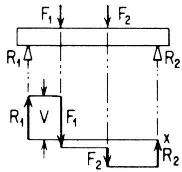
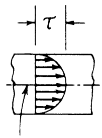
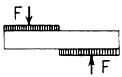
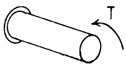
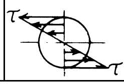
Case	Type of Loading	Illustration	Stress Distribution	Stress Equations
1	Direct tension		Uniform	$\sigma = \frac{F}{A}$ (9)
2	Direct compression		Uniform	$\sigma = -\frac{F}{A}$ (10)
3	Bending	 Bending moment diagram	 Neutral plane	$\sigma = \pm\frac{M}{Z} = \pm\frac{My}{I}$ (11)

Table of Simple Stresses *(Continued)*

Case	Type of Loading	Illustration	Stress Distribution	Stress Equations
4	Bending	 Shearing force diagram	 Neutral plane	For beams of rectangular cross-section: $\tau = \frac{3V}{2A}$ (12) For beams of solid circular cross-section: $\tau = \frac{4V}{3A}$ (13) For wide flange and I beams (approximately): $\tau = \frac{V}{a}$ (14)
5	Direct shear		Uniform	$\tau = \frac{F}{A}$ (15)
6	Torsion			$\tau = \frac{T}{Zp} = \frac{Tc}{J}$ (16)

SI metric units can be applied in the calculations in place of the English units of measurement without changes to the formulas. The SI units are the newton (N), which is the unit of force; the meter; the meter squared; the pascal (Pa) which is the newton per meter squared (N/M^2); and the newton-meter (N · m) for moment of force. Often in design work using the metric system, the millimeter is employed rather than the meter. In such instances, the dimensions can be converted to meters before the stress calculations are begun. Alternatively, the same formulas can be applied using millimeters in place of the meter, providing the treatment is consistent throughout. In such instances, stress and strength properties must be expressed in megapascals (MPa), which is the same as newtons per millimeter squared (N/mm^2), and moments in newton-millimeters ($N \cdot mm^2$). ***Note*****: $1\ N/mm^2 = 1\ N/10^{-6}m^2 = 10^6\ N/m^2$ = 1 meganewton/m^2 = 1 megapascal.**

For direct tension and direct compression loading, Cases 1 and 2 in the table on page 204, the force *F* must act along a line through the center of gravity of the section at which the stress is calculated. The equation for direct compression loading applies only to members for which the ratio of length to least radius of gyration is relatively small, approximately 20, otherwise the member must be treated as a column.

The table *Stresses and Deflections in Beams* starting on page 237 give equations for calculating stresses due to bending for common types of beams and conditions of loading. Where these tables are not applicable, stress may be calculated using Equation (11) in the table on page 204. In using this equation it is necessary to determine the value of the bending moment at the point where the stress is to be calculated. For beams of constant cross-section, stress is ordinarily calculated at the point coinciding with the maximum value of bending moment. Bending loading results in the characteristic stress distribution shown in the table for Case 3. It will be noted that the maximum stress values are at the surfaces farthest from the neutral plane. One of the surfaces is stressed in tension and the other in compression. It is for this reason that the ± sign is used in Equation (11). Numerous tables for evaluating section moduli are given in the section starting on page 217.

Shear stresses caused by bending have maximum values at neutral planes and zero values at the surfaces farthest from the neutral axis, as indicated by the stress distribution diagram shown for Case 4 in the . Values for V in Equations (12), (13) and (14) can be determined from shearing force diagrams. The shearing force diagram shown in Case 4 corresponds to the bending moment diagram for Case 3. As shown in this diagram, the value taken for V is represented by the greatest vertical distance from the x axis. The shear stress caused by direct shear loading, Case 5, has a uniform distribution. However, the shear stress caused by torsion loading, Case 6, has a zero value at the axis and a maximum value at the surface farthest from the axis.

Deflections.—For direct tension and direct compression loading on members with uniform cross sections, deflection can be calculated using Equation (17). For direct tension loading, e is an elongation; for direct compression loading, e is a contraction. Deflection is in inches when the load F is in pounds, the length L over which deflection occurs is in inches, the cross-sectional area A is in square inches, and the modulus of elasticity E is in pounds per square inch. The angular deflection of members with uniform circular cross sections subject to torsion loading can be calculated with Equation (18).

$$e = FL/AE \qquad (17) \qquad\qquad \theta = TL/GJ \qquad (18)$$

The angular deflection θ is in radians when the torsional moment T is in inch-pounds, the length L over which the member is twisted is in inches, the modulus of rigidity G is in pounds per square inch, and the polar moment of inertia J is in inches^4.

Metric SI units can be used in Equations (17) and (18), where F = force in newtons (N); L = length over which deflection or twisting occurs in meters; A = cross-sectional area in meters squared; E = the modulus of elasticity in (newtons per meter squared); θ = radians; T = the torsional moment in newton-meters (N·m); G = modulus of rigidity, in pascals; and J = the polar moment of inertia in meters^4. If the load (F) is applied as a weight, it should be noted that the weight of a mass M kilograms is Mg newtons, where g = 9.81 m/s^2. Millimeters can be used in the calculations in place of meters, providing the treatment is consistent throughout.

Combined Stresses.—A member may be loaded in such a way that a combination of simple stresses acts at a point. Three general cases occur, examples of which are shown in the accompanying illustration Fig. 10.

Superposition of stresses: Fig. 10 at (1) illustrates a common situation that results in simple stresses combining by superposition at points **a** and **b**. The equal and opposite forces F_1 will cause a compressive stress $\sigma_1 = -F_1/A$. Force F_2 will cause a bending moment M to exist in the plane of points **a** and **b**. The resulting stress $\sigma_2 = \pm M/Z$. The combined stress at point **a**,

$$\sigma_a' = -\frac{F_1}{A} - \frac{M}{Z} \qquad (19) \qquad \text{and at } \mathbf{b}, \qquad \sigma_b' = -\frac{F_1}{A} + \frac{M}{Z} \qquad (20)$$

where the minus sign indicates a compressive stress and the plus sign a tensile stress. Thus, the stress at **a** will be compressive and at **b** either tensile or compressive depending on which term in the equation for σ_b' has the greatest value.

Normal stresses at right angles: This is shown in Fig. 10 at (2). This combination of stresses occurs, for example, in tanks subjected to internal or external pressure. The principle normal stresses are $\sigma_x = F_1/A_1$, $\sigma_y = F_2/A_2$, and $\sigma_z = 0$ in this plane stress problem. Determine the values of these three stresses with their signs, order them algebraically, and then calculate the maximum shear stress:

$$\tau = (\sigma_{largest} - \sigma_{smallest})/2 \qquad (21)$$

Normal and shear stresses: The example in Fig. 10 at (3) shows a member subjected to a torsional shear stress, $\tau = T/Z_p$, and a direct compressive stress, $\sigma = -F/A$. At some point **a** on the member the principal normal stresses are calculated using the equation,

$$\sigma' = \frac{\sigma}{2} \pm \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} \tag{22}$$

The maximum shear stress is calculated by using the equation,

$$\tau' = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} \tag{23}$$

The point **a** should ordinarily be selected where stress is a maximum value. For the example shown in the figure at (3), the point **a** can be anywhere on the cylindrical surface because the combined stress has the same value anywhere on that surface.

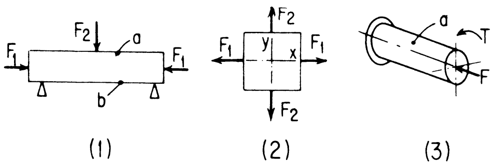


Fig. 10. Types of Combined Loading

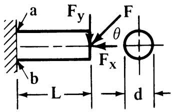
Tables of Combined Stresses.—Beginning on page 208, these tables list equations for maximum nominal tensile or compressive (normal) stresses, and maximum nominal shear stresses for common machine elements. These equations were derived using general Equations (19), (20), (22), and (23). The equations apply to the critical points indicated on the figures. Cases 1 through 4 are cantilever beams. These may be loaded with a combination of a vertical and horizontal force, or by a single oblique force. If the single oblique force F and the angle θ are given, then horizontal and vertical forces can be calculated using the equations $F_x = F \cos\theta$ and $F_y = F \sin\theta$. In cases 9 and 10 of the table, the equations for σ_a' can give a tensile and a compressive stress because of the ± sign in front of the radical. Equations involving direct compression are valid only if machine elements have relatively short lengths with respect to their sections, otherwise column equations apply.

Calculation of worst stress condition: Stress failure can occur at any critical point if either the tensile, compressive, or shear stress properties of the material are exceeded by the corresponding working stress. It is necessary to evaluate the factor of safety for each possible failure condition.

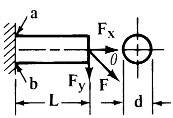
The following rules apply to calculations using equations in the , and to calculations based on Equations (19) and (20). *Rule 1:* For every calculated normal stress there is a corresponding induced shear stress; the value of the shear stress is equal to half that of the normal stress. *Rule 2:* For every calculated shear stress there is a corresponding induced normal stress; the value of the normal stress is equal to that of the shear stress. The tables of combined stresses includes equations for calculating both maximum nominal tensile or compressive stresses, and maximum nominal shear stresses.

Formulas for Combined Stresses

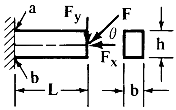
(1) Circular cantilever beam in direct compression and bending:

Type of Beam and Loading	Maximum Nominal Tens. or Comp. Stress	Maximum Nominal Shear Stress
	$\sigma_a' = \frac{1.273}{d^2}\left(\frac{8LF_y}{d} - F_x\right)$	$\tau_a' = 0.5\sigma_a'$
	$\sigma_b' = -\frac{1.273}{d^2}\left(\frac{8LF_y}{d} + F_x\right)$	$\tau_b' = 0.5\sigma_b'$

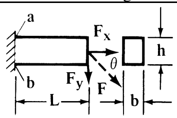
(2) Circular cantilever beam in direct tension and bending:

Type of Beam and Loading	Maximum Nominal Tens. or Comp. Stress	Maximum Nominal Shear Stress
	$\sigma_a' = \frac{1.273}{d^2}\left(F_x + \frac{8LF_y}{d}\right)$	$\tau_a' = 0.5\sigma_a'$
	$\sigma_b' = \frac{1.273}{d^2}\left(F_x - \frac{8LF_y}{d}\right)$	$\tau_b' = 0.5\sigma_b'$

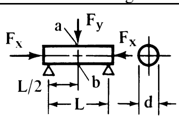
(3) Rectangular cantilever beam in direct compression and bending:

Type of Beam and Loading	Maximum Nominal Tens. or Comp. Stress	Maximum Nominal Shear Stress
	$\sigma_a' = \frac{1}{bh}\left(\frac{6LF_y}{h} - F_x\right)$	$\tau_a' = 0.5\sigma_a'$
	$\sigma_b' = -\frac{1}{bh}\left(\frac{6LF_y}{h} + F_x\right)$	$\tau_b' = 0.5\sigma_b'$

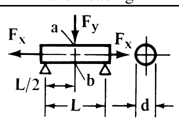
(4) Rectangular cantilever beam in direct tension and bending:

Type of Beam and Loading	Maximum Nominal Tens. or Comp. Stress	Maximum Nominal Shear Stress
	$\sigma_a' = \frac{1}{bh}\left(F_x + \frac{6LF_y}{h}\right)$	$\tau_a' = 0.5\sigma_a'$
	$\sigma_b' = \frac{1}{bh}\left(F_x - \frac{6LF_y}{h}\right)$	$\tau_b' = 0.5\sigma_b'$

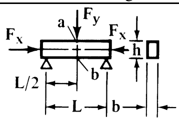
(5) Circular beam or shaft in direct compression and bending:

Type of Beam and Loading	Maximum Nominal Tens. or Comp. Stress	Maximum Nominal Shear Stress
	$\sigma_a' = -\frac{1.273}{d^2}\left(\frac{2LF_y}{d} + F_x\right)$	$\tau_a' = 0.5\sigma_a'$
	$\sigma_b' = \frac{1.273}{d^2}\left(\frac{2LF_y}{d} - F_x\right)$	$\tau_b' = 0.5\sigma_b'$

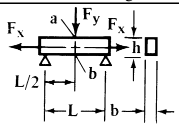
(6) Circular beam or shaft in direct tension and bending:

Type of Beam and Loading	Maximum Nominal Tens. or Comp. Stress	Maximum Nominal Shear Stress
	$\sigma_a' = \frac{1.273}{d^2}\left(F_x - \frac{2LF_y}{d}\right)$	$\tau_a' = 0.5\sigma_a'$
	$\sigma_b' = \frac{1.273}{d^2}\left(F_x + \frac{2LF_y}{d}\right)$	$\tau_b' = 0.5\sigma_b'$

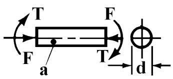
(7) Rectangular beam or shaft in direct compression and bending:

Type of Beam and Loading	Maximum Nominal Tens. or Comp. Stress	Maximum Nominal Shear Stress
	$\sigma_a' = -\frac{1}{bh}\left(\frac{3LF_y}{2h} + F_x\right)$ $\sigma_b' = \frac{1}{bh}\left(-\frac{3LF_y}{2h} - F_x\right)$	$\tau_a' = 0.5\sigma_a'$ $\tau_b' = 0.5\sigma_b'$

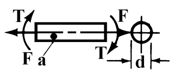
(8) Rectangular beam or shaft in direct tension and bending:

Type of Beam and Loading	Maximum Nominal Tens. or Comp. Stress	Maximum Nominal Shear Stress
	$\sigma_a' = \frac{1}{bh}\left(F_x - \frac{3LF_y}{2h}\right)$ $\sigma_b' = \frac{1}{bh}\left(F_x + \frac{3LF_y}{2h}\right)$	$\tau_a' = 0.5\sigma_a'$ $\tau_b' = 0.5\sigma_b'$

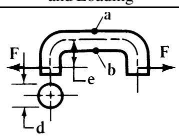
(9) Circular shaft in direct compression and torsion:

Type of Beam and Loading	Maximum Nominal Tens. or Comp. Stress	Maximum Nominal Shear Stress
 a anywhere on surface	$\sigma_a' =$ $-\frac{0.637}{d^2}\left[F \pm \sqrt{F^2 + \left(\frac{8T}{d}\right)^2}\right]$	$\tau_a' =$ $-\frac{0.637}{d^2}\sqrt{F^2 + \left(\frac{8T}{d}\right)^2}$

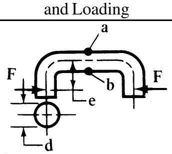
(10) Circular shaft in direct tension and torsion:

Type of Beam and Loading	Maximum Nominal Tens. or Comp. Stress	Maximum Nominal Shear Stress
 a anywhere on surface	$\sigma_a' =$ $-\frac{0.637}{d^2}\left[F \pm \sqrt{F^2 + \left(\frac{8T}{d}\right)^2}\right]$	$\tau_a' =$ $-\frac{0.637}{d^2}\sqrt{F^2 + \left(\frac{8T}{d}\right)^2}$

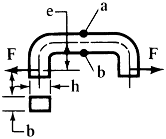
(11) Offset link, circular cross section, in direct tension:

Type of Beam and Loading	Maximum Nominal Tens. or Comp. Stress	Maximum Nominal Shear Stress
	$\sigma_a' = \frac{1.273F}{d^2}\left(1 - \frac{8e}{d}\right)$ $\sigma_b' = \frac{1.273F}{d^2}\left(1 + \frac{8e}{d}\right)$	$\tau_a' = 0.5\sigma_a'$ $\tau_b' = 0.5\sigma_b'$

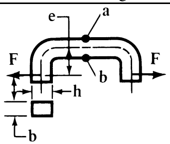
(12) Offset link, circular cross section, in direct compression:

Type of Beam and Loading	Maximum Nominal Tens. or Comp. Stress	Maximum Nominal Shear Stress
	$\sigma_a' = \frac{1.273F}{d^2}\left(\frac{8e}{d} - 1\right)$ $\sigma_b' = -\frac{1.273F}{d^2}\left(\frac{8e}{d} + 1\right)$	$\tau_a' = 0.5\sigma_a'$ $\tau_b' = 0.5\sigma_b'$

(13) Offset link, rectangular section, in direct tension:

Type of Beam and Loading	Maximum Nominal Tens. or Comp. Stress	Maximum Nominal Shear Stress
	$\sigma_a' = \frac{F}{bh}\left(1 - \frac{6e}{h}\right)$ $\sigma_b' = \frac{F}{bh}\left(1 + \frac{6e}{h}\right)$	$\tau_a' = 0.5\sigma_a'$ $\tau_b' = 0.5\sigma_b'$

(14) Offset link, rectangular section, in direct compression:

Type of Beam and Loading	Maximum Nominal Tens. or Comp. Stress	Maximum Nominal Shear Stress
	$\sigma_a' = \frac{F}{bh}\left(1 - \frac{6e}{h}\right)$ $\sigma_b' = \frac{F}{bh}\left(1 + \frac{6e}{h}\right)$	$\tau_a' = 0.5\sigma_a'$ $\tau_b' = 0.5\sigma_b'$

Formulas from the simple and combined stress tables, as well as tension and shear factors, can be applied without change in calculations using metric SI units. Stresses are given in newtons per meter squared (N/m^2) or in N/mm^2.

Three-Dimensional Stress.—Three-dimensional or triaxial stress occurs in assemblies such as a shaft press-fitted into a gear bore or in pipes and cylinders subjected to internal or external fluid pressure. Triaxial stress also occurs in two-dimensional stress problems if the loads produce normal stresses that are either both tensile or both compressive. In either case the calculated maximum shear stress, based on the corresponding two-dimensional theory, will be less than the true maximum value because of three-dimensional effects. Therefore, if the stress analysis is to be based on the maximum-shear-stress theory of failure, the triaxial stress cubic equation should first be used to calculate the three principal stresses and from these the true maximum shear stress. The following procedure provides the principal maximum normal tensile and compressive stresses and the true maximum shear stress at any point on a body subjected to any combination of loads.

The basis for the procedure is the stress cubic equation

$$S^3 - AS^2 + BS - C = 0$$

in which:

$$A = S_x + S_y + S_z$$
$$B = S_xS_y + S_yS_z + S_zS_x - S_{xy}{}^2 - S_{yz}{}^2 - S_{zx}{}^2$$
$$C = S_xS_yS_z + 2S_{xy}S_{yz}S_{zx} - S_xS_{yz}{}^2 - S_yS_{zx}{}^2 - S_zS_{xy}{}^2$$

and S_x, S_y, etc., are as shown in Fig. 1.

The coordinate system *XYZ* in Fig. 1 shows the positive directions of the normal and shear stress components on an elementary cube of material. Only six of the nine components shown are needed for the calculations: the normal stresses S_x, S_y, and S_z on three of the faces of the cube; and the three shear stresses S_{xy}, S_{yz}, and S_{zx}. The remaining three shear stresses are known because $S_{yx} = S_{xy}$, $S_{zy} = S_{yz}$, and $S_{xz} = S_{zx}$. The normal stresses S_x, S_y, and S_z are shown as positive (tensile) stresses; the opposite direction is negative (compressive). The first subscript of each shear stress identifies the coordinate axis perpendicular to the plane of the shear stress; the second subscript identifies the axis to which the stress is parallel. Thus, S_{xy}, is the shear stress in the *YZ* plane to which the *X* axis is perpendicular, and the stress is parallel to the *Y* axis.

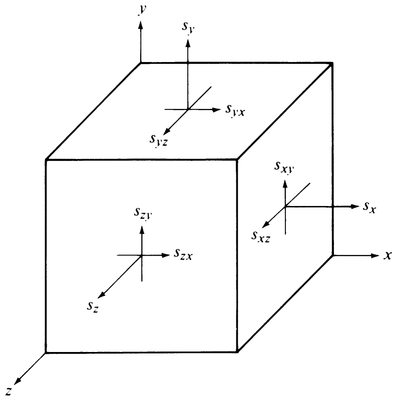


Fig. 1. *XYZ* Coordinate System Showing Positive Directions of Stresses

Step 1. Draw a diagram of the hardware to be analyzed, such as the shaft shown in Fig. 2, and show the applied loads *P*, *T*, and any others.

Step 2. For any point at which the stresses are to be analyzed, draw a coordinate diagram similar to Fig. 1 and show the magnitudes of the stresses resulting from the applied loads (these stresses may be calculated by using standard basic equations from strength of materials, and should include any stress concentration factors).

Step 3. Substitute the values of the six stresses S_x, S_y, S_z, S_{xy}, S_{yz}, and S_{zx}, including zero values, into the formulas for the quantities *A* through *K*. The quantities *I*, *J*, and *K* represent the principal normal stresses at the point analyzed. As a check, if the algebraic sum $I + J + K$ equals *A*, within rounding errors, then the calculations up to this point should be correct.

$$D = A^2/3 - B$$

$$E = A \times B/3 - C - 2 \times A^3/27$$

$$F = \sqrt{(D^3/27)}$$

$$G = \arccos(-E/(2 \times F))$$

$$H = \sqrt{(D/3)}$$

$$I = 2 \times H \times \cos(G/3) + A/3$$

$$J = 2 \times H \times [\cos(G/3 + 120°)] + A/3$$

$$K = 2 \times H \times [\cos(G/3 + 240°)] + A/3$$

Step 4. Calculate the true maximum shear stress, $S_{s(\max)}$ using the formula

$$S_{s(\max)} = 0.5 \times (S_{\text{large}} - S_{\text{small}})$$

in which S_{large} is equal to the algebraically largest of the calculated principal stresses *I*, *J*, or *K* and S_{small} is algebraically the smallest.

The maximum principal normal stresses and the maximum true shear stress calculated above may be used with any of the various theories of failure.

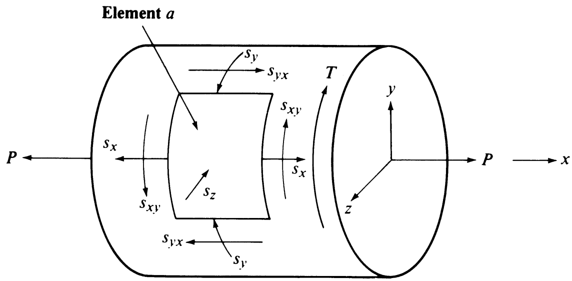


Fig. 2. Example of Triaxial Stress on an Element *a* of Shaft Surface Caused by Load *P*, Torque *T*, and 5000 psi Hydraulic Pressure

Example: A torque *T* on the shaft in Fig. 2 causes a shearing stress S_{xy} of 8000 psi in the outer fibers of the shaft; and the loads *P* at the ends of the shaft produce a tensile stress S_x of 4000 psi. The shaft passes through a hydraulic cylinder so that the shaft circumference is subjected to the hydraulic pressure of 5000 psi in the cylinder, causing compressive stresses S_y and S_z of – 5000 psi on the surface of the shaft. Find the maximum shear stress at any point *A* on the surface of the shaft.

From the statement of the problem $S_x = +4000$ psi, $S_y = -5000$ psi, $S_z = -5000$ psi, $S_{xy} = +8000$ psi, $S_{yz} = 0$ psi, and $S_{zx} = 0$ psi.

$A = 4000 - 5000 - 5000 = -6000$

$B = (4000 \times -5000) + (-5000 \times -5000) + (-5000 \times 4000) - 8000^2 - 0^2 - 0^2 = -7.9 \times 10^7$

$C = (4000 \times -5000 \times -5000) + 2 \times 8000 \times 0 \times 0 - (4000 \times 0^2) - (-5000 \times 0^2) - (-5000 \times 8000^2) = 4.2 \times 10^{11}$

$D = A^2/3 - B = 9.1 \times 10^7$

$E = A \times B/3 - C - 2 \times A^3/27 = -2.46 \times 10^{11}$

$F = \sqrt{(D^3/27)} = 1.6706 \times 10^{11}$

$G = \arccos(-E/(2 \times F)) = 42.586$ degrees, $H = \sqrt{(D/3)} = 5507.57$

$I = 2 \times H \times \cos(G/3 + A/3 = 8678.8$, say, 8680 psi

$J = 2 \times H \times [\cos(G/3 + 120°)] + A/3 = -9678.78$, say, – 9680 psi

$K = 2 \times H [\cos(G/3 + 240°)] + A/3 = -5000$ psi

Check: 8680 + (– 9680) + (– 5000) = – 6000 within rounding error.

$S_{s(\max)} = 0.5 \times (8680 - (-9680)) = 9180$ psi

Sample Calculations.—The following examples illustrate some typical strength of materials calculations, using both English and metric SI units of measurement.

Example 1(a): A round bar made from SAE 1025 low carbon steel is to support a direct tension load of 50,000 pounds. Using a factor of safety of 4, and assuming that the stress concentration factor *K* = 1, a suitable standard diameter is to be determined. Calculations are to be based on a yield strength of 40,000 psi.

Because the factor of safety and strength of the material are known, the allowable working stress s_w may be calculated using Equation (1): 40,000/4 = 10,000 psi. The relationship between working stress s_w and nominal stress σ is given by Equation (2). Since *K* = 1, σ = 10,000 psi. Applying Equation (9) in the , the area of the bar can be solved for: A = 50,000/10,000 or 5 square inches. The next largest standard diameter corresponding to this area is $2\frac{9}{16}$ inches.

Example 1(b): **A similar example to that given in 1(a), using metric SI units is as follows. A round steel bar of 300 meganewtons/meter ^{2} yield strength, is to withstand a direct tension of 200 kilonewtons. Using a safety factor of 4, and assuming that the stress concentration factor $K = 1$, a suitable diameter is to be determined.**

Because the factor of safety and the strength of the material are known, the allowable working stress s_w may be calculated using Equation (1): 300⁄4 = 75 mega-newtons/meter^2. The relationship between working stress and nominal stress σ is given by Equation (2). Since $K = 1$, σ = 75 MN/m^2. Applying Equation (9) in the , the area of the bar can be determined from:

$$A = \frac{200 \text{ kN}}{75 \text{ MN/m}^2} = \frac{200{,}000 \text{ N}}{75{,}000{,}000 \text{ N/m}^2} = 0.00267 \text{m}^2$$

The diameter corresponding to this area is 0.058 meters, or approximately 0.06 m.

Millimeters can be employed in the calculations in place of meters, providing the treatment is consistent throughout. In this instance the diameter would be 60 mm.

Note: **If the tension in the bar is produced by hanging a mass of M kilograms from its end, the value is Mg newtons, where g = approximately 9.81 meters per second^2.**

Example 2(a): What would the total elongation of the bar in Example 1(a) be if its length were 60 inches? Applying Equation (17),

$$e = \frac{50{,}000 \times 60}{5.157 \times 30{,}000{,}000} = 0.019 \text{ inch}$$

Example 2(b): **What would be the total elongation of the bar in Example 1(b) if its length were 1.5 meters? The problem is solved by applying Equation (17) in which F = 200 kilonewtons; L = 1.5 meters; $A = \pi 0.06^2/4 = 0.00283$ m^2. Assuming a modulus of elasticity E of 200 giganewtons/meter^2, then the calculation is:**

$$e = \frac{200{,}000 \times 1.5}{0.00283 \times 200{,}000{,}000{,}000} = 0.000530 \text{ m}$$

The calculation is less unwieldy if carried out using millimeters in place of meters; then F = 200 kN; L = 1500 mm; A = 2830 mm^2, and E = 200,000 N/mm^2. Thus:

$$e = \frac{200{,}000 \times 1500}{2830 \times 200{,}000} = 0.530 \text{ mm}$$

Example 3(a): Determine the size for the section of a square bar which is to be held firmly at one end and is to support a load of 3000 pounds at the outer end. The bar is to be 30 inches long and is to be made from SAE 1045 medium carbon steel with a yield point of 60,000 psi. A factor of safety of 3 and a stress concentration factor of 1.3 are to be used.

From Equation (1) the allowable working stress s_w = 60,000⁄3 = 20,000 psi. The applicable equation relating working stress and nominal stress is Equation (2); hence, σ = 20,000⁄1.3 = 15,400 psi. The member must be treated as a cantilever beam subject to a bending moment of 30 × 3000 or 90,000 inch-pounds. Solving Equation (11) in the for section modulus: Z = 90,000⁄15,400 = 5.85 inch^3. The section modulus for a square section with neutral axis equidistant from either side is $a^3/6$, where a is the dimension of the square, so $a = \sqrt[3]{35.1} = 3.27$ inches. The size of the bar can therefore be 3$\frac{5}{16}$ inches.

Example 3(b): **A similar example to that given in Example 3(a), using metric SI units is as follows. Determine the size for the section of a square bar which is to be held firmly at one end and is to support a load of 1600 kilograms at the outer end. The bar is to be 1 meter long, and is to be made from steel with a yield strength of 500 newtons/mm^2. A factor of safety of 3, and a stress concentration factor of 1.3 are to be used. The calculation can be performed using millimeters throughout.**

From Equation (1) the allowable working stress s_w = 500 N/mm²/3 = 167 N/mm². The formula relating working stress and nominal stress is Equation (2); hence σ = 167/1.3 = 128 N/mm². Since a mass of 1600 kg equals a weight of 1600 *g* newtons, where *g* = 9.81 meters/second², the force acting on the bar is 15,700 newtons. The bending moment on the bar, which must be treated as a cantilever beam, is thus 1000 mm × 15,700 N = 15,700,000 N · mm. Solving Equation (11) in the for section modulus: *Z* = *M*/σ = 15,700,000/128 = 123,000 mm³. Since the section modulus for a square section with neutral axis equidistant from either side is *a*³/6, where *a* is the dimension of the square,

$$a = \sqrt[3]{6 \times 123{,}000} = 90.4 \text{ mm}$$

Example 4(a): Find the working stress in a 2-inch diameter shaft through which a transverse hole ¼ inch in diameter has been drilled. The shaft is subject to a torsional moment of 80,000 inch-pounds and is made from hardened steel so that the index of sensitivity $q = 0.2$.

The polar section modulus is calculated using the equation shown in the stress concentration curve for a Round Shaft in Torsion with Transverse Hole, page 202.

$$\frac{J}{c} = Z_p = \frac{\pi \times 2^3}{16} - \frac{2^2}{4 \times 6} = 1.4 \text{ inches}^3$$

The nominal shear stress due to the torsion loading is computed using Equation (16) in the :

$$\tau = 80{,}000 / 1.4 = 57{,}200 \text{ psi}$$

Referring to the previously mentioned stress concentration curve on page 202, K_t is 2.82 since *d*/*D* is 0.125. The stress concentration factor may now be calculated by means of Equation (8): $K = 1 + 0.2(2.82 - 1) = 1.36$. Working stress calculated with Equation (3) is $s_w = 1.36 \times 57{,}200 = 77{,}800$ psi.

***Example 4(b):* A similar example to that given in 4(a), using metric SI units is as follows. Find the working stress in a 50 mm diameter shaft through which a transverse hole 6 mm in diameter has been drilled. The shaft is subject to a torsional moment of 8000 newton-meters, and has an index of sensitivity of *q* = 0.2. If the calculation is made in millimeters, the torsional moment is 8,000,000 N · mm.**

The polar section modulus is calculated using the equation shown in the stress concentration curve for a Round Shaft with Transverse Hole, page 202:

$$\frac{J}{c} = Z_p = \frac{\pi \times 50^3}{16} - \frac{6 \times 50^2}{6}$$

$$= 24{,}544 - 2500 = 22{,}044 \text{ mm}^3$$

The nominal shear stress due to torsion loading is computed using Equation (16) in the :

$$\tau = 8{,}000{,}000 / 22{,}000 = 363 \text{ N/mm}^2 = 363 \text{ megapascals}$$

Referring to the previously mentioned stress concentration curve on page 202, K_t is 2.85, since *a*/*d* = 6/50 = 0.12. The stress concentration factor may now be calculated by means of Equation (8): $K = 1 + 0.2(2.85 - 1) = 1.37$. From Equation (3), working stress $s_w = 1.37 \times 363 = 497$ N/mm² = 497 megapascals.

Example 5(a): For Case 3 in the *Tables of Combined Stresses*, calculate the least factor of safety for a 5052-H32 aluminum beam is 10 inches long, one inch wide, and 2 inches high. Yield strengths are 23,000 psi tension; 21,000 psi compression; 13,000 psi shear. The stress concentration factor is 1.5; F_y is 600 lbs; F_x 500 lbs.

From *Tables of Combined Stresses*, Case 3:

$$\sigma_b' = -\frac{1}{1 \times 2}\left(\frac{6 \times 10 \times 600}{2} + 500\right) = -9250 \text{ psi (in compression)}$$

The other formulas for Case 3 give $\sigma_a' = 8750$ psi (in tension); $\tau_a' + 4375$ psi, and $\tau_b' +$ 4625 psi. Using equation (4) for the nominal compressive stress of 9250 psi: $S_w = 1.5 \times 9250 = 13{,}900$ psi. From Equation (1) $f_s = 21{,}000/13{,}900 = 1.51$. Applying Equations (1), (4) and (5) in appropriate fashion to the other calculated nominal stress values for tension and shear will show that the factor of safety of 1.51, governed by the compressive stress at *b* on the beam, is minimum.

***Example 5(b):* What maximum *F* can be applied in Case 3 if the aluminum beam is 200 mm long; 20 mm wide; 40 mm high; θ = 30°; $f_s = 2$, governing for compression, *K* = 1.5, and $S_m = 144\text{N/mm}^2$ for compression.**

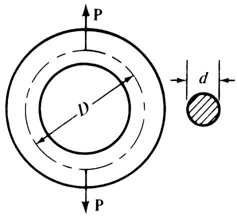
From Equation (1) $S_w = -144\text{N/mm}^2$. Therefore, from Equation (4), $\sigma_b' = -72/1.5 = -48\text{N/mm}^2$. Since $F_x = F \cos 30° = 0.866F$, and $F_y = F \sin 30° = 0.5F$:

$$-48 = -\frac{1}{20 \times 40}\left(0.866F + \frac{6 \times 200 \times 0.5F}{40}\right)$$

$$F = 2420 \text{ N}$$

Stresses and Deflections in a Loaded Ring.—For *thin* rings, that is, rings in which the dimension *d* shown in the accompanying diagram is small compared with *D*, the maximum stress in the ring is due primarily to bending moments produced by the forces *P*. The maximum stress due to bending is:

$$S = \frac{PDd}{4\pi I} \qquad (1)$$



For a ring of circular cross section where *d* is the diameter of the bar from which the ring is made,

$$S = \frac{1.621PD}{d^3} \quad \text{or} \quad P = \frac{0.617Sd^3}{D} \qquad (2)$$

The increase in the vertical diameter of the ring due to load *P* is:

$$\text{Increase in vertical diameter} = \frac{0.0186PD^3}{EI} \text{ inches} \quad (3)$$

The *decrease* in the horizontal diameter will be about 92% of the increase in the vertical diameter given by Formula (3). In the above formulas, *P* = load on ring in pounds; *D* = mean diameter of ring in inches; *S* = tensile stress in pounds per square inch, *I* = moment of inertia of section in inches^4; and *E* = modulus of elasticity of material in pounds per square inch.

Strength of Taper Pins.—The mean diameter of taper pin required to safely transmit a known torque, may be found from the formulas:

$$d = 1.13\sqrt{\frac{T}{DS}} \quad (1) \qquad \text{and} \qquad d = 283\sqrt{\frac{\text{HP}}{NDS}} \quad (2)$$

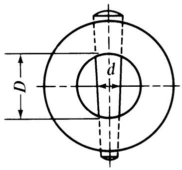
in which formulas *T* = torque in inch-pounds; *S* = safe unit stress in pounds per square inch; HP = horsepower transmitted; *N* = number of revolutions per minute; and *d* and *D* denote dimensions shown in the figure.

Formula (1) can be used with metric SI units where *d* and *D* denote dimensions shown in the figure in millimeters; *T* = torque in newton-millimeters (N · mm); and *S* = safe unit stress in newtons per millimeter^2 (N/mm^2). Formula (2) is replaced by:

$$d = 110.3\sqrt{\frac{\text{Power}}{NDS}}$$

where *d* and *D* denote dimensions shown in the figure in millimeters; *S* = safe unit stress in N/mm^2; *N* = number of revolutions per minute, and Power = power transmitted in watts.

Examples: A lever secured to a 2-inch round shaft by a steel tapered pin (dimension $d = \frac{3}{8}$ inch) has a pull of 50 pounds at a 30-inch radius from shaft center. Find *S*, the unit working stress on the pin. By rearranging Formula (1):



$$S = \frac{1.27T}{Dd^2} = \frac{1.27 \times 50 \times 30}{2 \times \left(\frac{3}{8}\right)^2} = 6770$$

pounds per square inch (nearly), which is a safe unit working stress for machine steel in shear.

Let P = 50 pounds, R = 30 inches, D = 2 inches, and S = 6000 pounds unit working stress. Using Formula (1) to find *d*:

$$d = 1.13\sqrt{\frac{T}{DS}} = 1.13\sqrt{\frac{50 \times 30}{2 \times 6000}} = 1.13\sqrt{\frac{1}{8}} = 0.4 \text{ inch}$$

A similar example using SI units is as follows: A lever secured to a 50 mm round shaft by a steel tapered pin (*d* = 10 mm) has a pull of 200 newtons at a radius of 800 mm. Find *S*, the working stress on the pin. By rearranging Formula (1):

$$S = \frac{1.27T}{Dd^2} = \frac{1.27 \times 200 \times 800}{50 \times 10^2} = 40.6 \text{ N/mm}^2 = 40.6 \text{ megapascals}$$

If a shaft of 50 mm diameter is to transmit power of 12 kilowatts at a speed of 500 rpm, find the mean diameter of the pin for a material having a safe unit stress of 40 N/mm^2. Using the formula:

$$d = 110.3\sqrt{\frac{\text{Power}}{NDS}} \qquad \text{then } d = 110.3\sqrt{\frac{12{,}000}{500 \times 50 \times 40}}$$

$$= 110.3 \times 0.1096 = 12.09 \text{ mm}$$

MOMENT OF INERTIA

Calculating Moment of Inertia

Moment of Inertia of Built-up Sections.—The usual method of calculating the moment of inertia of a built-up section involves the calculations of the moment of inertia for each element of the section about its own neutral axis, and the transferring of this moment of inertia to the previously found neutral axis of the whole built-up section. A much simpler method that can be used in the case of any section which can be divided into rectangular elements bounded by lines parallel and perpendicular to the neutral axis is the so-called tabular method based upon the formula: $I = b(h_1^3 - h^3)/3$ in which I = the moment of inertia about axis *DE*, Fig. 1, and b, h and h_1 are dimensions as given in the same illustration.

The method may be illustrated by applying it to the section shown in Fig. 2, and for simplicity of calculation shown "massed" in Fig. 3. The calculation may then be tabulated as shown in the accompanying table. The distance from the axis *DE* to the neutral axis *xx* (which will be designated as d) is found by dividing the sum of the geometrical moments by the area. The moment of inertia about the neutral axis is then found in the usual way by subtracting the area multiplied by d^2 from the moment of inertia about the axis *DE*.

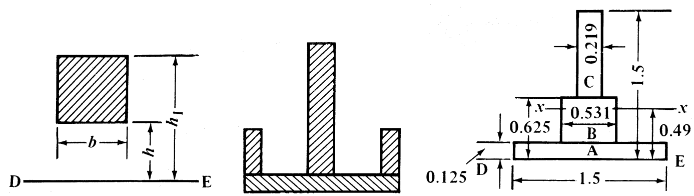


Fig. 1. Fig. 2. Fig. 3.

Tabulated Calculation of Moment of Inertia

Section	Breadth b	Height h_1	Area $b(h_1 - h)$	h_1^2	Moment $\frac{b(h_1^2 - h^2)}{2}$	h_1^3	I about axis *DE* $\frac{b(h_1^3 - h^3)}{3}$
A	1.500	0.125	0.187	0.016	0.012	0.002	0.001
B	0.531	0.625	0.266	0.391	0.100	0.244	0.043
C	0.219	1.500	0.191	2.250	0.203	3.375	0.228
			$A = 0.644$		$M = 0.315$		$I_{DE} = 0.272$

The distance d from *DE*, the axis through the base of the configuration, to the neutral axis *xx* is:

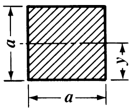
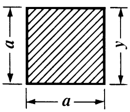
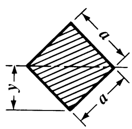
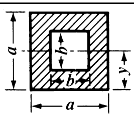
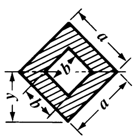
$$d = \frac{M}{A} = \frac{0.315}{0.644} = 0.49$$

The moment of inertia of the entire section with reference to the neutral axis *xx* is:

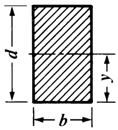
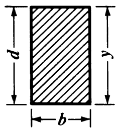
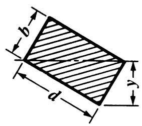
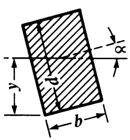
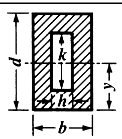
$$I_N = I_{DE} - Ad^2$$
$$= 0.272 - 0.644 \times 0.49^2$$
$$= 0.117$$

Formulas for Moments of Inertia, Section Moduli, etc.—On the following pages are given formulas for the moments of inertia and other properties of forty-two different cross-sections. The formulas give the area of the section A, and the distance y from the neutral axis to the extreme fiber, for each example. Where the formulas for the section modulus and radius of gyration are very lengthy, the formula for the section modulus, for example, has been simply given as $I \div y$. The radius of gyration is sometimes given as $\sqrt{I \div A}$ to save space.

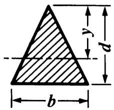
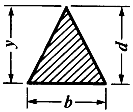
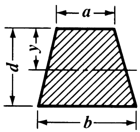
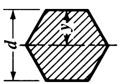
Moments of Inertia, Section Moduli, and Radii of Gyration

Section A = area y = distance from axis to extreme fiber	Moment of Inertia I	Section Modulus $Z = \frac{I}{y}$	Radius of Gyration $k = \sqrt{\frac{I}{A}}$
	Square and Rectangular Sections		
 $A = a^2 \quad y = a/2$	$\frac{a^4}{12}$	$\frac{a^3}{6}$	$\frac{a}{\sqrt{12}} = 0.289a$
 $A = a^2 \quad y = a$	$\frac{a^4}{3}$	$\frac{a^3}{3}$	$\frac{a}{\sqrt{3}} = 0.577a$
 $A = a^2$ $y = \frac{a}{\sqrt{2}} = 0.707a$	$\frac{a^4}{12}$	$\frac{a^3}{6\sqrt{2}} = 0.118a^3$	$\frac{a}{\sqrt{12}} = 0.289a$
 $A = a^2 - b^2 \quad y = a/2$	$\frac{a^4 - b^4}{12}$	$\frac{a^4 - b^4}{6a}$	$\sqrt{\frac{a^2 + b^2}{12}}$ $= 0.289\sqrt{a^2 + b^2}$
 $A = a^2 - b^2$ $y = \frac{a}{\sqrt{2}} = 0.707a$	$\frac{a^4 - b^4}{12}$	$\frac{\sqrt{2}(a^4 - b^4)}{12a}$ $= 0.118\frac{a^4 - b^4}{a}$	$\sqrt{\frac{a^2 + b^2}{12}}$ $= 0.289\sqrt{a^2 + b^2}$

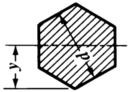
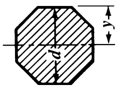
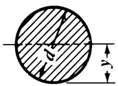
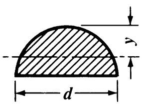
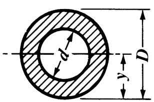
Moments of Inertia, Section Moduli, and Radii of Gyration *(Continued)*

Section A = area y = distance from axis to extreme fiber	Moment of Inertia I	Section Modulus $Z = \frac{I}{y}$	Radius of Gyration $k = \sqrt{\frac{I}{A}}$
	Square and Rectangular Sections *(Continued)*		
 $A = bd \quad y = d/2$	$\frac{bd^3}{12}$	$\frac{bd^2}{6}$	$\frac{d}{\sqrt{12}} = 0.289d$
 $A = bd \quad y = d$	$\frac{bd^3}{3}$	$\frac{bd^2}{3}$	$\frac{d}{\sqrt{3}} = 0.577d$
 $A = bd$ $y = \frac{bd}{\sqrt{b^2 + d^2}}$	$\frac{b^3 d^3}{6(b^2 + d^2)}$	$\frac{b^2 d^2}{6\sqrt{b^2 + d^2}}$	$\frac{bd}{\sqrt{6(b^2 + d^2)}}$ $= 0.408\frac{bd}{\sqrt{b^2 + d^2}}$
 $A = bd$ $y = \frac{1}{2}(d \cos\alpha + b \sin\alpha)$	$\frac{bd}{12}(d^2 \cos^2\alpha + b^2 \sin^2\alpha)$	$\frac{bd}{6} \times \left(\frac{d^2 \cos^2\alpha + b^2 \sin^2\alpha}{d \cos\alpha + b \sin\alpha}\right)$	$\sqrt{\frac{d^2 \cos^2\alpha + b^2 \sin^2\alpha}{12}}$ $= 0.289 \times \sqrt{d^2 \cos^2\alpha + b^2 \sin^2\alpha}$
 $A = bd - hk$ $y = d/2$	$\frac{bd^3 - hk^3}{12}$	$\frac{bd^3 - hk^3}{6d}$	$\sqrt{\frac{bd^3 - hk^3}{12(bd - hk)}}$ $= 0.289\sqrt{\frac{bd^3 - hk^3}{bd - hk}}$

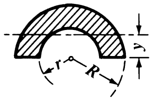
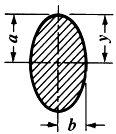
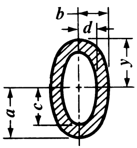
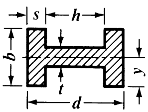
Moments of Inertia, Section Moduli, and Radii of Gyration

Section	Area of Section, A	Distance from Neutral Axis to Extreme Fiber, y	Moment of Inertia, I	Section Modulus, $Z = I/y$	Radius of Gyration, $k = \sqrt{I/A}$
Triangular Sections					
	$\frac{1}{2}bd$	$\frac{2}{3}d$	$\frac{bd^3}{36}$	$\frac{bd^2}{24}$	$\frac{d}{\sqrt{18}} = 0.236d$
	$\frac{1}{2}bd$	d	$\frac{bd^3}{12}$	$\frac{bd^2}{12}$	$\frac{d}{\sqrt{6}} = 0.408d$
Polygon Sections					
	$\frac{d(a+b)}{2}$	$\frac{d(a+2b)}{3(a+b)}$	$\frac{d^3(a^2+4ab+b^2)}{36(a+b)}$	$\frac{d^2(a^2+4ab+b^2)}{12(a+2b)}$	$\sqrt{\frac{d^2(a^2+4ab+b^2)}{18(a+b)^2}}$
	$\frac{3d^2 \tan 30°}{2} = 0.866d^2$	$\frac{d}{2}$	$\frac{A}{12}\left[\frac{d^2(1+2\cos^2 30°)}{4\cos^2 30°}\right] = 0.06d^4$	$\frac{A}{6}\left[\frac{d(1+2\cos^2 30°)}{4\cos^2 30°}\right] = 0.12d^3$	$\sqrt{\frac{d^2(1+2\cos^2 30°)}{48\cos^2 30°}} = 0.264d$

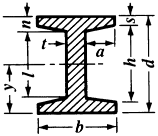
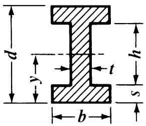
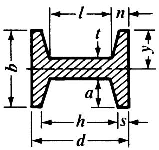
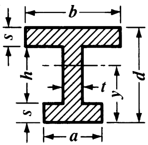
Moments of Inertia, Section Moduli, and Radii of Gyration *(Continued)*

Section	Area of Section, A	Distance from Neutral Axis to Extreme Fiber, y	Moment of Inertia, I	Section Modulus, $Z = I/y$	Radius of Gyration, $k = \sqrt{I/A}$
	$\frac{3d^2\tan 30°}{2} = 0.866d^2$	$\frac{d}{2\cos 30°} = 0.577d$	$\frac{A}{12}\left[\frac{d^2(1+2\cos^2 30°)}{4\cos^2 30°}\right] = 0.06d^4$	$\frac{A}{6.9}\left[\frac{d(1+2\cos^2 30°)}{4\cos^2 30°}\right] = 0.104d^3$	$\sqrt{\frac{d^2(1+2\cos^2 30°)}{48\cos^2 30°}} = 0.264d$
	$2d^2\tan 22\tfrac{1}{2} = 0.828d^2$	$\frac{d}{2}$	$\frac{A}{12}\left[\frac{d^2(1+2\cos^2 22\tfrac{1}{2}°)}{4\cos^2 22\tfrac{1}{2}°}\right] = 0.055d^4$	$\frac{A}{6}\left[\frac{d(1+2\cos^2 22\tfrac{1}{2}°)}{4\cos^2 22\tfrac{1}{2}°}\right] = 0.109d^3$	$\sqrt{\frac{d^2(1+2\cos^2 22\tfrac{1}{2}°)}{48\cos^2 22\tfrac{1}{2}°}} = 0.257d$
Circular, Elliptical, and Circular Arc Sections					
	$\frac{\pi d^2}{4} = 0.7854d^2$	$\frac{d}{2}$	$\frac{\pi d^4}{64} = 0.049d^4$	$\frac{\pi d^3}{32} = 0.098d^3$	$\frac{d}{4}$
	$\frac{\pi d^2}{8} = 0.393d^2$	$\frac{(3\pi - 4)d}{6\pi} = 0.288d$	$\frac{(9\pi^2 - 64)d^4}{1152\pi} = 0.007d^4$	$\frac{(9\pi^2 - 64)d^3}{192(3\pi - 4)} = 0.024d^3$	$\frac{\sqrt{(9\pi^2 - 64)d^2}}{12\pi} = 0.132d$
	$\frac{\pi(D^2 - d^2)}{4} = 0.7854(D^2 - d^2)$	$\frac{D}{2}$	$\frac{\pi(D^4 - d^4)}{64} = 0.049(D^4 - d^4)$	$\frac{\pi(D^4 - d^4)}{32D} = 0.098\frac{D^4 - d^4}{D}$	$\frac{\sqrt{D^2 + d^2}}{4}$

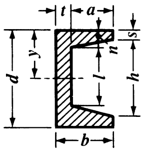
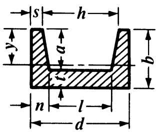
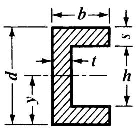
Moments of Inertia, Section Moduli, and Radii of Gyration *(Continued)*

Section	Area of Section, A	Distance from Neutral Axis to Extreme Fiber, y	Moment of Inertia, I	Section Modulus, $Z = I/y$	Radius of Gyration, $k = \sqrt{I/A}$
	$\frac{\pi(R^2 - r^2)}{2}$ $= 1.5708(R^2 - r^2)$	$\frac{4(R^3 - r^3)}{3\pi(R^2 - r^2)}$ $= 0.424\frac{R^3 - r^3}{R^2 - r^2}$	$0.1098(R^4 - r^4)$ $-\frac{0.283R^2r^2(R - r)}{R + r}$	$\frac{I}{y}$	$\sqrt{\frac{I}{A}}$
	$\pi ab = 3.1416ab$	a	$\frac{\pi a^3 b}{4} = 0.7854a^3b$	$\frac{\pi a^2 b}{4} = 0.7854a^2b$	$\frac{a}{2}$
	$\pi(ab - cd)$ $= 3.1416(ab - cd)$	a	$\frac{\pi}{4}(a^3b - c^3d)$ $= 0.7854(a^3b - c^3d)$	$\frac{\pi(a^3b - c^3d)}{4a}$ $= 0.7854\frac{a^3b - c^3d}{a}$	$\frac{1}{2}\sqrt{\frac{a^3b - c^3d}{ab - cd}}$
I-Sections					
	$bd - h(b - t)$	$\frac{b}{2}$	$\frac{2sb^3 + ht^3}{12}$	$\frac{2sb^3 + ht^3}{6b}$	$\sqrt{\frac{2sb^3 + ht^3}{12[bd - h(b - t)]}}$

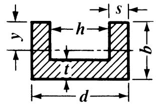
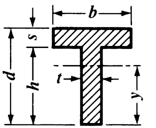
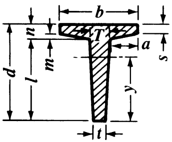
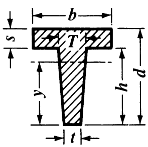
Moments of Inertia, Section Moduli, and Radii of Gyration *(Continued)*

Section	Area of Section, A	Distance from Neutral Axis to Extreme Fiber, y	Moment of Inertia, I	Section Modulus, $Z = I/y$	Radius of Gyration, $k = \sqrt{I/A}$
	$dt + 2a(s + n)$	$\frac{d}{2}$	$\frac{1}{12}\left[bd^3 - \frac{1}{4g}(h^4 - l^4)\right]$ in which g = slope of flange = $(h - l)/(b - t) = \frac{1}{6}$ for standard I-beams.	$\frac{1}{6d}\left[bd^3 - \frac{1}{4g}(h^4 - l^4)\right]$	$\sqrt{\frac{\frac{1}{12}\left[bd^3 - \frac{1}{4g}(h^4 - l^4)\right]}{dt + 2a(s + n)}}$
	$bd - h(b - t)$	$\frac{d}{2}$	$\frac{bd^3 - h^3(b - t)}{12}$	$\frac{bd^3 - h^3(b - t)}{6d}$	$\sqrt{\frac{bd^3 - h^3(b - t)}{12[bd - h(b - t)]}}$
	$dt + 2a(s + n)$	$\frac{b}{2}$	$\frac{1}{12}\left[b^3(d - h) + lt^3 + \frac{g}{4}(b^4 - t^4)\right]$ in which g = slope of flange = $(h - l)/(b - t) = \frac{1}{6}$ for standard I-beams.	$\frac{1}{6b}\left[b^3(d - h) + lt^3 + \frac{g}{4}(b^4 - t^4)\right]$	$\sqrt{\frac{I}{A}}$
	$bs + ht + as$	$d - [td^2 + s^2(b - t) + s(a - t)(2d - s)]\ 2A$	$\frac{1}{3}[b(d - y)^3 + ay^3 - (b - t)(d - y - s)^3 - (a - t)(y - s)^3]$	$\frac{I}{y}$	$\sqrt{\frac{I}{A}}$

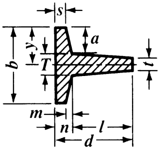
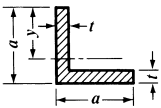
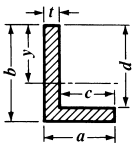
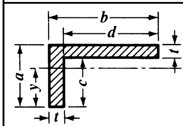
Moments of Inertia, Section Moduli, and Radii of Gyration *(Continued)*

Section	Area of Section, A	Distance from Neutral Axis to Extreme Fiber, y	Moment of Inertia, I	Section Modulus, $Z = I/y$	Radius of Gyration, $k = \sqrt{I/A}$
		C-Sections			
	$dt + a(s + n)$	$\frac{d}{2}$	$\frac{1}{12}\left[bd^3 - \frac{1}{8g}(h^4 - l^4)\right]$ g = slope of flange $= \frac{h-l}{2(b-t)} = \frac{1}{6}$ for standard channels.	$\frac{1}{6d}\left[bd^3 - \frac{1}{8g}(h^4 - l^4)\right]$	$\sqrt{\frac{\frac{1}{12}\left[bd^3 - \frac{1}{8g}(h^4 - l^4)\right]}{dt + a(s+n)}}$
	$dt + 2a(s + n)$	$b - \left[b^2 s + \frac{ht^2}{2}\right.$ $+ \frac{g}{3}(b-t)^2$ $\left.\times (b + 2t)\right] \div A$ g = slope of flange $= \frac{h-l}{2(b-t)}$	$\frac{1}{3}\left[2sb^3 + lt^3 + \frac{g}{2}(b^4 - t^4)\right]$ $-A(b-y)^2$ g = slope of flange $= \frac{h-l}{2(b-t)} = \frac{1}{6}$ for standard channels.	$\frac{I}{y}$	$\sqrt{\frac{I}{A}}$
	$bd - h(b - t)$	$\frac{d}{2}$	$\frac{bd^3 - h^3(b-t)}{12}$	$\frac{bd^3 - h^3(b-t)}{6d}$	$\sqrt{\frac{bd^3 - h^3(b-t)}{12[bd - h(b-t)]}}$

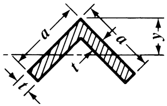
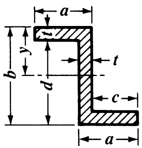
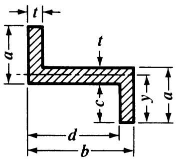
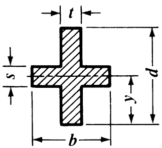
Moments of Inertia, Section Moduli, and Radii of Gyration *(Continued)*

Section	Area of Section, A	Distance from Neutral Axis to Extreme Fiber, y	Moment of Inertia, I	Section Modulus, $Z = I/y$	Radius of Gyration, $k = \sqrt{I/A}$
	$bd - h(b - t)$	$b - \dfrac{2b^2s + ht^2}{2bd - 2h(b - t)}$	$\dfrac{2sb^3 + ht^3}{3} - A(b - y)^2$	$\dfrac{I}{y}$	$\sqrt{\dfrac{I}{A}}$
T-Sections					
	$bs + ht$	$d - \dfrac{d^2t + s^2(b - t)}{2(bs + ht)}$	$\frac{1}{3}[ty^3 + b(d - y)^3 - (b - t)(d - y - s)^3]$	$\dfrac{I}{y}$	$\sqrt{\dfrac{1}{3(bs + ht)}[ty^3 + b(d - y)^3 - (b - t)(d - y - s)^3]}$
	$\dfrac{l(T + t)}{2} + Tn + a(s + n)$	$d - [3s^2(b - T) + 2am(m + 3s) + 3Td^2 - l(T - t)(3d - l)] \div 6A$	$\frac{1}{12}[l^3(T + 3t) + 4bn^3 - 2am^3] - A(d - y - n)^2$	$\dfrac{I}{y}$	$\sqrt{\dfrac{I}{A}}$
	$bs + \dfrac{h(T + t)}{2}$	$d - [3bs^2 + 3ht(d + s) + h(T - t)(h + 3s)]\ 6A$	$\frac{1}{12}[4bs^3 + h^3(3t + T)] - A(d - y - s)^2$	$\dfrac{I}{y}$	$\sqrt{\dfrac{I}{A}}$

Moments of Inertia, Section Moduli, and Radii of Gyration *(Continued)*

Section	Area of Section, A	Distance from Neutral Axis to Extreme Fiber, y	Moment of Inertia, I	Section Modulus, $Z = I/y$	Radius of Gyration, $k = \sqrt{I/A}$
	$\frac{l(T+t)}{2} + Tn + a(s+n)$	$\frac{b}{2}$	$\frac{sb^3 + mT^3 + lt^3}{12} + \frac{am[2a^2 + (2a+3T)^2]}{36} + \frac{l(T-t)[(T-t)^2 + 2(T+2t)^2]}{144}$	$\frac{I}{y}$	$\sqrt{\frac{I}{A}}$
L-, Z-, and X-Sections					
	$t(2a - t)$	$a - \frac{a^2 + at - t^2}{2(2a - t)}$	$\frac{1}{3}[ty^3 + a(a - y)^3 - (a - t)(a - y - t)^3]$	$\frac{I}{y}$	$\sqrt{\frac{I}{A}}$
	$t(a + b - t)$	$b - \frac{t(2d + a) + d^2}{2(d + a)}$	$\frac{1}{3}[ty^3 + a(b - y)^3 - (a - t)(b - y - t)^3]$	$\frac{I}{y}$	$\sqrt{\frac{1}{3t(a+b-t)}[ty^3 + a(b-y)^3 - (a-t)(b-y-t)^3]}$
	$t(a + b - t)$	$a - \frac{t(2c + b) + c^2}{2(c + b)}$	$\frac{1}{3}[ty^3 + b(a - y)^3 - (b - t)(a - y - t)^3]$	$\frac{I}{y}$	$\sqrt{\frac{1}{3t(a+b-t)}[ty^3 + b(a-y)^3 - (b-t)(a-y-t)^3]}$

Moments of Inertia, Section Moduli, and Radii of Gyration *(Continued)*

Section	Area of Section, A	Distance from Neutral Axis to Extreme Fiber, y	Moment of Inertia, I	Section Modulus, $Z = I/y$	Radius of Gyration, $k = \sqrt{I/A}$
	$t(2a - t)$	$\frac{a^2 + at - t^2}{2(2a - t)\cos 45°}$	$\frac{A}{12}[7(a^2 + b^2) - 12y^2]$ $-2ab^2(a - b)$ in which $b = (a - t)$	$\frac{I}{y}$	$\sqrt{\frac{I}{A}}$
	$t[b + 2(a - t)]$	$\frac{b}{2}$	$\frac{ab^3 - c(b - 2t)^3}{12}$	$\frac{ab^3 - c(b - 2t)^3}{6b}$	$\sqrt{\frac{ab^3 - c(b - 2t)^3}{12t[b + 2(a - t)]}}$
	$t[b + 2(a - t)]$	$\frac{2a - t}{2}$	$\frac{b(a + c)^3 - 2c^3d - 6a^2cd}{12}$	$\frac{b(a + c)^3 - 2c^3d - 6a^2cd}{6(2a - t)}$	$\sqrt{\frac{b(a + c)^3 - 2c^3d - 6a^2cd}{12t[b + 2(a - t)]}}$
	$dt + s(b - t)$	$\frac{d}{2}$	$\frac{td^3 + s^3(b - t)}{12}$	$\frac{td^3 - s^3(b - t)}{6d}$	$\sqrt{\frac{td^3 + s^3(b - t)}{12[td + s(b - t)]}}$

Tabulated Moments of Inertia and Section Moduli for Rectangles and Round Shafts

Moments of Inertia and Section Moduli for Rectangles (Metric Units)

Moments of inertia and section modulus values shown here are for rectangles 1 millimeter wide. To obtain moment of inertia or section modulus for rectangle of given side length, multiply appropriate table value by given width. (See the text starting on page 217 for basic formulas.)

Length of Side (mm)	Moment of Inertia	Section Modulus	Length of Side (mm)	Moment of Inertia	Section Modulus	Length of Side (mm)	Moment of Inertia	Section Modulus
5	10.4167	4.16667	56	14634.7	522.667	107	102087	1908.17
6	18.0000	6.00000	57	15432.8	541.500	108	104976	1944.00
7	28.5833	8.16667	58	16259.3	560.667	109	107919	1980.17
8	42.6667	10.6667	59	17114.9	580.167	110	110917	2016.67
9	60.7500	13.5000	60	18000.0	600.000	111	113969	2053.50
10	83.3333	16.6667	61	18915.1	620.167	112	117077	2090.67
11	110.917	20.1667	62	19860.7	640.667	113	120241	2128.17
12	144.000	24.0000	63	20837.3	661.500	114	123462	2166.00
13	183.083	28.1667	64	21845.3	682.667	115	126740	2204.17
14	228.667	32.6667	65	22885.4	704.167	116	130075	2242.67
15	281.250	37.5000	66	23958.0	726.000	117	133468	2281.50
16	341.333	42.6667	67	25063.6	748.167	118	136919	2320.67
17	409.417	48.1667	68	26202.7	770.667	119	140430	2360.17
18	486.000	54.0000	69	27375.8	793.500	120	144000	2400.00
19	571.583	60.1667	70	28583.3	816.667	121	147630	2440.17
20	666.667	66.6667	71	29825.9	840.167	122	151321	2480.67
21	771.750	73.5000	72	31104.0	864.000	123	155072	2521.50
22	887.333	80.6667	73	32418.1	888.167	124	158885	2562.67
23	1013.92	88.1667	74	33768.7	912.667	125	162760	2604.17
24	1152.00	96.0000	75	35156.3	937.500	126	166698	2646.00
25	1302.08	104.1667	76	36581.3	962.667	127	170699	2688.17
26	1464.67	112.6667	77	38044.4	988.167	128	174763	2730.67
27	1640.25	121.5000	78	39546.0	1014.00	130	183083	2816.67
28	1829.33	130.6667	79	41086.6	1040.17	132	191664	2904.00
29	2032.42	140.167	80	42666.7	1066.67	135	205031	3037.50
30	2250.00	150.000	81	44286.8	1093.50	138	219006	3174.00
31	2482.58	160.167	82	45947.3	1120.67	140	228667	3266.67
32	2730.67	170.667	83	47648.9	1148.17	143	243684	3408.17
33	2994.75	181.500	84	49392.0	1176.00	147	264710	3601.50
34	3275.33	192.667	85	51177.1	1204.17	150	281250	3750.00
35	3572.92	204.167	86	53004.7	1232.67	155	310323	4004.17
36	3888.00	216.000	87	54875.3	1261.50	160	341333	4266.67
37	4221.08	228.167	88	56789.3	1290.67	165	374344	4537.50
38	4572.67	240.667	89	58747.4	1320.17	170	409417	4816.67
39	4943.25	253.500	90	60750.0	1350.00	175	446615	5104.17
40	5333.33	266.667	91	62797.6	1380.17	180	486000	5400.00
41	5743.42	280.167	92	64890.7	1410.67	185	527635	5704.17
42	6174.00	294.000	93	67029.8	1441.50	190	571583	6016.67
43	6625.58	308.167	94	69215.3	1472.67	195	617906	6337.50
44	7098.67	322.667	95	71447.9	1504.17	200	666667	6666.67
45	7593.75	337.500	96	73728.0	1536.00	210	771750	7350.00
46	8111.33	352.667	97	76056.1	1568.17	220	887333	8066.67
47	8651.92	368.167	98	78432.7	1600.67	230	1013917	8816.67
48	9216.00	384.000	99	80858.3	1633.50	240	1152000	9600.00
49	9804.08	400.167	100	83333.3	1666.67	250	1302083	10416.7
50	10416.7	416.667	101	85858.4	1700.17	260	1464667	11266.7
51	11054.3	433.500	102	88434.0	1734.00	270	1640250	12150.0
52	11717.3	450.667	103	91060.6	1768.17	280	1829333	13066.7
53	12406.4	468.167	104	93738.7	1802.67	290	2032417	14016.7
54	13122.0	486.000	105	96468.8	1837.50	300	2250000	15000.0
55	13864.6	504.167	106	99251.3	1872.67	…	…	…

Section Moduli for Rectangles

Length of Side	Section Modulus	Length of Side	Section Modulus	Length of Side	Section Modulus	Length of Side	Section Modulus
1⁄8	0.0026	2 3⁄4	1.26	12	24.00	25	104.2
3⁄16	0.0059	3	1.50	12 1⁄2	26.04	26	112.7
1⁄4	0.0104	3 1⁄4	1.76	13	28.17	27	121.5
5⁄16	0.0163	3 1⁄2	2.04	13 1⁄2	30.38	28	130.7
3⁄8	0.0234	3 3⁄4	2.34	14	32.67	29	140.2
7⁄16	0.032	4	2.67	14 1⁄2	35.04	30	150.0
1⁄2	0.042	4 1⁄2	3.38	15	37.5	32	170.7
5⁄8	0.065	5	4.17	15 1⁄2	40.0	34	192.7
3⁄4	0.094	5 1⁄2	5.04	16	42.7	36	216.0
7⁄8	0.128	6	6.00	16 1⁄2	45.4	38	240.7
1	0.167	6 1⁄2	7.04	17	48.2	40	266.7
1 1⁄8	0.211	7	8.17	17 1⁄2	51.0	42	294.0
1 1⁄4	0.260	7 1⁄2	9.38	18	54.0	44	322.7
1 3⁄8	0.315	8	10.67	18 1⁄2	57.0	46	352.7
1 1⁄2	0.375	8 1⁄2	12.04	19	60.2	48	384.0
1 5⁄8	0.440	9	13.50	19 1⁄2	63.4	50	416.7
1 3⁄4	0.510	9 1⁄2	15.04	20	66.7	52	450.7
1 7⁄8	0.586	10	16.67	21	73.5	54	486.0
2	0.67	10 1⁄2	18.38	22	80.7	56	522.7
2 1⁄4	0.84	11	20.17	23	88.2	58	560.7
2 1⁄2	1.04	11 1⁄2	22.04	24	96.0	60	600.0

Section modulus values are shown for rectangles 1 inch wide. To obtain section modulus for rectangle of given side length, multiply value in table by given width.

Section Moduli and Moments of Inertia for Round Shafts

Dia.	Section Modulus	Moment of Inertia	Dia.	Section Modulus	Moment of Inertia	Dia.	Section Modulus	Moment of Inertia
1⁄8	0.00019	0.00001	27⁄64	0.00737	0.00155	23⁄32	0.03645	0.01310
9⁄64	0.00027	0.00002	7⁄16	0.00822	0.00180	47⁄64	0.03888	0.01428
5⁄32	0.00037	0.00003	29⁄64	0.00913	0.00207	3⁄4	0.04142	0.01553
11⁄64	0.00050	0.00004	15⁄32	0.01011	0.00237	49⁄64	0.04406	0.01687
3⁄16	0.00065	0.00006	31⁄64	0.01116	0.00270	25⁄32	0.04681	0.01829
13⁄64	0.00082	0.00008	1⁄2	0.01227	0.00307	51⁄64	0.04968	0.01979
7⁄32	0.00103	0.00011	33⁄64	0.01346	0.00347	13⁄16	0.05266	0.02139
15⁄64	0.00126	0.00015	17⁄32	0.01472	0.00391	53⁄64	0.05576	0.02309
1⁄4	0.00153	0.00019	35⁄64	0.01606	0.00439	27⁄32	0.05897	0.02488
17⁄64	0.00184	0.00024	9⁄16	0.01747	0.00491	55⁄64	0.06231	0.02677
9⁄32	0.00218	0.00031	37⁄64	0.01897	0.00548	7⁄8	0.06577	0.02877
19⁄64	0.00257	0.00038	19⁄32	0.02055	0.00610	57⁄64	0.06936	0.03089
5⁄16	0.00300	0.00047	39⁄64	0.02222	0.00677	29⁄32	0.07307	0.03311
21⁄64	0.00347	0.00057	5⁄8	0.02397	0.00749	59⁄64	0.07692	0.03545
11⁄32	0.00399	0.00069	41⁄64	0.02581	0.00827	15⁄16	0.08089	0.03792
23⁄64	0.00456	0.00082	21⁄32	0.02775	0.00910	61⁄64	0.08501	0.04051
3⁄8	0.00518	0.00097	43⁄64	0.02978	0.01000	31⁄32	0.08926	0.04323
25⁄64	0.00585	0.00114	11⁄16	0.03190	0.01097	63⁄64	0.09364	0.04609
13⁄32	0.00658	0.00134	45⁄64	0.03413	0.01200	…	…	…

In this and succeeding tables, the *Polar Section Modulus* for a shaft of given diameter can be obtained by multiplying its section modulus by 2. Similarly, its *Polar Moment of Inertia* can be obtained by multiplying its moment of inertia by 2.

Section Moduli and Moments of Inertia for Round Shafts (English or Metric Units)

Dia.	Section Modulus	Moment of Inertia	Dia.	Section Modulus	Moment of Inertia	Dia.	Section Modulus	Moment of Inertia
1.00	0.0982	0.0491	1.50	0.3313	0.2485	2.00	0.7854	0.7854
1.01	0.1011	0.0511	1.51	0.3380	0.2552	2.01	0.7972	0.8012
1.02	0.1042	0.0531	1.52	0.3448	0.2620	2.02	0.8092	0.8173
1.03	0.1073	0.0552	1.53	0.3516	0.2690	2.03	0.8213	0.8336
1.04	0.1104	0.0574	1.54	0.3586	0.2761	2.04	0.8335	0.8501
1.05	0.1136	0.0597	1.55	0.3656	0.2833	2.05	0.8458	0.8669
1.06	0.1169	0.0620	1.56	0.3727	0.2907	2.06	0.8582	0.8840
1.07	0.1203	0.0643	1.57	0.3799	0.2982	2.07	0.8708	0.9013
1.08	0.1237	0.0668	1.58	0.3872	0.3059	2.08	0.8835	0.9188
1.09	0.1271	0.0693	1.59	0.3946	0.3137	2.09	0.8963	0.9366
1.10	0.1307	0.0719	1.60	0.4021	0.3217	2.10	0.9092	0.9547
1.11	0.1343	0.0745	1.61	0.4097	0.3298	2.11	0.9222	0.9730
1.12	0.1379	0.0772	1.62	0.4174	0.3381	2.12	0.9354	0.9915
1.13	0.1417	0.0800	1.63	0.4252	0.3465	2.13	0.9487	1.0104
1.14	0.1455	0.0829	1.64	0.4330	0.3551	2.14	0.9621	1.0295
1.15	0.1493	0.0859	1.65	0.4410	0.3638	2.15	0.9757	1.0489
1.16	0.1532	0.0889	1.66	0.4491	0.3727	2.16	0.9894	1.0685
1.17	0.1572	0.0920	1.67	0.4572	0.3818	2.17	1.0032	1.0885
1.18	0.1613	0.0952	1.68	0.4655	0.3910	2.18	1.0171	1.1087
1.19	0.1654	0.0984	1.69	0.4739	0.4004	2.19	1.0312	1.1291
1.20	0.1696	0.1018	1.70	0.4823	0.4100	2.20	1.0454	1.1499
1.21	0.1739	0.1052	1.71	0.4909	0.4197	2.21	1.0597	1.1710
1.22	0.1783	0.1087	1.72	0.4996	0.4296	2.22	1.0741	1.1923
1.23	0.1827	0.1124	1.73	0.5083	0.4397	2.23	1.0887	1.2139
1.24	0.1872	0.1161	1.74	0.5172	0.4500	2.24	1.1034	1.2358
1.25	0.1917	0.1198	1.75	0.5262	0.4604	2.25	1.1183	1.2581
1.26	0.1964	0.1237	1.76	0.5352	0.4710	2.26	1.1332	1.2806
1.27	0.2011	0.1277	1.77	0.5444	0.4818	2.27	1.1484	1.3034
1.28	0.2059	0.1318	1.78	0.5537	0.4928	2.28	1.1636	1.3265
1.29	0.2108	0.1359	1.79	0.5631	0.5039	2.29	1.1790	1.3499
1.30	0.2157	0.1402	1.80	0.5726	0.5153	2.30	1.1945	1.3737
1.31	0.2207	0.1446	1.81	0.5822	0.5268	2.31	1.2101	1.3977
1.32	0.2258	0.1490	1.82	0.5919	0.5386	2.32	1.2259	1.4221
1.33	0.2310	0.1536	1.83	0.6017	0.5505	2.33	1.2418	1.4468
1.34	0.2362	0.1583	1.84	0.6116	0.5627	2.34	1.2579	1.4717
1.35	0.2415	0.1630	1.85	0.6216	0.5750	2.35	1.2741	1.4971
1.36	0.2470	0.1679	1.86	0.6317	0.5875	2.36	1.2904	1.5227
1.37	0.2524	0.1729	1.87	0.6420	0.6003	2.37	1.3069	1.5487
1.38	0.2580	0.1780	1.88	0.6523	0.6132	2.38	1.3235	1.5750
1.39	0.2637	0.1832	1.89	0.6628	0.6264	2.39	1.3403	1.6016
1.40	0.2694	0.1886	1.90	0.6734	0.6397	2.40	1.3572	1.6286
1.41	0.2752	0.1940	1.91	0.6841	0.6533	2.41	1.3742	1.6559
1.42	0.2811	0.1996	1.92	0.6949	0.6671	2.42	1.3914	1.6836
1.43	0.2871	0.2053	1.93	0.7058	0.6811	2.43	1.4087	1.7116
1.44	0.2931	0.2111	1.94	0.7168	0.6953	2.44	1.4262	1.7399
1.45	0.2993	0.2170	1.95	0.7280	0.7098	2.45	1.4438	1.7686
1.46	0.3055	0.2230	1.96	0.7392	0.7244	2.46	1.4615	1.7977
1.47	0.3119	0.2292	1.97	0.7506	0.7393	2.47	1.4794	1.8271
1.48	0.3183	0.2355	1.98	0.7621	0.7545	2.48	1.4975	1.8568
1.49	0.3248	0.2419	1.99	0.7737	0.7698	2.49	1.5156	1.8870

Section Moduli and Moments of Inertia for Round Shafts (English or Metric Units)

Dia.	Section Modulus	Moment of Inertia	Dia.	Section Modulus	Moment of Inertia	Dia.	Section Modulus	Moment of Inertia
2.50	1.5340	1.9175	3.00	2.6507	3.9761	3.50	4.2092	7.3662
2.51	1.5525	1.9483	3.01	2.6773	4.0294	3.51	4.2454	7.4507
2.52	1.5711	1.9796	3.02	2.7041	4.0832	3.52	4.2818	7.5360
2.53	1.5899	2.0112	3.03	2.7310	4.1375	3.53	4.3184	7.6220
2.54	1.6088	2.0432	3.04	2.7582	4.1924	3.54	4.3552	7.7087
2.55	1.6279	2.0755	3.05	2.7855	4.2479	3.55	4.3922	7.7962
2.56	1.6471	2.1083	3.06	2.8130	4.3038	3.56	4.4295	7.8844
2.57	1.6665	2.1414	3.07	2.8406	4.3604	3.57	4.4669	7.9734
2.58	1.6860	2.1749	3.08	2.8685	4.4175	3.58	4.5054	8.0631
2.59	1.7057	2.2089	3.09	2.8965	4.4751	3.59	4.5424	8.1536
2.60	1.7255	2.2432	3.10	2.9247	4.5333	3.60	4.5804	8.2248
2.61	1.7455	2.2779	3.11	2.9531	4.5921	3.61	4.6187	8.3368
2.62	1.7656	2.3130	3.12	2.9817	4.6514	3.62	4.6572	8.4295
2.63	1.7859	2.3485	3.13	3.0105	4.7114	3.63	4.6959	8.5231
2.64	1.8064	2.3844	3.14	3.0394	4.7719	3.64	4.7348	8.6174
2.65	1.8270	2.4208	3.15	3.0685	4.8329	3.65	4.7740	8.7125
2.66	1.8478	2.4575	3.16	3.0979	4.8946	3.66	4.8133	8.8083
2.67	1.8687	2.4947	3.17	3.1274	4.9569	3.67	4.8529	8.9050
2.68	1.8897	2.5323	3.18	3.1570	5.0197	3.68	4.8926	9.0025
2.69	1.9110	2.5703	3.19	3.1869	5.0831	3.69	4.9326	9.1007
2.70	1.9324	2.6087	3.20	3.2170	5.1472	3.70	4.9728	9.1998
2.71	1.9539	2.6476	3.21	3.2472	5.2118	3.71	5.0133	9.2996
2.72	1.9756	2.6869	3.22	3.2777	5.2771	3.72	5.0539	9.4003
2.73	1.9975	2.7266	3.23	3.3083	5.3429	3.73	5.0948	9.5018
2.74	2.0195	2.7668	3.24	3.3391	5.4094	3.74	5.1359	9.6041
2.75	2.0417	2.8074	3.25	3.3702	5.4765	3.75	5.1772	9.7072
2.76	2.0641	2.8484	3.26	3.4014	5.5442	3.76	5.2187	9.8112
2.77	2.0866	2.8899	3.27	3.4328	5.6126	3.77	5.2605	9.9160
2.78	2.1093	2.9319	3.28	3.4643	5.6815	3.78	5.3024	10.0216
2.79	2.1321	2.9743	3.29	3.4961	5.7511	3.79	5.3446	10.1281
2.80	2.1551	3.0172	3.30	3.5281	5.8214	3.80	5.3870	10.2354
2.81	2.1783	3.0605	3.31	3.5603	5.8923	3.81	5.4297	10.3436
2.82	2.2016	3.1043	3.32	3.5926	5.9638	3.82	5.4726	10.4526
2.83	2.2251	3.1486	3.33	3.6252	6.0360	3.83	5.5156	10.5625
2.84	2.2488	3.1933	3.34	3.6580	6.1088	3.84	5.5590	10.6732
2.85	2.2727	3.2385	3.35	3.6909	6.1823	3.85	5.6025	10.7848
2.86	2.2967	3.2842	3.36	3.7241	6.2564	3.86	5.6463	10.8973
2.87	2.3208	3.3304	3.37	3.7574	6.3313	3.87	5.6903	11.0107
2.88	2.3452	3.3771	3.38	3.7910	6.4067	3.88	5.7345	11.1249
2.89	2.3697	3.4242	3.39	3.8247	6.4829	3.89	5.7789	11.2401
2.90	2.3944	3.4719	3.40	3.8587	6.5597	3.90	5.8236	11.3561
2.91	2.4192	3.5200	3.41	3.8928	6.6372	3.91	5.8685	11.4730
2.92	2.4443	3.5686	3.42	3.9272	6.7154	3.92	5.9137	11.5908
2.93	2.4695	3.6178	3.43	3.9617	6.7943	3.93	5.9591	11.7095
2.94	2.4948	3.6674	3.44	3.9965	6.8739	3.94	6.0047	11.8292
2.95	2.5204	3.7176	3.45	4.0314	6.9542	3.95	6.0505	11.9497
2.96	2.5461	3.7682	3.46	4.0666	7.0352	3.96	6.0966	12.0712
2.97	2.5720	3.8194	3.47	4.1019	7.1168	3.97	6.1429	12.1936
2.98	2.5981	3.8711	3.48	4.1375	7.1992	3.98	6.1894	12.3169
2.99	2.6243	3.9233	3.49	4.1733	7.2824	3.99	6.2362	12.4412

Section Moduli and Moments of Inertia for Round Shafts (English or Metric Units)

Dia.	Section Modulus	Moment of Inertia	Dia.	Section Modulus	Moment of Inertia	Dia.	Section Modulus	Moment of Inertia
4.00	6.2832	12.566	4.50	8.946	20.129	5.00	12.272	30.680
4.01	6.3304	12.693	4.51	9.006	20.308	5.01	12.346	30.926
4.02	6.3779	12.820	4.52	9.066	20.489	5.02	12.420	31.173
4.03	6.4256	12.948	4.53	9.126	20.671	5.03	12.494	31.423
4.04	6.4736	13.077	4.54	9.187	20.854	5.04	12.569	31.673
4.05	6.5218	13.207	4.55	9.248	21.039	5.05	12.644	31.925
4.06	6.5702	13.337	4.56	9.309	21.224	5.06	12.719	32.179
4.07	6.6189	13.469	4.57	9.370	21.411	5.07	12.795	32.434
4.08	6.6678	13.602	4.58	9.432	21.599	5.08	12.870	32.691
4.09	6.7169	13.736	4.59	9.494	21.788	5.09	12.947	32.949
4.10	6.7663	13.871	4.60	9.556	21.979	5.10	13.023	33.209
4.11	6.8159	14.007	4.61	9.618	22.170	5.11	13.100	33.470
4.12	6.8658	14.144	4.62	9.681	22.363	5.12	13.177	33.733
4.13	6.9159	14.281	4.63	9.744	22.558	5.13	13.254	33.997
4.14	6.9663	14.420	4.64	9.807	22.753	5.14	13.332	34.263
4.15	7.0169	14.560	4.65	9.871	22.950	5.15	13.410	34.530
4.16	7.0677	14.701	4.66	9.935	23.148	5.16	13.488	34.799
4.17	7.1188	14.843	4.67	9.999	23.347	5.17	13.567	35.070
4.18	7.1702	14.986	4.68	10.063	23.548	5.18	13.645	35.342
4.19	7.2217	15.130	4.69	10.128	23.750	5.19	13.725	35.616
4.20	7.2736	15.275	4.70	10.193	23.953	5.20	13.804	35.891
4.21	7.3257	15.420	4.71	10.258	24.158	5.21	13.884	36.168
4.22	7.3780	15.568	4.72	10.323	24.363	5.22	13.964	36.446
4.23	7.4306	15.716	4.73	10.389	24.571	5.23	14.044	36.726
4.24	7.4834	15.865	4.74	10.455	24.779	5.24	14.125	37.008
4.25	7.5364	16.015	4.75	10.522	24.989	5.25	14.206	37.291
4.26	7.5898	16.166	4.76	10.588	25.200	5.26	14.288	37.576
4.27	7.6433	16.319	4.77	10.655	25.412	5.27	14.369	37.863
4.28	7.6972	16.472	4.78	10.722	25.626	5.28	14.451	38.151
4.29	7.7513	16.626	4.79	10.790	25.841	5.29	14.533	38.441
4.30	7.8056	16.782	4.80	10.857	26.058	5.30	14.616	38.732
4.31	7.8602	16.939	4.81	10.925	26.275	5.31	14.699	39.025
4.32	7.9150	17.096	4.82	10.994	26.495	5.32	14.782	39.320
4.33	7.9701	17.255	4.83	11.062	26.715	5.33	14.866	39.617
4.34	8.0254	17.415	4.84	11.131	26.937	5.34	14.949	39.915
4.35	8.0810	17.576	4.85	11.200	27.160	5.35	15.034	40.215
4.36	8.1369	17.738	4.86	11.270	27.385	5.36	15.118	40.516
4.37	8.1930	17.902	4.87	11.339	27.611	5.37	15.203	40.819
4.38	8.2494	18.066	4.88	11.409	27.839	5.38	15.288	41.124
4.39	8.3060	18.232	4.89	11.480	28.068	5.39	15.373	41.431
4.40	8.3629	18.398	4.90	11.550	28.298	5.40	15.459	41.739
4.41	8.4201	18.566	4.91	11.621	28.530	5.41	15.545	42.049
4.42	8.4775	18.735	4.92	11.692	28.763	5.42	15.631	42.361
4.43	8.5351	18.905	4.93	11.764	28.997	5.43	15.718	42.675
4.44	8.5931	19.077	4.94	11.835	29.233	5.44	15.805	42.990
4.45	8.6513	19.249	4.95	11.907	29.471	5.45	15.892	43.307
4.46	8.7097	19.423	4.96	11.980	29.710	5.46	15.980	43.626
4.47	8.7684	19.597	4.97	12.052	29.950	5.47	16.068	43.946
4.48	8.8274	19.773	4.98	12.125	30.192	5.48	16.156	44.268
4.49	8.8867	19.951	4.99	12.198	30.435	5.49	16.245	44.592

Section Moduli and Moments of Inertia for Round Shafts (English or Metric Units)

Dia.	Section Modulus	Moment of Inertia	Dia.	Section Modulus	Moment of Inertia	Dia.	Section Modulus	Moment of Inertia
5.5	16.3338	44.9180	30	2650.72	39760.8	54.5	15892.4	433068
6	21.2058	63.6173	30.5	2785.48	42478.5	55	16333.8	449180
6.5	26.9612	87.6241	31	2924.72	45333.2	55.5	16783.4	465738
7	33.6739	117.859	31.5	3068.54	48329.5	56	17241.1	482750
7.5	41.4175	155.316	32	3216.99	51471.9	56.5	17707.0	500223
8	50.2655	201.062	32.5	3370.16	54765.0	57	18181.3	518166
8.5	60.2916	256.239	33	3528.11	58213.8	57.5	18663.9	536588
9	71.5694	322.062	33.5	3690.92	61822.9	58	19155.1	555497
9.5	84.1726	399.820	34	3858.66	65597.2	58.5	19654.7	574901
10	98.1748	490.874	34.5	4031.41	69541.9	59	20163.0	594810
10.5	113.650	596.660	35	4209.24	73661.8	59.5	20680.0	615230
11	130.671	718.688	35.5	4392.23	77962.1	60	21205.8	636173
11.5	149.312	858.541	36	4580.44	82448.0	60.5	21740.3	657645
12	169.646	1017.88	36.5	4773.96	87124.7	61	22283.8	679656
12.5	191.748	1198.42	37	4972.85	91997.7	61.5	22836.3	702215
13	215.690	1401.98	37.5	5177.19	97072.2	62	23397.8	725332
13.5	241.547	1630.44	38	5387.05	102354	62.5	23968.4	749014
14	269.392	1885.74	38.5	5602.50	107848	63	24548.3	773272
14.5	299.298	2169.91	39	5823.63	113561	63.5	25137.4	798114
15	331.340	2485.05	39.5	6050.50	119497	64	25735.9	823550
15.5	365.591	2833.33	40	6283.19	125664	64.5	26343.8	849589
16	402.124	3216.99	40.5	6521.76	132066	65	26961.2	876241
16.5	441.013	3638.36	41	6766.30	138709	65.5	27588.2	903514
17	482.333	4099.83	41.5	7016.88	145600	66	28224.9	931420
17.5	526.155	4603.86	42	7273.57	152745	66.5	28871.2	959967
18	572.555	5153.00	42.5	7536.45	160150	67	29527.3	989166
18.5	621.606	5749.85	43	7805.58	167820	67.5	30193.3	1019025
19	673.381	6397.12	43.5	8081.05	175763	68	30869.3	1049556
19.5	727.954	7097.55	44	8362.92	183984	68.5	31555.2	1080767
20	785.398	7853.98	44.5	8651.27	192491	69	32251.3	1112670
20.5	845.788	8669.33	45	8946.18	201289	69.5	32957.5	1145273
21	909.197	9546.56	45.5	9247.71	210385	70	33673.9	1178588
21.5	975.698	10488.8	46	9555.94	219787	70.5	34400.7	1212625
22	1045.36	11499.0	46.5	9870.95	229499	71	35137.8	1247393
22.5	1118.27	12580.6	47	10192.8	239531	71.5	35885.4	1282904
23	1194.49	13736.7	47.5	10521.6	249887	72	36643.5	1319167
23.5	1274.10	14970.7	48	10857.3	260576	72.5	37412.3	1356194
24	1357.17	16286.0	48.5	11200.2	271604	73	38191.7	1393995
24.5	1443.77	17686.2	49	11550.2	282979	73.5	38981.8	1432581
25	1533.98	19174.8	49.5	11907.4	294707	74	39782.8	1471963
25.5	1627.87	20755.4	50	12271.8	306796	74.5	40594.6	1512150
26	1725.52	22431.8	50.5	12643.7	319253	75	41417.5	1553156
26.5	1827.00	24207.7	51	13023.0	332086	75.5	42251.4	1594989
27	1932.37	26087.0	51.5	13409.8	345302	76	43096.4	1637662
27.5	2041.73	28073.8	52	13804.2	358908	76.5	43952.6	1681186
28	2155.13	30171.9	52.5	14206.2	372913	77	44820.0	1725571
28.5	2272.66	32385.4	53	14616.0	387323	77.5	45698.8	1770829
29	2394.38	34718.6	53.5	15033.5	402147	78	46589.0	1816972
29.5	2520.38	37175.6	54	15459.0	417393	78.5	47490.7	1864011

Section Moduli and Moments of Inertia for Round Shafts (English or Metric Units)

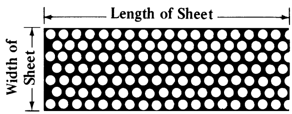
Dia.	Section Modulus	Moment of Inertia	Dia.	Section Modulus	Moment of Inertia	Dia.	Section Modulus	Moment of Inertia
79	48404.0	1911958	103.5	108848	5632890	128	205887	13176795
79.5	49328.9	1960823	104	110433	5742530	128.5	208310	13383892
80	50265.5	2010619	104.5	112034	5853762	129	210751	13593420
80.5	51213.9	2061358	105	113650	5966602	129.5	213211	13805399
81	52174.1	2113051	105.5	115281	6081066	130	215690	14019848
81.5	53146.3	2165710	106	116928	6197169	130.5	218188	14236786
82	54130.4	2219347	106.5	118590	6314927	131	220706	14456231
82.5	55126.7	2273975	107	120268	6434355	131.5	223243	14678204
83	56135.1	2329605	107.5	121962	6555469	132	225799	14902723
83.5	57155.7	2386249	108	123672	6678285	132.5	228374	15129808
84	58188.6	2443920	108.5	125398	6802818	133	230970	15359478
84.5	59233.9	2502631	109	127139	6929085	133.5	233584	15591754
85	60291.6	2562392	109.5	128897	7057102	134	236219	15826653
85.5	61361.8	2623218	110	130671	7186884	134.5	238873	16064198
86	62444.7	2685120	110.5	132461	7318448	135	241547	16304406
86.5	63540.1	2748111	111	134267	7451811	135.5	244241	16547298
87	64648.4	2812205	111.5	136089	7586987	136	246954	16792893
87.5	65769.4	2877412	112	137928	7723995	136.5	249688	17041213
88	66903.4	2943748	112.5	139784	7862850	137	252442	17292276
88.5	68050.2	3011223	113	141656	8003569	137.5	255216	17546104
89	69210.2	3079853	113.5	143545	8146168	138	258010	17802715
89.5	70383.2	3149648	114	145450	8290664	138.5	260825	18062131
90	71569.4	3220623	114.5	147372	8437074	139	263660	18324372
90.5	72768.9	3292791	115	149312	8585414	139.5	266516	18589458
91	73981.7	3366166	115.5	151268	8735703	140	269392	18857410
91.5	75207.9	3440759	116	153241	8887955	140.5	272288	19128248
92	76447.5	3516586	116.5	155231	9042189	141	275206	19401993
92.5	77700.7	3593659	117	157238	9198422	141.5	278144	19678666
93	78967.6	3671992	117.5	159262	9356671	142	281103	19958288
93.5	80248.1	3751598	118	161304	9516953	142.5	284083	20240878
94	81542.4	3832492	118.5	163363	9679286	143	287083	20526460
94.5	82850.5	3914688	119	165440	9843686	143.5	290105	20815052
95	84172.6	3998198	119.5	167534	10010172	144	293148	21106677
95.5	85508.6	4083038	120	169646	10178760	144.5	296213	21401356
96	86858.8	4169220	120.5	171775	10349469	145	299298	21699109
96.5	88223.0	4256760	121	173923	10522317	145.5	302405	21999959
97	89601.5	4345671	121.5	176088	10697321	146	305533	22303926
97.5	90994.2	4435968	122	178270	10874498	146.5	308683	22611033
98	92401.3	4527664	122.5	180471	11053867	147	311854	22921300
98.5	93822.8	4620775	123	182690	11235447	147.5	315047	23234749
99	95258.9	4715315	123.5	184927	11419254	148	318262	23551402
99.5	96709.5	4811298	124	187182	11605307	148.5	321499	23871280
100	98174.8	4908739	124.5	189456	11793625	149	324757	24194406
100.5	99654.8	5007652	125	191748	11984225	149.5	328037	24520802
101	101150	5108053	125.5	194058	12177126	150	331340	24850489
101.5	102659	5209956	126	196386	12372347	…	…	…
102	104184	5313376	126.5	198734	12569905	…	…	…
102.5	105723	5418329	127	201100	12769820	…	…	…
103	107278	5524828	127.5	203484	12972110	…	…	…

Strength and Stiffness of Perforated Metals.— It is common practice to use perforated metals in equipment enclosures to provide cooling by the flow of air or fluids. If the perforated material is to serve also as a structural member, then calculations of stiffness and strength must be made that take into account the effect of the perforations on the strength of the panels.

The accompanying table provides equivalent or effective values of the yield strength S^*; modulus of elasticity E^*; and Poisson's ratio v^* of perforated metals in terms of the values for solid material. The S^*/S and E^*/E ratios, given in the accompanying table for the standard round hole staggered pattern, can be used to determine the safety margins or deflections for perforated metal use as compared to the unperforated metal for any geometry or loading condition.

Perforated material has different strengths depending on the direction of loading; therefore, values of S^*/S in the table are given for the width (strongest) and length (weakest) directions. Also, the effective elastic constants are for plane stress conditions and apply to the in-plane loading of thin perforated sheets; the bending stiffness is greater. However, since most loading conditions involve a combination of bending and stretching, it is more convenient to use the same effective elastic constants for these combined loading conditions. The plane stress effective elastic constants given in the table can be conservatively used for all loading conditions.

Mechanical Properties of Materials Perforated with Round Holes in IPA Standard Staggered Hole Pattern



IPA No.	Perforation Diam. (in.)	Center Distance (in.)	Open Area (%)	S^*/S Width (in.)	S^*/S Length (in.)	E^*/E	v^*
100	0.020	(625)	20	0.530	0.465	0.565	0.32
106	1/16	1/8	23	0.500	0.435	0.529	0.33
107	5/64	7/64	46	0.286	0.225	0.246	0.38
108	5/64	1/8	36	0.375	0.310	0.362	0.35
109	3/32	5/32	32	0.400	0.334	0.395	0.34
110	3/32	3/16	23	0.500	0.435	0.529	0.33
112	1/10	5/32	36	0.360	0.296	0.342	0.35
113	1/8	3/16	40	0.333	0.270	0.310	0.36
114	1/8	7/32	29	0.428	0.363	0.436	0.33
115	1/8	1/4	23	0.500	0.435	0.529	0.33
116	5/32	7/32	46	0.288	0.225	0.249	0.38
117	5/32	1/4	36	0.375	0.310	0.362	0.35
118	3/16	1/4	51	0.250	0.192	0.205	0.42
119	3/16	5/16	33	0.400	0.334	0.395	0.34
120	1/4	5/16	58	0.200	0.147	0.146	0.47
121	1/4	3/8	40	0.333	0.270	0.310	0.36
122	1/4	7/16	30	0.428	0.363	0.436	0.33
123	1/4	1/2	23	0.500	0.435	0.529	0.33
124	3/8	1/2	51	0.250	0.192	0.205	0.42
125	3/8	9/16	40	0.333	0.270	0.310	0.36
126	3/8	5/8	33	0.400	0.334	0.395	0.34
127	7/16	5/8	45	0.300	0.239	0.265	0.38
128	1/2	11/16	47	0.273	0.214	0.230	0.39
129	9/16	3/4	51	0.250	0.192	0.205	0.42

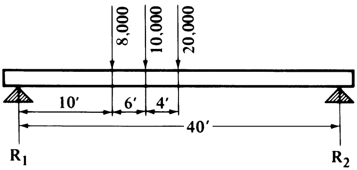
Value in parentheses specifies holes per square inch instead of center distance. S^*/S = ratio of yield strength of perforated to unperforated material; E^*/E = ratio of modulus of elasticity of perforated to unperforated material; v^* = Poisson's ratio for given percentage of open area.

IPA is Industrial Perforators Association.

BEAMS

Beam Calculations

Reaction at the Supports.—When a beam is loaded by vertical loads or forces, the sum of the reactions at the supports equals the sum of the loads. In a simple beam, when the loads are symmetrically placed with reference to the supports, or when the load is uniformly distributed, the reaction at each end will equal one-half of the sum of the loads. When the loads are not symmetrically placed, the reaction at each support may be ascertained from the fact that the algebraic sum of the moments must equal zero. In the accompanying illustration, if moments are taken about the support to the left, then: $R_2 \times 40 - 8000 \times 10 - 10{,}000 \times 16 - 20{,}000 \times 20 = 0$; $R_2 = 16{,}000$ pounds. In the same way, moments taken about the support at the right give $R_1 = 22{,}000$ pounds.



The sum of the reactions equals 38,000 pounds, which is also the sum of the loads. If part of the load is uniformly distributed over the beam, this part is first equally divided between the two supports, or the uniform load may be considered as concentrated at its center of gravity.

If metric SI units are used for the calculations, distances may be expressed in meters or millimeters, providing the treatment is consistent, and loads in newtons. ***Note:*** **If the load is given in kilograms, the value referred to is the mass. A mass of *M* kilograms has a weight (applies a force) of *Mg* newtons, where *g* = approximately 9.81 meters per second².**

Stresses and Deflections in Beams.—On the following pages are given an extensive table of formulas for stresses and deflections in beams, shafts, etc. It is assumed that all the dimensions are in inches, all loads in pounds, and all stresses in pounds per square inch. **The formulas are also valid using metric SI units, with all dimensions in millimeters, all loads in newtons, and stresses and moduli in newtons per millimeter² (N/mm²).** ***Note:*** **A load due to the weight of a mass of *M* kilograms is *Mg* newtons, where *g* = approximately 9.81 meters per second².** In the tables:

E = modulus of elasticity of the material

I = moment of inertia of the cross-section of the beam

Z = section modulus of the cross-section of the beam = I ÷ distance from neutral axis to extreme fiber

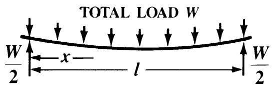
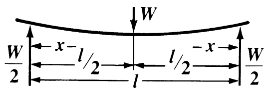
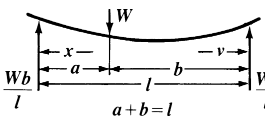
W = load on beam

s = stress in extreme fiber, or maximum stress in the cross-section considered, due to load W. A positive value of s denotes tension in the upper fibers and compression in the lower ones (as in a cantilever). A negative value of s denotes the reverse (as in a beam supported at the ends). The greatest safe load is that value of W which causes a maximum stress equal to, but not exceeding, the greatest safe value of s

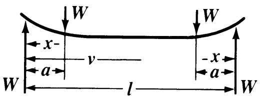
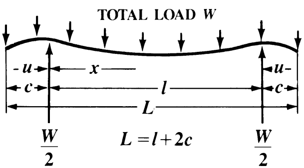
y = deflection measured from the position occupied if the load causing the deflection were removed. A positive value of y denotes deflection below this position; a negative value, deflection upward

u, v, w, x = variable distances along the beam from a given support to any point

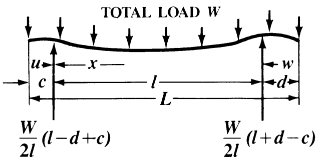
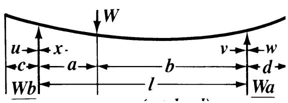
Stresses and Deflections in Beams

Type of Beam	Stresses: General Formula for Stress at any Point	Stresses: Stresses at Critical Points	Deflections: General Formula for Deflection at any Point[a]	Deflections: Deflections at Critical Points[a]
Case 1. — Supported at Both Ends, Uniform Load				
	$s = -\frac{W}{2Zl}x(l-x)$	Stress at center, $-\frac{Wl}{8Z}$ If cross-section is constant, this is the maximum stress.	$y = \frac{Wx(l-x)}{24EIl}[l^2 + x(l-x)]$	Maximum deflection, at center, $\frac{5}{384}\frac{Wl^3}{EI}$
Case 2. — Supported at Both Ends, Load at Center				
	Between each support and load, $s = -\frac{Wx}{2Z}$	Stress at center, $-\frac{Wl}{4Z}$ If cross-section is constant, this is the maximum stress.	Between each support and load, $y = \frac{Wx}{48EI}(3l^2 - 4x^2)$	Maximum deflection, at load, $\frac{Wl^3}{48EI}$
Case 3. — Supported at Both Ends, Load at any Point				
	For segment of length a, $s = -\frac{Wbx}{Zl}$ For segment of length b, $s = -\frac{Wav}{Zl}$	Stress at load, $-\frac{Wab}{Zl}$ If cross-section is constant, this is the maximum stress.	For segment of length a, $y = \frac{Wbx}{6EIl}(l^2 - x^2 - b^2)$ For segment of length b, $y = \frac{Wav}{6EIl}(l^2 - v^2 - a^2)$	Deflection at load, $\frac{Wa^2b^2}{3EIl}$ Let a be the length of the shorter segment and b of the longer one. The maximum deflection $\frac{Wav_1^3}{3EIl}$ is in the longer segment, at $v = b\sqrt{\frac{1}{3} + \frac{2a}{3b}} = v_1$

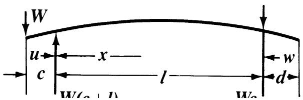
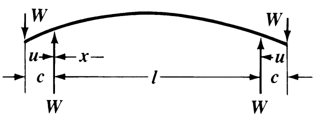
Stresses and Deflections in Beams *(Continued)*

Type of Beam	Stresses: General Formula for Stress at any Point	Stresses: Stresses at Critical Points	Deflections: General Formula for Deflection at any Point[a]	Deflections: Deflections at Critical Points[a]
Case 4. — Supported at Both Ends, Two Symmetrical Loads				
	Between each support and adjacent load, $s = -\frac{Wx}{Z}$ Between loads, $s = -\frac{Wa}{Z}$	Stress at each load, and at all points between, $-\frac{Wa}{Z}$	Between each support and adjacent load, $y = \frac{Wx}{6EI}[3a(l-a) - x^2]$ Between loads, $y = \frac{Wa}{6EI}[3v(l-v) - a^2]$	Maximum deflection at center, $\frac{Wa}{24EI}(3l^2 - 4a^2)$ Deflection at loads $\frac{Wa^2}{6EI}(3l - 4a)$
Case 5. — Both Ends Overhanging Supports Symmetrically, Uniform Load				
	Between each support and adjacent end, $s = \frac{W}{2Zl}(c-u)^2$ Between supports, $s = \frac{W}{2ZL}(c^2 - x(l-x))$	Stress at each support, $\frac{Wc^2}{2ZL}$ Stress at center, $\frac{W}{2ZL}(c^2 - \frac{1}{4}l^2)$ If cross-section is constant, the greater of these is the maximum stress. If l is greater than $2c$, the stress is zero at points $\sqrt{\frac{1}{4}l^2 - c^2}$ on both sides of the center. If cross-section is constant and if $l = 2.828c$, the stresses at supports and center are equal and opposite, and are $\pm\frac{WL}{46.62Z}$	Between each support and adjacent end, $y = \frac{Wu}{24EIL}[6c^2(l+u) - u^2(4c-u) - l^3]$ Between supports, $y = \frac{Wx(l-x)}{24EIL}[x(l-x) + l^2 - 6c^2]$	Deflection at ends, $\frac{Wc}{24EIL}[3c^2(c+2l) - l^3]$ Deflection at center, $\frac{Wl^2}{384EIL}(5l^2 - 24c^2)$ If l is between $2c$ and $2.449c$, there are maximum upward deflections at points $\sqrt{3(\frac{1}{4}l^2 - c^2)}$ on both sides of the center, which are, $-\frac{W}{96EIL}(6c^2 - l^2)^2$

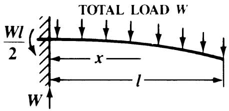
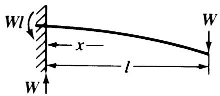
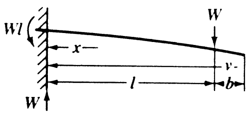
Stresses and Deflections in Beams *(Continued)*

Type of Beam	Stresses		Deflections	
	General Formula for Stress at any Point	Stresses at Critical Points	General Formula for Deflection at any Point[a]	Deflections at Critical Points[a]
Case 6. — Both Ends Overhanging Supports Unsymmetrically, Uniform Load				
 	For overhanging end of length c, $s = \frac{W}{2ZL}(c-u)^2$ Between supports, $s = \frac{W}{2ZL}\left\{c^2\left(\frac{l-x}{l}\right) + d^2\frac{x}{l} - x(l-x)\right\}$ For overhanging end of length d, $s = \frac{W}{2ZL}(d-w)^2$	Stress at support next end of length c, $\frac{Wc^2}{2ZL}$ Critical stress between supports is at $x = \frac{l^2+c^2-d^2}{2l} = x_1$ and is $\frac{W}{2ZL}(c^2 - x_1^2)$ Stress at support next end of length d, $\frac{Wd^2}{2ZL}$ If cross-section is constant, the greatest of these three is the maximum stress. If $x_1 > c$, the stress is zero at points $\sqrt{x_1^2 - c^2}$ on both sides of $x = x_1$.	For overhanging end of length c, $y = \frac{Wu}{24EIL}[2l(d^2+2c^2) + 6c^2u - u^2(4c-u) - l^3]$ Between supports, $y = \frac{Wx(l-x)}{24EIL}\{x(l-x) + l^2 - 2(d^2+c^2) - \frac{2}{l}[d^2x + c^2(l-x)]\}$ For overhanging end of length d, $y = \frac{Ww}{24EIL}[2l(c^2+2d^2) + 6d^2w - w^2(4d-w) - l^3]$	Deflection at end c, $\frac{Wc}{24EIL}[2l(d^2+2c^2) + 3c^3 - l^3]$ Deflection at end d, $\frac{Wd}{24EIL}[2l(c^2+2d^2) + 3d^3 - l^3]$ This case is so complicated that convenient general expressions for the critical deflections between supports cannot be obtained.
Case 7. — Both Ends Overhanging Supports, Load at any Point Between				
 	Between supports: For segment of length a, $s = -\frac{Wbx}{Zl}$ For segment of length b, $s = -\frac{Wav}{Zl}$ Beyond supports $s = 0$.	Stress at load, $-\frac{Wab}{Zl}$ If cross-section is constant, this is the maximum stress.	Between supports, same as Case 3. For overhanging end of length c, $y = -\frac{Wabu}{6EIl}(l+b)$ For overhanging end of length d, $y = -\frac{Wabw}{6EIl}(l+a)$	Between supports, same as Case 3. Deflection at end c, $-\frac{Wabc}{6EIl}(l+b)$ Deflection at end d, $-\frac{Wabd}{6EIl}(l+a)$

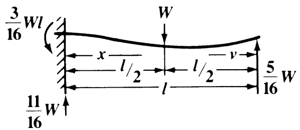
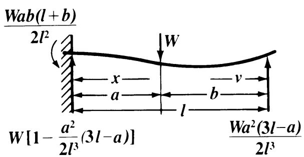
Stresses and Deflections in Beams *(Continued)*

Type of Beam	Stresses: General Formula for Stress at any Point	Stresses: Stresses at Critical Points	Deflections: General Formula for Deflection at any Point[a]	Deflections: Deflections at Critical Points[a]
Case 8. — Both Ends Overhanging Supports, Single Overhanging Load				
	Between load and adjacent support, $s = \frac{W}{Z}(c-u)$ Between supports, $s = \frac{Wc}{Zl}(l-x)$ Between unloaded end and adjacent supports, $s = 0$.	Stress at support adjacent to load, $\frac{Wc}{Z}$ If cross-section is constant, this is the maximum stress. Stress is zero at other support.	Between load and adjacent support, $y = \frac{Wu}{6EI}(3cu - u^2 + 2cl)$ Between supports, $y = -\frac{Wcx}{6EIl}(l-x)(2l-x)$ Between unloaded end and adjacent support, $y = \frac{Wclw}{6EI}$	Deflection at load, $\frac{Wc^2}{3EI}(c+l)$ Maximum upward deflection is at $x = .42265l$, and is $-\frac{Wcl^2}{15.55EI}$ Deflection at unloaded end, $\frac{Wcld}{6EI}$
Case 9. — Both Ends Overhanging Supports, Symmetrical Overhanging Loads				
	Between each load and adjacent support, $s = \frac{W}{Z}(c-u)$ Between supports, $s = \frac{Wc}{Z}$	Stress at supports and at all points between, $\frac{Wc}{Z}$ If cross-section is constant, this is the maximum stress.	Between each load and adjacent support, $y = \frac{Wu}{6EI}[3c(l+u) - u^2]$ Between supports, $y = -\frac{Wcx}{2EI}(l-x)$	Deflections at loads, $\frac{Wc^2}{6EI}(2c+3l)$ Deflection at center, $-\frac{Wcl^2}{8EI}$
			The above expressions involve the usual approximations of the theory of flexure, and hold only for small deflections. Exact expressions for deflections of any magnitude are as follows: Between supports the curve is a circle of radius $r = \frac{EI}{Wc}$; $y = \sqrt{r^2 - \tfrac{1}{4}l^2} - \sqrt{r^2 - (\tfrac{1}{2}l - x)^2}$ Deflection at center, $\sqrt{r^2 - \tfrac{1}{4}l^2} - r$	

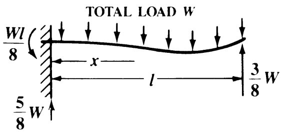
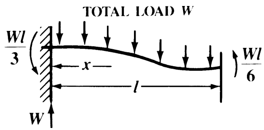
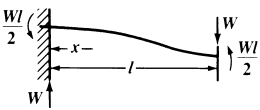
Stresses and Deflections in Beams *(Continued)*

<table>
<tr><th rowspan="2">Type of Beam</th><th colspan="2">Stresses</th><th colspan="2">Deflections</th></tr>
<tr><th>General Formula for Stress at any Point</th><th>Stresses at Critical Points</th><th>General Formula for Deflection at any Point[a]</th><th>Deflections at Critical Points[a]</th></tr>
<tr><td colspan="5">Case 10. — Fixed at One End, Uniform Load</td></tr>
<tr><td>
</td><td>$s = \frac{W}{2Zl}(l-x)^2$</td><td>Stress at support,
$\frac{Wl}{2Z}$
If cross-section is constant, this is the maximum stress.</td><td>$y = \frac{Wx^2}{24EIl}[2l^2 + (2l-x)^2]$</td><td>Maximum deflection, at end,
$\frac{Wl^3}{8EI}$</td></tr>
<tr><td colspan="5">Case 11. — Fixed at One End, Load at Other</td></tr>
<tr><td>
</td><td>$s = \frac{W}{Z}(l-x)$</td><td>Stress at support,
$\frac{Wl}{Z}$
If cross-section is constant, this is the maximum stress.</td><td>$y = \frac{Wx^2}{6EI}(3l-x)$</td><td>Maximum deflection, at end,
$\frac{Wl^3}{3EI}$</td></tr>
<tr><td colspan="5">Case 12. — Fixed at One End, Intermediate Load</td></tr>
<tr><td>
</td><td>Between support and load,
$s = \frac{W}{Z}(l-x)$
Beyond load, $s = 0$.</td><td>Stress at support,
$\frac{Wl}{Z}$
If cross-section is constant, this is the maximum stress.</td><td>Between support and load,
$y = \frac{Wx^2}{6EI}(3l-x)$
Beyond load,
$y = \frac{Wl^2}{6EI}(3v-l)$</td><td>Deflection at load,
$\frac{Wl^3}{3EI}$
Maximum deflection, at end,
$\frac{Wl^2}{6EI}(2l+3b)$</td></tr>
</table>

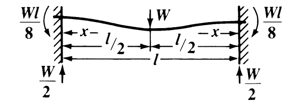
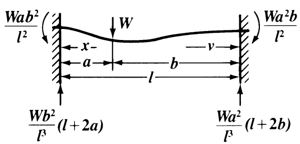
Stresses and Deflections in Beams *(Continued)*

Type of Beam	Stresses: General Formula for Stress at any Point	Stresses: Stresses at Critical Points	Deflections: General Formula for Deflection at any Point[a]	Deflections: Deflections at Critical Points[a]
Case 13. — Fixed at One End, Supported at the Other, Load at Center				
	Between point of fixture and load, $s = \frac{W}{16Z}(3l - 11x)$ Between support and load, $s = -\frac{5}{16}\frac{Wv}{Z}$	Maximum stress at point of fixture, $\frac{3}{16}\frac{Wl}{Z}$ Stress is zero at $x = \frac{3}{11}l$ Greatest negative stress at center, $-\frac{5}{32}\frac{Wl}{Z}$	Between point of fixture and load, $y = \frac{Wx^2}{96EI}(9l - 11x)$ Between support and load, $y = \frac{Wv}{96EI}(3l^2 - 5v^2)$	Maximum deflection is at $v = 0.4472l$, and is $\frac{Wl^3}{107.33EI}$ Deflection at load, $\frac{7}{768}\frac{Wl^3}{EI}$
Case 14. — Fixed at One End, Supported at the Other, Load at any Point				
$m = (l + a)(l + b) + al$ $n = al(l + b)$ 	Between point of fixture and load, $s = \frac{Wb}{2Zl^3}(n - mx)$ Between support and load, $s = -\frac{Wa^2v}{2Zl^3}(3l - a)$	Greatest positive stress, at point of fixture, $\frac{Wab}{2Zl^2}(l + b)$ Greatest negative stress, at load, $-\frac{Wa^2b}{2Zl^3}(3l - a)$ If $a < 0.5858l$, the first is the maximum stress. If $a = 0.5858l$, the two are equal and are $\pm\frac{Wl}{5.83Z}$ If $a > 0.5858l$, the second is the maximum stress. Stress is zero at $x = \frac{n}{m}$	Between point of fixture and load, $y = \frac{Wx^2b}{12EIl^3}(3n - mx)$ Between support and load, $y = \frac{Wa^2v}{12EIl^3}[3l^2b - v^2(3l - a)]$	Deflection at load, $\frac{Wa^3b^2}{12EIl^3}(3l + b)$ If $a < 0.5858l$, maximum deflection is $\frac{Wa^2b}{6EI}\sqrt{\frac{b}{2l + b}}$ and located between load and support, at $v = l\sqrt{\frac{b}{2l + b}}$ If $a = 0.5858l$, maximum deflection is at load and is $\frac{Wl^3}{101.9EI}$ If $a > 0.5858l$, maximum deflection is $\frac{Wbn^3}{3EIm^2l^3}$ and located between load and point of fixture, at $x = \frac{2n}{m}$

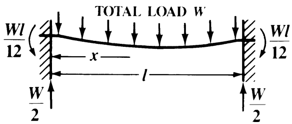
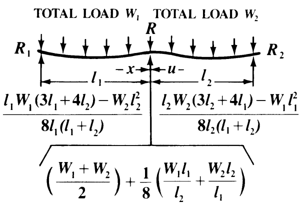
Stresses and Deflections in Beams *(Continued)*

Type of Beam	Stresses		Deflections	
	General Formula for Stress at any Point	Stresses at Critical Points	General Formula for Deflection at any Point[a]	Deflections at Critical Points[a]
Case 15. — Fixed at One End, Supported at the Other, Uniform Load				
	$s = \frac{W(l-x)}{2Zl}(\frac{1}{4}l - x)$	Maximum stress at point of fixture, $\frac{Wl}{8Z}$ Stress is zero at $x = \frac{1}{4}l$. Greatest negative stress is at $x = \frac{5}{8}l$ and is $-\frac{9}{128}\frac{Wl}{Z}$	$y = \frac{Wx^2(l-x)}{48EIl}(3l - 2x)$	Maximum deflection is at $x = 0.5785l$, and is $\frac{Wl^3}{185EI}$ Deflection at center, $\frac{Wl^3}{192EI}$ Deflection at point of greatest negative stress, at $x = \frac{5}{8}l$ is $\frac{Wl^3}{187EI}$
Case 16. — Fixed at One End, Free but Guided at the Other, Uniform Load				
	$s = \frac{Wl}{Z}\left\{\frac{1}{3} - \frac{x}{l} + \frac{1}{2}\left(\frac{x}{l}\right)^2\right\}$	Maximum stress, at support, $\frac{Wl}{3Z}$ Stress is zero at $x = 0.4227l$ Greatest negative stress, at free end, $-\frac{Wl}{6Z}$	$y = \frac{Wx^2}{24EIl}(2l - x)^2$	Maximum deflection, at free end, $\frac{Wl^3}{24EI}$
Case 17. — Fixed at One End, Free but Guided at the Other, with Load				
	$s = \frac{W}{Z}(\frac{1}{2}l - x)$	Stress at support, $\frac{Wl}{2Z}$ Stress at free end $-\frac{Wl}{2Z}$ These are the maximum stresses and are equal and opposite. Stress is zero at center.	$y = \frac{Wx^2}{12EI}(3l - 2x)$	Maximum deflection, at free end, $\frac{Wl^3}{12EI}$

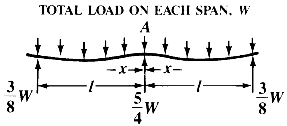
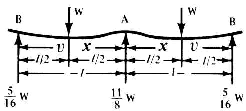
Stresses and Deflections in Beams *(Continued)*

Type of Beam	Stresses: General Formula for Stress at any Point	Stresses: Stresses at Critical Points	Deflections: General Formula for Deflection at any Point[a]	Deflections: Deflections at Critical Points[a]
Case 18. — Fixed at Both Ends, Load at Center				
	Between each end and load, $s = \frac{W}{2Z}(\frac{1}{4}l - x)$	Stress at ends $\frac{Wl}{8Z}$ at load $-\frac{Wl}{8Z}$ These are the maximum stresses and are equal and opposite. Stress is zero at $x = \frac{1}{4}l$	$y = \frac{Wx^2}{48EI}(3l - 4x)$	Maximum deflection, at load, $\frac{Wl^3}{192EI}$
Case 19. — Fixed at Both Ends, Load at any Point				
	For segment of length a, $s = \frac{Wb^2}{Zl^3}[al - x(l + 2a)]$ For segment of length b, $\frac{Wl}{8Z}$	Stress at end next segment of length a, $\frac{Wab^2}{Zl^2}$ Stress at end next segment of length b, $\frac{Wa^2b}{Zl^2}$ Maximum stress is at end next shorter segment. Stress is zero at $x = \frac{al}{l + 2a}$ and $v = \frac{bl}{l + 2b}$ Greatest negative stress, at load, $-\frac{2Wa^2b^2}{Zl^3}$	For segment of length a, $y = \frac{Wx^2b^2}{6EIl^3}[2a(l - x) + l(a - x)]$ For segment of length b, $y = \frac{Wv^2a^2}{6EIl^3}[2b(l - v) + l(b - v)]$	Deflection at load, $\frac{Wa^3b^3}{3EIl^3}$ Let b be the length of the longer segment and a of the shorter one. The maximum deflection is in the longer segment, at $v = \frac{2bl}{l + 2b}$ and is $x = \frac{l_1}{W_1}(W_1 - R_1)$

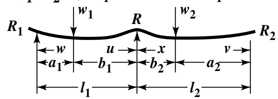
Stresses and Deflections in Beams *(Continued)*

Type of Beam	Stresses		Deflections	
	General Formula for Stress at any Point	Stresses at Critical Points	General Formula for Deflection at any Point[a]	Deflections at Critical Points[a]
Case 20. — Fixed at Both Ends, Uniform Load				
	$s = \frac{Wl}{2Z}\left\{\frac{1}{6} - \frac{x}{l} + \left(\frac{x}{l}\right)^2\right\}$	Maximum stress, at ends, $x = \frac{2n}{m}$ and is $\frac{Wbn^3}{3EIm^2l^3}$ Stress is zero at $x = 0.7887l$ and at $x = 0.2113l$ Greatest negative stress, at center, $-\frac{Wl}{24Z}$	$y = \frac{Wx^2}{24EIl}(l-x)^2$	Maximum deflection, at center, $\frac{Wl^3}{384EI}$
Case 21. — Continuous Beam, with Two Unequal Spans, Unequal, Uniform Loads				
	Between R_1 and R, $s = \frac{l_1 - x}{Z}\left\{\frac{(l_1 - x)W_1}{2l_1} - R_1\right\}$ Between R_2 and R, $s = \frac{l_2 - u}{Z}\left\{\frac{(l_2 - u)W_2}{2l_2} - R_2\right\}$	Stress at support R, $\frac{W_1 l_1^2 + W_2 l_2^2}{8Z(l_1 + l_2)}$ Greatest stress in the first span is at $x = \frac{l_1}{W_1}(W_1 - R_1)$ and is $v = \frac{2bl}{l + 2b}$ Greatest stress in the second span is at $u = \frac{l_2}{W_2}(W_2 - R_2)$ and is, $-\frac{R_2^2 l_2}{2ZW_2}$	Between R_1 and R, $y = \frac{x(l_1 - x)}{24EI}\left\{(2l_1 - x)(4R_1 - W_1) - \frac{W_1(l_1 - x)^2}{l_1}\right\}$ Between R_2 an R, $y = \frac{u(l_2 - u)}{24EI}\left\{(2l_2 - u)(4R_2 - W_2) - \frac{W_2(l_2 - u)^2}{l_2}\right\}$	This case is so complicated that convenient general expressions for the critical deflections cannot be obtained.

Stresses and Deflections in Beams *(Continued)*

Type of Beam	Stresses: General Formula for Stress at any Point	Stresses: Stresses at Critical Points	Deflections: General Formula for Deflection at any Point[a]	Deflections: Deflections at Critical Points[a]
Case 22. — Continuous Beam, with Two Equal Spans, Uniform Load				
	$s = \frac{W(l-x)}{2Zl}(\frac{1}{4}l - x)$	Maximum stress at point $u = \frac{l_2}{W_2}(W_2 - R_2)$ Stress is zero at $x = \frac{5}{8}l$ Greatest negative stress is at $x = \frac{5}{8}l$ and is, $-\frac{9}{128}\frac{Wl}{Z}$	$y = \frac{Wx^2(l-x)}{48EIl}(3l - 2x)$	Maximum deflection is at $x =$ 0.5785l, and is $\frac{Wl^3}{185EI}$ Deflection at center of span, $\frac{Wl^3}{192EI}$ Deflection at point of greatest negative stress, at $x = \frac{5}{8}l$ is $\frac{Wl^3}{187EI}$
Case 23. — Continuous Beam, with Two Equal Spans, Equal Loads at Center of Each				
	Between point A and load, $s = \frac{W}{16Z}(3l - 11x)$ Between point B and load, $s = -\frac{5}{16}\frac{Wv}{Z}$	Maximum stress at point A, $\frac{3}{16}\frac{Wl}{Z}$ Stress is zero at $x = \frac{3}{11}l$ Greatest negative stress at center of span, $-\frac{5}{32}\frac{Wl}{Z}$	Between point A and load, $y = \frac{Wx^2}{96EI}(9l - 11x)$ Between point B and load, $y = \frac{Wv}{96EI}(3l^2 - 5v^2)$	Maximum deflection is at $v =$ 0.4472l, and is $\frac{Wl^3}{107.33EI}$ Deflection at load, $\frac{7}{768}\frac{Wl^3}{EI}$

Stresses and Deflections in Beams *(Continued)*

Type of Beam	Stresses: General Formula for Stress at any Point	Stresses: Stresses at Critical Points	Deflections: General Formula for Deflection at any Point[a]	Deflections: Deflections at Critical Points[a]
Case 24. — Continuous Beam, with Two Unequal Spans, Unequal Loads at any Point of Each				
$m = \frac{1}{2(l_1 + l_2)}\left(\frac{W_1 a_1 b_1}{l_1}(l_1 + a_1) + \frac{W_2 a_2 b_2}{l_2}(l_2 + a_2)\right)$  $\frac{W_1 b_1 - m}{l_1} = r_1$ $\frac{W_1 a_1 + m}{l_1} + \frac{W_2 b_2 - m}{l_2} = r$ $\frac{W_2 b_2 - m}{l_2} = r_2$	Between R_1 and W_1, $s = -\frac{w r_1}{Z}$ Between R and W_1, $s =$ $\frac{1}{l_1 Z}[m(l_1 - u) - W_1 a_1 u]$ Between R and W_2, $s =$ $\frac{1}{l_2 Z}[m(l_2 - x) - W_2 a_2 x]$ Between R_2 and W_2, $s = -\frac{v r_2}{Z}$	Stress at load W_1, $-\frac{a_1 r_1}{Z}$ Stress at support R, $\frac{m}{Z}$ Stress at load W_2, $-\frac{a_2 r_2}{Z}$ The greatest of these is the maximum stress.	Between R_1 and W_1, $y = \frac{w}{6EI}\left\{(l_1 - w)(l_1 + w)r_1 - \frac{W_1 b_1^3}{l_1}\right\}$ Between R and W_1, $y = \frac{u}{6EIl_1}[W_1 a_1 b_1(l_1 + a_1)$ $- W_1 a_1 u^2 - m(2l_1 - u)(l_1 - u)]$ Between R and W_2 $y = \frac{x}{6EIl_2}[W_2 a_2 b_2(l_2 + a_2)$ $- W_2 a_2 x^2 - m(2l_2 - x)(l_2 - x)]$ Between R_2 and W_2, $y = \frac{v}{6EI}\left\{(l_2 - v)(l_2 + v)r_2 - \frac{W_2 b_2^3}{l_2}\right\}$	Deflection at load W_1, $\frac{a_1 b_1}{6EIl_1}[2a_1 b_1 W_1$ $- m(l_1 + a_1)]$ Deflection at load W_2, $\frac{a_2 b_2}{6EIl_2}[2a_2 b_2 W_2$ $- m(l_2 + a_2)]$ This case is so complicated that convenient general expressions for the maximum deflections cannot be obtained.

[a] The deflections apply only to cases where the cross section of the beam is constant for its entire length.

In the diagrammatical illustrations of the beams and their loading, the values indicated near, but below, the supports are the "reactions" or upward forces at the supports. For Cases 1 to 12, inclusive, the reactions, as well as the formulas for the stresses, are the same whether the beam is of constant or variable cross-section. For the other cases, the reactions and the stresses given are for constant cross-section beams only.

The bending moment at any point in inch-pounds is $s \times Z$ and can be found by omitting the divisor Z in the formula for the stress given in the tables. A positive value of the bending moment denotes tension in the upper fibers and compression in the lower ones. A negative value denotes the reverse, The value of W corresponding to a given stress is found by transposition of the formula. For example, in Case 1, the stress at the critical point is $s = -Wl \div 8Z$. From this formula we find $W = -8Zs \div l$. Of course, the negative sign of W may be ignored.

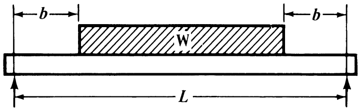
If there are several kinds of loads, as, for instance, a uniform load and a load at any point, or separate loads at different points, the total stress and the total deflection at any point is found by adding together the various stresses or deflections at the point considered due to each load acting by itself. If the stress or deflection due to any one of the loads is negative, it must be subtracted instead of added.

Deflection of Beam Uniformly Loaded for Part of Its Length.—In the following formulas, lengths are in inches, weights in pounds. W = total load; L = total length between supports; E = modulus of elasticity; I = moment of inertia of beam section; a = fraction of length of beam at each end, that is not loaded = $b \div L$; f = deflection.

$$f = \frac{WL^3}{384EI(\qquad)}(\qquad\qquad)$$

The expression for maximum bending moment is: $M_{max} = \frac{1}{8}WL(1 + 2a)$.

These formulas apply to simple beams resting on supports at the ends.

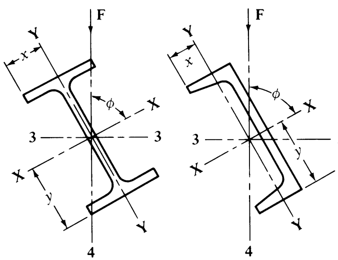


If the formulas are used with metric SI units, W = total load in newtons; L = total length between supports in millimeters; E = modulus of elasticity in newtons per millimeter^2; I = moment of inertia of beam section in millimeters^4; a = fraction of length of beam at each end, that is not loaded = $b \div L$; and f = deflection in millimeters. The bending moment M_{max} is in newton-millimeters (N · mm).

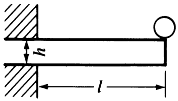
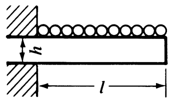
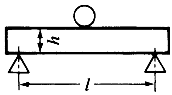
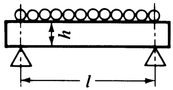
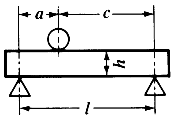
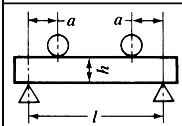
***Note:* A load due to the weight of a mass of M kilograms is Mg newtons, where g = approximately 9.81 meters per second ^{2}.**

Bending Stress Due to an Oblique Transverse Force.—The following illustration shows a beam and a channel being subjected to a transverse force acting at an angle ϕ to the center of gravity. To find the bending stress, the moments of inertia I around axes 3-3 and 4-4 are computed from the following equations: $I_3 = I_x\sin^2\phi + I_y\cos^2\phi$, and $I_4 = I_x\cos^2\phi + I_y\sin^2\phi$.

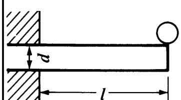
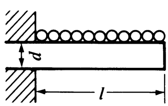
The computed bending stress f_b is then found from $f_b = M\left(\frac{y}{I_x}\sin\phi + \frac{x}{I_y}\cos\phi\right)$ where M is the bending moment due to force F.



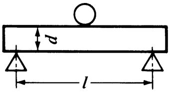
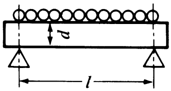
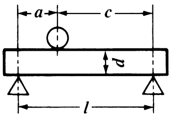
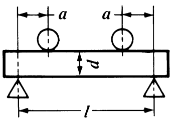
Rectangular Solid Beams

Style of Loading and Support	Diameter of Beam, d inch (mm)	Beam Height, h inch (mm)	Stress in Extreme Fibers, f lb/in^2 (N/mm^2)	Beam Length, l inch (mm)	Total Load, W lb (N)
	Beam fixed at one end, loaded at the other				
	$\frac{6lW}{fh^2} = b$	$\sqrt{\frac{6lW}{bf}} = h$	$\frac{6lW}{bh^2} = f$	$\frac{bfh^2}{6W} = l$	$\frac{bfh^2}{6l} = W$
	Beam fixed at one end, uniformly loaded				
	$\frac{3lW}{fh^2} = b$	$\sqrt{\frac{3lW}{bf}} = h$	$\frac{3lW}{bh^2} = f$	$\frac{bfh^2}{3W} = l$	$\frac{bfh^2}{3l} = W$
	Beam supported at both ends, single load in middle				
	$\frac{3lW}{2fh^2} = b$	$\sqrt{\frac{3lW}{2bf}} = h$	$\frac{3lW}{2bh^2} = f$	$\frac{2bfh^2}{3W} = l$	$\frac{2bfh^2}{3l} = W$
	Beam supported at both ends, uniformly loaded				
	$\frac{3lW}{4fh^2} = b$	$\sqrt{\frac{3lW}{4bf}} = h$	$\frac{3lW}{4bh^2} = f$	$\frac{4bfh^2}{3W} = l$	$\frac{4bfh^2}{3l} = W$
	Beam supported at both ends, single unsymmetrical load				
	$\frac{6Wac}{fh^2l} = b$	$\sqrt{\frac{6Wac}{bfl}} = h$	$\frac{6Wac}{bh^2l} = f$	$a + c = l$	$\frac{bh^2fl}{6ac} = W$
	Beam supported at both ends, two symmetrical loads				
	$\frac{3Wa}{fh^2} = b$	$\sqrt{\frac{3Wa}{bf}} = h$	$\frac{3Wa}{bh^2} = f$	l, any length $\frac{bh^2f}{3W} = a$	$\frac{bh^2f}{3a} = W$

Round Solid Beams

Style of Loading and Support	Diameter of Beam, d inch (mm)	Stress in Extreme Fibers, f lb/in^2 (N/mm^2)	Beam Length, l inch (mm)	Total Load, W lb (N)
	Beam fixed at one end, loaded at the other			
	$\sqrt[3]{\frac{10.18lW}{f}} = d$	$\frac{10.18lW}{d^3} = f$	$\frac{d^3f}{10.18W} = l$	$\frac{d^3f}{10.18l} = W$
	Beam fixed at one end, uniformly loaded			
	$\sqrt[3]{\frac{5.092Wl}{f}} = d$	$\frac{5.092Wl}{d^3} = f$	$\frac{d^3f}{5.092W} = l$	$\frac{d^3f}{5.092l} = W$

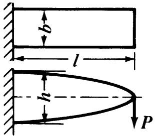
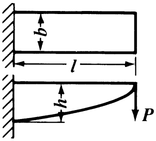
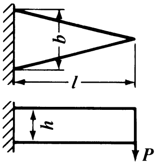
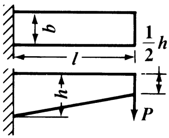
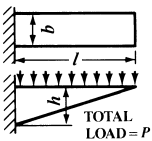
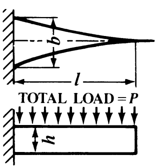
Round Solid Beams *(Continued)*

Style of Loading and Support	Diameter of Beam, d inch (mm)	Stress in Extreme Fibers, f lb/in^2 (N/mm^2)	Beam Length, l inch (mm)	Total Load, W lb (N)
	Beam supported at both ends, single load in middle			
	$\sqrt[3]{\frac{2.546Wl}{f}} = d$	$\frac{2.546Wl}{d^3} = f$	$\frac{d^3f}{2.546W} = l$	$\frac{d^3f}{2.546l} = W$
	Beam supported at both ends, uniformly loaded			
	$\sqrt[3]{\frac{1.273Wl}{f}} = d$	$\frac{1.273Wl}{d^3} = f$	$\frac{d^3f}{1.273W} = l$	$\frac{d^3f}{1.273l} = W$
	Beam supported at both ends, single unsymmetrical load			
	$\sqrt[3]{\frac{10.18Wac}{fl}} = d$	$\frac{10.18Wac}{d^3l} = f$	$a + c = l$	$\frac{d^3fl}{10.18ac} = W$
	Beam supported at both ends, two symmetrical loads			
	$\sqrt[3]{\frac{5.092Wa}{f}} = d$	$\frac{5.092Wa}{d^3} = f$	l, any length $\frac{d^3f}{5.092W} = a$	$\frac{d^3f}{5.092a} = W$

Beams of Uniform Strength Throughout Their Length.—The bending moment in a beam is generally not uniform throughout its length, but varies. Therefore, a beam of uniform cross-section which is made strong enough at its most strained section, will have an excess of material at every other section. Sometimes it may be desirable to have the cross-section uniform, but at other times the metal can be more advantageously distributed if the beam is so designed that its cross-section varies from point to point, so that it is at every point just great enough to take care of the bending stresses at that point. A table is given showing beams in which the load is applied in different ways and which are supported by different methods, and the shape of the beam required for uniform strength is indicated. It should be noted that the shape given is the theoretical shape required to resist bending only. It is apparent that sufficient cross-section of beam must also be added either at the points of support (in beams supported at both ends), or at the point of application of the load (in beams loaded at one end), to take care of the vertical shear.

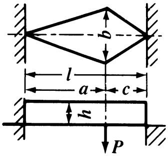
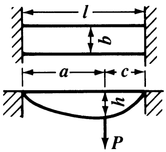
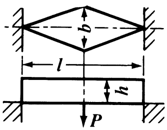
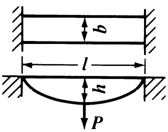
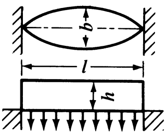
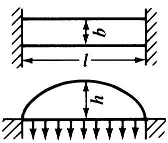
It should be noted that the theoretical shapes of the beams given in the two tables that follow are based on the stated assumptions of uniformity of width or depth of cross-section, and unless these are observed in the design, the theoretical outlines do not apply without modifications. For example, in a cantilever with the load at one end, the outline is a parabola only when the width of the beam is uniform. It is not correct to use a strictly parabolic shape when the thickness is not uniform, as, for instance, when the beam is made of an I- or T-section. In such cases, some modification may be necessary; but it is evident that whatever the shape adopted, the correct depth of the section can be obtained by an investigation of the bending moment and the shearing load at a number of points, and then a line can be drawn through the points thus ascertained, which will provide for a beam of practically uniform strength whether the cross-section be of uniform width or not.

Beams of Uniform Strength Throughout Their Length

Type of Beam	Description	Formula[a]
	Load at one end. Width of beam uniform. Depth of beam decreasing towards loaded end. Outline of beam-shape, parabola with vertex at loaded end.	$P = \frac{Sbh^2}{6l}$
	Load at one end. Width of beam uniform. Depth of beam decreasing towards loaded end. Outline of beam, one-half of a parabola with vertex at loaded end. Beam may be reversed so that upper edge is parabolic.	$P = \frac{Sbh^2}{6l}$
	Load at one end. Depth of beam uniform. Width of beam decreasing towards loaded end. Outline of beam triangular, with apex at loaded end.	$P = \frac{Sbh^2}{6l}$
	Beam of *approximately* uniform strength. Load at one end. Width of beam uniform. Depth of beam decreasing towards loaded end, but not tapering to a sharp point.	$P = \frac{Sbh^2}{6l}$
	Uniformly distributed load. Width of beam uniform. Depth of beam decreasing towards outer end. Outline of beam, right-angled trian-gle.	$P = \frac{Sbh^2}{3l}$
	Uniformly distributed load. Depth of beam uniform. Width of beam gradually decreasing towards outer end. Outline of beam is formed by two parabolas which tangent each other at their vertexes at the outer end of the beam.	$P = \frac{Sbh^2}{3l}$

[a] In the formulas, *P* = load in pounds; *S* = safe stress in pounds per square inch; and *a*, *b*, *c*, *h*, and *l* are in inches. **If metric SI units are used, *P* is in newtons; *S* = safe stress in N/mm^2; and *a*, *b*, *c*, *h*, and *l* are in millimeters.**

Beams of Uniform Strength Throughout Their Length

Type of Beam	Description	Formula[a]
	Beam supported at both ends. Load concentrated at any point. Depth of beam uniform. Width of beam maximum at point of loading. Outline of beam, two triangles with apexes at points of support.	$P = \frac{Sbh^2 l}{6ac}$
	Beam supported at both ends. Load concentrated at any point. Width of beam uniform. Depth of beam maximum at point of loading. Outline of beam is formed by two parabolas with their vertexes at points of support.	$P = \frac{Sbh^2 l}{6ac}$
	Beam supported at both ends. Load concentrated in the middle. Depth of beam uniform. Width of beam maximum at point of loading. Outline of beam, two triangles with apexes at points of support.	$P = \frac{2Sbh^2}{3l}$
	Beam supported at both ends. Load concentrated at center. Width of beam uniform. Depth of beam maximum at point of loading. Outline of beam, two parabolas with vertices at points of support.	$P = \frac{2Sbh^2}{3l}$
	Beam supported at both ends. Load uniformly distributed. Depth of beam uniform. Width of beam maximum at center. Outline of beam, two parabolas with vertexes at middle of beam.	$P = \frac{4Sbh^2}{3l}$
	Beam supported at both ends. Load uniformly distributed. Width of beam uniform. Depth of beam maximum at center. Outline of beam one-half of an ellipse.	$P = \frac{4Sbh^2}{3l}$

[a] For details of English and metric SI units used in the formulas, see footnote on page 251.

Deflection as a Limiting Factor in Beam Design.—For some applications, a beam must be stronger than required by the maximum load it is to support, in order to prevent excessive deflection. Maximum allowable deflections vary widely for different classes of service, so a general formula for determining them cannot be given. When exceptionally stiff girders are required, one rule is to limit the deflection to 1 inch per 100 feet of span; hence, if l = length of span in inches, deflection = $l \div 1200$. According to another formula, deflection limit = $l \div 360$ where beams are adjacent to materials like plaster which would be broken by excessive beam deflection. Some machine parts of the beam type must be very rigid to maintain alignment under load. For example, the deflection of a punch press column may be limited to 0.010 inch or less. These examples merely illustrate variations in practice. It is impracticable to give general formulas for determining the allowable deflection in any specific application, because the allowable amount depends on the conditions governing each class of work.

Procedure in Designing for Deflection: Assume that a deflection equal to $l \div 1200$ is to be the limiting factor in selecting a wide-flange (W-shape) beam having a span length of 144 inches. Supports are at both ends and load at center is 15,000 pounds. Deflection y is to be limited to 144 ÷ 1200 = 0.12 inch. According to the formula on page 237 (Case 2), in which W = load on beam in pounds, l = length of span in inches, E = modulus of elasticity of material, I = moment of inertia of cross section:

$$\text{Deflection } y = \frac{Wl^3}{48EI} \quad \text{hence, } I = \frac{Wl^3}{48yE} = \frac{15{,}000 \times 144^3}{48 \times 0.12 \times 29{,}000{,}000} = 268.1$$

A structural wide-flange beam having a depth of 12 inches and weighing 35 pounds per foot has a moment of inertia I of 285 and a section modulus (Z or S) Of 45.6 (see *Steel Wide-Flange Sections—3* on page 2491)). Checking now for maximum stress s (Case 2, page 237):

$$s = \frac{Wl}{4Z} = \frac{15{,}000 \times 144}{4 \times 46.0} = 11{,}842 \text{ lbs. per sq. in.}$$

Although deflection is the limiting factor in this case, the maximum stress is checked to make sure that it is within the allowable limit. As the limiting deflection is decreased, for a given load and length of span, the beam strength and rigidity must be increased, and, consequently, the maximum stress is decreased. Thus, in the preceding example, if the maximum deflection is 0.08 inch instead of 0.12 inch, then the calculated value for the moment of inertia I will be 402; hence a W 12 × 53 beam having an I value of 426 could be used (nearest value above 402). The maximum stress then would be reduced to 7640 pounds per square inch and the calculated deflection is 0.076 inch.

A similar example using metric SI units is as follows. Assume that a deflection equal to $l \div 1000$ millimeters is to be the limiting factor in selecting a W-beam having a span length of 5 meters. Supports are at both ends and the load at the center is 30 kilonewtons. Deflection y is to be limited to 5000 ÷ 1000 = 5 millimeters. The formula on page 237 (Case 2) is applied, and W = load on beam in newtons; l = length of span in mm; E = modulus of elasticity (assume 200,000 N/mm^2 in this example); and I = moment of inertia of cross-section in millimeters^4. Thus,

$$\text{Deflection } y = \frac{Wl^3}{48EI}$$

hence

$$I = \frac{Wl^3}{48yE} = \frac{30{,}000 \times 5000^3}{48 \times 5 \times 200{,}000} = 78{,}125{,}000 \text{ mm}^4$$

Although deflection is the limiting factor in this case, the maximum stress is checked to make sure that it is within the allowable limit, using the formula from page 237 (Case 2):

$$s = \frac{Wl}{4Z}$$

The units of s are newtons per square millimeter; W is the load in newtons; l is the length in mm; and Z = section modulus of the cross-section of the beam = I ÷ distance in mm from neutral axis to extreme fiber.

Curved Beams.—The formula $S = Mc/I$ used to compute stresses due to bending of beams is based on the assumption that the beams are straight before any loads are applied. In beams having initial curvature, however, the stresses may be considerably higher than predicted by the ordinary straight-beam formula because the effect of initial curvature is to shift the neutral axis of a curved member in from the gravity axis toward the center of curvature (the concave side of the beam). This shift in the position of the neutral axis causes an increase in the stress on the concave side of the beam and decreases the stress at the outside fibers.

Hooks, press frames, and other machine members which as a rule have a rather pronounced initial curvature may have a maximum stress at the inside fibers of up to about 3½ times that predicted by the ordinary straight-beam formula.

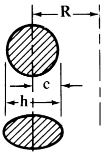
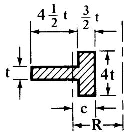
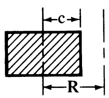
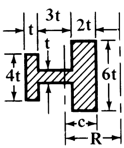
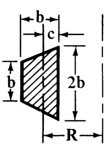
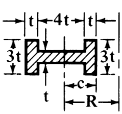
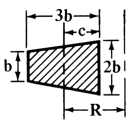
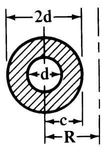
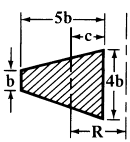
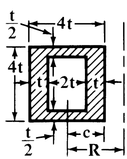
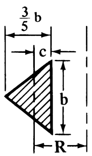
Stress Correction Factors for Curved Beams: A simple method for determining the maximum fiber stress due to bending of curved members consists of 1) calculating the maximum stress using the straight-beam formula $S = Mc/I$; and; and 2) multiplying the calculated stress by a stress correction factor. The table on page 255 gives stress correction factors for some of the common cross-sections and proportions used in the design of curved members..

An example in the application of the method using English units of measurement is given at the bottom of the table. **A similar example using metric SI units is as follows: The fiber stresses of a curved rectangular beam are calculated as 40 newtons per millimeter^2, using the straight beam formula, $S = Mc/I$. If the beam is 150 mm deep and its radius of curvature is 300 mm, what are the true stresses? R/c = 300/75 = 4. From the table on** page 255**, the K factors corresponding to R/c = 4 are 1.20 and 0.85. Thus, the inside fiber stress is 40 × 1.20 = 48 N/mm^2 = 48 megapascals; and the outside fiber stress is 40 × 0.85 = 34 N/mm^2 = 34 megapascals.**

Approximate Formula for Stress Correction Factor: The stress correction factors given in the table on page 255 were determined by Wilson and Quereau and published in the University of Illinois Engineering Experiment Station Circular No. 16, "A Simple Method of Determining Stress in Curved Flexural Members." In this same publication the authors indicate that the following empirical formula may be used to calculate the value of the stress correction factor for the *inside* fibers of sections not covered by the tabular data to within 5 per cent accuracy except in triangular sections where up to 10 per cent deviation may be expected. However, for most engineering calculations, this formula should prove satisfactory for general use in determining the factor for the inside fibers.

$$K = 1.00 + 0.5\frac{I}{bc^2}\left[\frac{1}{R-c} + \frac{1}{R}\right]$$

Values of the Stress Correction Factor *K* for Various Curved Beam Sections

Section	R/c	Factor *K* Inside Fiber	Factor *K* Outside Fiber	y_0[a]	Section	R/c	Factor *K* Inside Fiber	Factor *K* Outside Fiber	y_0[a]
	1.2	3.41	.54	.224*R*		1.2	3.63	.58	.418*R*
	1.4	2.40	.60	.151*R*		1.4	2.54	.63	.299*R*
	1.6	1.96	.65	.108*R*		1.6	2.14	.67	.229*R*
	1.8	1.75	.68	.084*R*		1.8	1.89	.70	.183*R*
	2.0	1.62	.71	.069*R*		2.0	1.73	.72	.149*R*
	3.0	1.33	.79	.030*R*		3.0	1.41	.79	.069*R*
	4.0	1.23	.84	.016*R*		4.0	1.29	.83	.040*R*
	6.0	1.14	.89	.0070*R*		6.0	1.18	.88	.018*R*
	8.0	1.10	.91	.0039*R*		8.0	1.13	.91	.010*R*
	10.0	1.08	.93	.0025*R*		10.0	1.10	.92	.0065*R*
	1.2	2.89	.57	.305*R*		1.2	3.55	.67	.409*R*
	1.4	2.13	.63	.204*R*		1.4	2.48	.72	.292*R*
	1.6	1.79	.67	.149*R*		1.6	2.07	.76	.224*R*
	1.8	1.63	.70	.112*R*		1.8	1.83	.78	.178*R*
	2.0	1.52	.73	.090*R*		2.0	1.69	.80	.144*R*
	3.0	1.30	.81	.041*R*		3.0	1.38	.86	.067*R*
	4.0	1.20	.85	.021*R*		4.0	1.26	.89	.038*R*
	6.0	1.12	.90	.0093*R*		6.0	1.15	.92	.018*R*
	8.0	1.09	.92	.0052*R*		8.0	1.10	.94	.010*R*
	10.0	1.07	.94	.0033*R*		10.0	1.08	.95	.0065*R*
	1.2	3.01	.54	.336*R*		1.2	2.52	.67	.408*R*
	1.4	2.18	.60	.229*R*		1.4	1.90	.71	.285*R*
	1.6	1.87	.65	.168*R*		1.6	1.63	.75	.208*R*
	1.8	1.69	.68	.128*R*		1.8	1.50	.77	.160*R*
	2.0	1.58	.71	.102*R*		2.0	1.41	.79	.127*R*
	3.0	1.33	.80	.046*R*		3.0	1.23	.86	.058*R*
	4.0	1.23	.84	.024*R*		4.0	1.16	.89	.030*R*
	6.0	1.13	.88	.011*R*		6.0	1.10	.92	.013*R*
	8.0	1.10	.91	.0060*R*		8.0	1.07	.94	.0076*R*
	10.0	1.08	.93	.0039*R*		10.0	1.05	.95	.0048*R*
	1.2	3.09	.56	.336*R*		1.2	3.28	.58	.269*R*
	1.4	2.25	.62	.229*R*		1.4	2.31	.64	.182*R*
	1.6	1.91	.66	.168*R*		1.6	1.89	.68	.134*R*
	1.8	1.73	.70	.128*R*		1.8	1.70	.71	.104*R*
	2.0	1.61	.73	.102*R*		2.0	1.57	.73	.083*R*
	3.0	1.37	.81	.046*R*		3.0	1.31	.81	.038*R*
	4.0	1.26	.86	.024*R*		4.0	1.21	.85	.020*R*
	6.0	1.17	.91	.011*R*		6.0	1.13	.90	.0087*R*
	8.0	1.13	.94	.0060*R*		8.0	1.10	.92	.0049*R*
	10.0	1.11	.95	.0039*R*		10.0	1.07	.93	.0031*R*
	1.2	3.14	.52	.352*R*		1.2	2.63	.68	.399*R*
	1.4	2.29	.54	.243*R*		1.4	1.97	.73	.280*R*
	1.6	1.93	.62	.179*R*		1.6	1.66	.76	.205*R*
	1.8	1.74	.65	.138*R*		1.8	1.51	.78	.159*R*
	2.0	1.61	.68	.110*R*		2.0	1.43	.80	.127*R*
	3.0	1.34	.76	.050*R*		3.0	1.23	.86	.058*R*
	4.0	1.24	.82	.028*R*		4.0	1.15	.89	.031*R*
	6.0	1.15	.87	.012*R*		6.0	1.09	.92	.014*R*
	8.0	1.12	.91	.0060*R*		8.0	1.07	.94	.0076*R*
	10.0	1.10	.93	.0039*R*		10.0	1.06	.95	.0048*R*
	1.2	3.26	.44	.361*R*	*Example:* The fiber stresses of a curved rectangular beam are calculated as 5000 psi using the straight beam formula, $S = Mc/I$. If the beam is 8 inches deep and its radius of curvature is 12 inches, what are the true stresses? $R/c = 12/4 = 3$. The factors in the table corresponding to $R/c = 3$ are 0.81 and 1.30. Outside fiber stress = 5000 × 0.81 = 4050 psi; inside fiber stress = 5000 × 1.30 = 6500 psi.				
	1.4	2.39	.50	.251*R*					
	1.6	1.99	.54	.186*R*					
	1.8	1.78	.57	.144*R*					
	2.0	1.66	.60	.116*R*					
	3.0	1.37	.70	.052*R*					
	4.0	1.27	.75	.029*R*					
	6.0	1.16	.82	.013*R*					
	8.0	1.12	.86	.0060*R*					
	10.0	1.09	.88	.0039*R*					

[a] y_0 is the distance from the centroidal axis to the neutral axis of curved beams subjected to pure bending and is measured from the centroidal axis toward the center of curvature.

(Use 1.05 instead of 0.5 in this formula for circular and elliptical sections.)

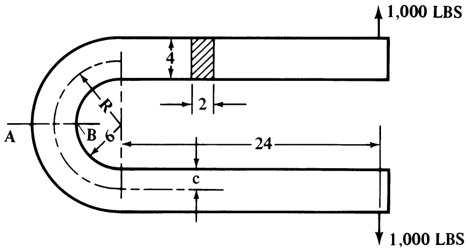
I = Moment of inertia of section about centroidal axis

b = maximum width of section

c = distance from centroidal axis to inside fiber, i.e., to the extreme fiber nearest the center of curvature

R = radius of curvature of centroidal axis of beam

Example: The accompanying diagram shows the dimensions of a clamp frame of rectangular cross-section. Determine the maximum stress at points A and B due to a clamping force of 1000 pounds.



The cross-sectional area = 2 × 4 = 8 square inches; the bending moment at section AB is 1000 (24 + 6 + 2) = 32,000 inch pounds; the distance from the center of gravity of the section at AB to point B is c = 2 inches; and using the formula on page 219, the moment of inertia of the section is $2 \times (4)^3 \div 12 = 10.667$ inches^4.

Using the straight-beam formula, page 254, the stress at points A and B due to the bending moment is:

$$S = \frac{Mc}{I} = \frac{32{,}000 \times 2}{10.667} = 6000 \text{ psi}$$

The stress at A is a compressive stress of 6000 psi and that at B is a tensile stress of 6000 psi.

These values must be corrected to account for the curvature effect. In the table on page 255 for $R/c = (6 + 2)/(2) = 4$, the value of K is found to be 1.20 and 0.85 for points B and A respectively. Thus, the actual stress due to bending at point B is 1.20 × 6000 = 7200 psi in tension and the stress at point A is 0.85 × 6000 = 5100 psi in compression.

To these stresses at A and B must be added, algebraically, the direct stress at section AB due to the 1000-pound clamping force. The direct stress on section AB will be a tensile stress equal to the clamping force divided by the section area. Thus 1000 ÷ 8 = 125 psi in tension.

The maximum unit stress at A is, therefore, 5100 − 125 = 4975 psi in compression and the maximum unit stress at B is 7200 + 125 = 7325 psi in tension.

The following is a similar calculation using metric SI units, assuming that it is required to determine the maximum stress at points A and B due to clamping force of 4 kilonewtons acting on the frame. The frame cross-section is 50 by 100 millimeters, the radius R = 200 mm, and the length of the straight portions is 600 mm. Thus, the cross-sectional area = 50 × 100 = 5000 mm^2; the bending moment at AB is 4000(600 + 200) = 3,200,000 newton-millimeters; the distance from the center of gravity of the section at AB to point B is c = 50 mm; and the moment of inertia of the section is, using the formula on page 219, $50 \times (100)^3$ = 4,170,000 mm^4.

Using the straight-beam formula, page 254, the stress at points A and B due to the bending moment is:

$$s = \frac{Mc}{I} = \frac{3{,}200{,}000 \times 50}{4{,}170{,}000}$$

$$= 38.4 \text{ newtons per millimeter}^2 = 38.4 \text{ megapascals}$$

The stress at *A* is a compressive stress of 38.4 N/mm², while that at *B* is a tensile stress of 38.4 N/mm². These values must be corrected to account for the curvature effect. From the table on page 255, the *K* factors are 1.20 and 0.85 for points *A* and *B* respectively, derived from *R*/*c* = 200/50 = 4. Thus, the actual stress due to bending at point *B* is 1.20 × 38.4 = 46.1 N/mm² (46.1 megapascals) in tension; and the stress at point *A* is 0.85 × 38.4 = 32.6 N/mm² (32.6 megapascals) in compression.

To these stresses at *A* and *B* must be added, algebraically, the direct stress at section *AB* due to the 4 kN clamping force. The direct stress on section *AB* will be a tensile stress equal to the clamping force divided by the section area. Thus, 4000/5000 = 0.8 N/mm ². The maximum unit stress at *A* is, therefore, 32.61 − 0.8 = 31.8 N/mm ² (31.8 megapascals) in compression, and the maximum unit stress at *B* is 46.1 + 0.8 = 46.9 N/mm ² (46.9 megapascals) in tension.

Stresses Produced by Shocks

Stresses in Beams Produced by Shocks.—Any elastic structure subjected to a shock will deflect until the product of the average resistance, developed by the deflection, and the distance through which it has been overcome, has reached a value equal to the energy of the shock. It follows that for a given shock, the average resisting stresses are inversely proportional to the deflection. If the structure were perfectly rigid, the deflection would be zero, and the stress infinite. The effect of a shock is, therefore, to a great extent dependent upon the elastic property (the springiness) of the structure subjected to the impact.

The energy of a body in motion, such as a falling body, may be spent in each of four ways:

1) In deforming the body struck as a whole.

2) In deforming the falling body as a whole.

3) In partial deformation of both bodies on the surface of contact (most of this energy will be transformed into heat).

4) Part of the energy will be taken up by the supports, if these are not perfectly rigid and inelastic.

How much energy is spent in the last three ways it is usually difficult to determine, and for this reason it is safest to figure as if the whole amount were spent as in Case 1. If a reliable judgment is possible as to what percentage of the energy is spent in other ways than the first, a corresponding fraction of the total energy can be assumed as developing stresses in the body subjected to shocks.

One investigation into the stresses produced by shocks led to the following conclusions:

1) A suddenly applied load will produce the same deflection, and, therefore, the same stress as a static load twice as great; and 2) The unit stress *p* (see formulas in the table "*Stresses Produced in Beams by Shocks*") for a given load producing a shock, varies directly as the square root of the modulus of elasticity *E*, and inversely as the square root of the length *L* of the beam and the area of the section.

Thus, for instance, if the sectional area of a beam is increased by four times, the unit stress will diminish only by half. This result is entirely different from those produced by static loads where the stress would vary inversely with the area, and within certain limits be practically independent of the modulus of elasticity.

In the table, the expression for the approximate value of *p*, which is applicable whenever the deflection of the beam is small as compared with the total height *h* through which the body producing the shock is dropped, is always the same for beams supported at both ends and subjected to shock at *any* point between the supports. In the formulas all dimensions are in inches and weights in pounds.

If metric SI units are used, p is in newtons per square millimeter; Q is in newtons; E = modulus of elasticity in N/mm^2; I = moment of inertia of section in millimeters^4; and h, a, and L in millimeters. *Note:* If Q is given in kilograms, the value referred to is mass. The weight Q of a mass M kilograms is Mg newtons, where g = approximately 9.81 meters per second^2.

Stresses Produced in Beams by Shocks

Method of Support and Point Struck by Falling Body	Fiber (Unit) Stress p produced by Weight Q Dropped Through a Distance h	Approximate Value of p
Supported at both ends; struck in center.	$p = \frac{QaL}{4I}\left(1 + \sqrt{1 + \frac{96hEI}{QL^3}}\right)$	$p = a\sqrt{\frac{6QhE}{LI}}$
Fixed at one end; struck at the other.	$p = \frac{QaL}{I}\left(1 + \sqrt{1 + \frac{6hEI}{QL^3}}\right)$	$p = a\sqrt{\frac{6QhE}{LI}}$
Fixed at both ends; struck in center.	$p = \frac{QaL}{8I}\left(1 + \sqrt{1 + \frac{384hEI}{QL^3}}\right)$	$p = a\sqrt{\frac{6QhE}{LI}}$
I = moment of inertia of section; a = distance of extreme fiber from neutral axis; L = length of beam; E = modulus of elasticity.		

Examples of How Formulas for Stresses Produced by Shocks are Derived: The general formula from which specific formulas for shock stresses in beams, springs, and other machine and structural members are derived is:

$$p = p_s\left(1 + \sqrt{1 + \frac{2h}{y}}\right) \quad (1)$$

In this formula, p = stress in pounds per square inch due to shock caused by impact of a moving load; p_s = stress in pounds per square inch resulting when moving load is applied statically; h = distance in inches that load falls before striking beam, spring, or other member; y = deflection, in inches, resulting from static load.

As an example of how Formula (1) may be used to obtain a formula for a specific application, suppose that the load W shown applied to the beam in Case 2 on page 237 were dropped on the beam from a height of h inches instead of being gradually applied (static loading). The maximum stress p_s due to load W for Case 2 is given as $Wl \div 4Z$ and the maximum deflection y is given as $Wl^3 \div 48\ EI$. Substituting these values in Formula (1),

$$p = \frac{Wl}{4Z}\left(1 + \sqrt{1 + \frac{2h}{Wl^3 \div 48EI}}\right) = \frac{Wl}{4Z}\left(1 + \sqrt{1 + \frac{96hEI}{Wl^3}}\right) \quad (2)$$

If in Formula (2) the letter Q is used in place of W and if Z, the section modulus, is replaced by its equivalent, $I \div$ distance a from neutral axis to extreme fiber of beam, then Formula (2) becomes the first formula given in the accompanying table *Stresses Produced in Beams by Shocks*

Stresses in Helical Springs Produced by Shocks.—A load suddenly applied on a spring will produce the same deflection, and, therefore, also the same unit stress, as a static load twice as great. When the load drops from a height h, the stresses are as given in the accompanying table. The approximate values are applicable when the deflection is small as compared with the height h. The formulas show that the fiber stress for a given shock will be greater in a spring made from a square bar than in one made from a round bar, if the diameter of coil be the same and the side of the square bar equals the diameter of the round bar. It is, therefore, more economical to use round stock for springs which must withstand shocks, due to the fact that the deflection for the same fiber stress for a square bar spring is smaller than that for a round bar spring, the ratio being as 4 to 5. The round bar spring is therefore capable of storing more energy than a square bar spring for the same stress.

Stresses Produced in Springs by Shocks

Form of Bar from Which Spring is Made	Fiber (Unit) Stress *f* Produced by Weight *Q* Dropped a Height *h* on a Helical Spring	Approximate Value of *f*
Round	$f = \frac{8QD}{\pi d^3}\left(1 + \sqrt{1 + \frac{Ghd^4}{4QD^3 n}}\right)$	$f = 1.27\sqrt{\frac{QhG}{Dd^2 n}}$
Square	$f = \frac{9QD}{4d^3}\left(1 + \sqrt{1 + \frac{Ghd^4}{0.9\pi(QD)^3 n}}\right)$	$f = 1.34\sqrt{\frac{QhG}{Dd^2 n}}$
G = modulus of elasticity for torsion; *d* = diameter or side of bar; *D* = mean diameter of spring; *n* = number of coils in spring.		

Shocks from Bodies in Motion.—The formulas given can be applied, in general, to shocks from bodies in motion. A body of weight *W* moving horizontally with the velocity of *v* feet per second, has a stored-up energy:

$$E_K = \frac{1}{2} \times \frac{Wv^2}{g} \text{ foot-pounds} \qquad \text{or} \qquad \frac{6Wv^2}{g} \text{ inch-po}$$

This expression may be substituted for *Qh* in the tables in the equations for unit stresses containing this quantity, and the stresses produced by the energy of the moving body thereby determined.

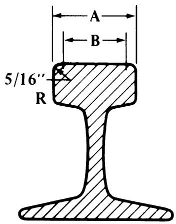
The formulas in the tables give the maximum value of the stresses, providing the designer with some definitive guidance even where there may be justification for assuming that only a part of the energy of the shock is taken up by the member under stress.

The formulas can also be applied using metric SI units. The stored-up energy of a body of mass *M* kilograms moving horizontally with the velocity of *v* meters per second is:

$$E_K = \tfrac{1}{2}Mv^2 \text{newton-meters}$$

This expression may be substituted for *Qh* in the appropriate equations in the tables. For calculation in millimeters, *Qh* = 1000 E_K newton-millimeters.

Size of Rail Necessary to Carry a Given Load.—The following formulas may be employed for determining the size of rail and wheel suitable for carrying a given load. Let, *A* = the width of the head of the rail in inches; *B* = width of the tread of the rail in inches; *C* = the wheel-load in pounds; *D* = the diameter of the wheel in inches.



Then the width of the tread of the rail in inches is found from the formula:

$$B = \frac{C}{1250D} \qquad (1)$$

The width A of the head equals $B + \frac{5}{8}$ inch. The diameter D of the smallest track wheel that will safely carry the load is found from the formula:

$$D = \frac{C}{A \times K} \qquad (2)$$

in which $K = 600$ to 800 for steel castings; $K = 300$ to 400 for cast iron.

As an example, assume that the wheel-load is 10,000 pounds; the diameter of the wheel is 20 inches; and the material is cast steel. Determine the size of rail necessary to carry this load. From Formula (1):

$$B = \frac{10{,}000}{1250 \times 20} = 0.4 \text{ inch}$$

Hence the width of the rail required equals 0.4 + ⅝ inch = 1.025 inch. Determine also whether a wheel 20 inches in diameter is large enough to safely carry the load. From Formula (2):

$$D = \frac{10{,}000}{1.025 \times 600} = 16\tfrac{1}{4} \text{ inches}$$

This is the smallest diameter of track wheel that will safely carry the load; hence a 20-inch wheel is ample.

American Railway Engineering Association Formulas.—The American Railway Engineering Association recommends for safe operation of steel cylinders rolling on steel plates that the allowable load p in pounds per inch of length of the cylinder should not exceed the value calculated from the formula

$$p = \frac{\text{y.s.} - 13{,}000}{20{,}000} 600\ d \text{ for diameter } d \text{ less than 25 inches}$$

This formula is based on steel having a yield strength, y.s., of 32,000 pounds per square inch. For roller or wheel diameters of up to 25 inches, the Hertz stress (contact stress) resulting from the calculated load p will be approximately 76,000 pounds per square inch.

For a 10-inch diameter roller the safe load per inch of roller length is

$$p = \frac{32{,}000 - 13{,}000}{20{,}000} 600 \times 10 = 5700 \text{ lbs per inch of length}$$

Therefore, to support a 10,000 pound load the roller or wheel would need to be 10,000/5700 = 1.75 inches wide.

COLUMNS

Columns

Strength of Columns or Struts.—Structural members which are subject to compression may be so long in proportion to the diameter or lateral dimensions that failure may be the result 1) of both compression and bending; and 2) of bending or buckling to such a degree that compression stress may be ignored.

In such cases, the *slenderness ratio* is important. This ratio equals the length l of the column in inches divided by the least radius of gyration r of the cross-section. Various formulas have been used for designing columns which are too slender to be designed for compression only.

Rankine or Gordon Formula.—This formula is generally applied when slenderness ratios range between 20 and 100, and sometimes for ratios up to 120. The notation, in English and metric SI units of measurement, is given on page 263.

$$p = \frac{S}{1 + K\left(\frac{l}{r}\right)^2} = \text{ultimate load, lbs. per sq. in.}$$

Factor K may be established by tests with a given material and end condition, and for the probable range of l/r. If determined by calculation, $K = S/C\pi^2E$. Factor C equals 1 for either rounded or pivoted column ends, 4 for fixed ends, and 1 to 4 for square flat ends. The factors 25,000, 12,500, etc., in the Rankine formulas, arranged as on page 263, equal $1/K$, and have been used extensively.

Straight-line Formula.—This general type of formula is often used in designing compression members for buildings, bridges, or similar structural work. It is convenient especially in designing a number of columns that are made of the same material but vary in size, assuming that factor B is known. This factor is determined by tests.

$$p = S_y - B\left(\frac{l}{r}\right) = \text{ultimate load, lbs. per sq. in.}$$

S_y equals yield point, lbs. per square inch, and factor B ranges from 50 to 100. Safe unit stress = p ÷ factor of safety.

Formulas of American Railway Engineering Association.—The formulas that follow apply to structural steel having an ultimate strength of 60,000 to 72,000 pounds per square inch.

For building columns having l/r ratios not greater than 120, allowable unit stress = $17{,}000 - 0.485\, l^2/r^2$. For columns having l/r ratios greater than 120, allowable unit stress

$$\text{allowable unit stress} = \frac{18{,}000}{1 + l^2/18{,}000r^2}$$

For bridge compression members centrally loaded and with values of l/r not greater than 140:

$$\text{Allowable unit stress, riveted ends} = 15{,}000 - \frac{1}{4}\frac{l^2}{r^2}$$

$$\text{Allowable unit stress, pin ends} = 15{,}000 - \frac{1}{3}\frac{l^2}{r^2}$$

Euler Formula.—This formula is for columns that are so slender that bending or buckling action predominates and compressive stresses are not taken into account.

$$P = \frac{C\pi^2 IE}{l^2} = \text{total ultimate load, in pounds}$$

The notation, in English and metric SI units of measurement, is given in the table *Rankine's and Euler's Formulas for Columns* on page 263. Factors C for different end conditions are included in the Euler formulas at the bottom of the table. According to a series of experiments, Euler formulas should be used if the values of l/r exceed the following ratios: Structural steel and flat ends, 195; hinged ends, 155; round ends, 120; cast iron with flat ends, 120; hinged ends, 100; round ends, 75; oak with flat ends, 130. The *critical slenderness ratio*, which marks the dividing line between the shorter columns and those slender enough to warrant using the Euler formula, depends upon the column material and its end conditions. If the Euler formula is applied when the slenderness ratio is too small, the *calculated* ultimate strength will exceed the yield point of the material and, obviously, will be incorrect.

Eccentrically Loaded Columns.—In the application of the column formulas previously referred to, it is assumed that the action of the load coincides with the axis of the column. If the load is offset relative to the column axis, the column is said to be eccentrically loaded, and its strength is then calculated by using a modification of the Rankine formula, the quantity cz/r^2 being added to the denominator, as shown in the table on the next page. This modified formula is applicable to columns having a slenderness ratio varying from 20 or 30 to about 100.

Machine Elements Subjected to Compressive Loads.—As in structural compression members, an unbraced machine member that is relatively slender (i.e., its length is more than, say, six times the least dimension perpendicular to its longitudinal axis) is usually designed as a column, because failure due to overloading (assuming a compressive load centrally applied in an axial direction) may occur by buckling or a combination of buckling and compression rather than by direct compression alone. In the design of unbraced steel machine "columns" which are to carry compressive loads applied along their longitudinal axes, two formulas are in general use:

(Euler)
$$P_{cr} = \frac{s_y A r^2}{Q} \quad (1)$$

(J. B. Johnson)
$$P_{cr} = A s_y\left(1 - \frac{Q}{4r^2}\right) \quad (2) \qquad \text{where} \qquad Q = \frac{s_y l^2}{n\pi^2 E} \quad (3)$$

In these formulas, P_{cr} = critical load in pounds that would result in failure of the column; A = cross-sectional area, square inches; S_y = yield point of material, pounds per square inch; r = least radius of gyration of cross-section, inches; E = modulus of elasticity, pounds per square inch; l = column length, inches; and n = coefficient for end conditions. For both ends fixed, $n = 4$; for one end fixed, one end free, $n = 0.25$; for one end fixed and the other end free but guided, $n = 2$; for round or pinned ends, free but guided, $n = 1$; and for flat ends, $n = 1$ to 4. It should be noted that these values of n represent ideal conditions that are seldom attained in practice; for example, for both ends fixed, a value of $n = 3$ to 3.5 may be more realistic than $n = 4$.

If metric SI units are used in these formulas, P_{cr} = critical load in newtons that would result in failure of the column; A = cross-sectional area, square millimeters; S_y = yield point of the material, newtons per square mm; r = least radius of gyration of cross-section, mm; E = modulus of elasticity, newtons per square mm; l = column length, mm; and n = a coefficient for end conditions. The coefficients given are valid for calculations in metric units.

Rankine's and Euler's Formulas for Columns

Symbol	Quantity	English Unit	Metric SI Units
p	Ultimate unit load	Lbs./sq. in.	Newtons/sq. mm.
P	Total ultimate load	Pounds	Newtons
S	Ultimate compressive strength of material	Lbs./sq. in.	Newtons/sq. mm.
l	Length of column or strut	Inches	Millimeters
r	Least radius of gyration	Inches	Millimeters
I	Least moment of inertia	Inches^4	Millimeters^4
r^2	Moment of inertia/area of section	Inches^2	Millimeters^2
E	Modulus of elasticity of material	Lbs./sq. in.	Newtons/sq. mm.
c	Distance from neutral axis of cross-section to side under compression	Inches	Millimeters
z	Distance from axis of load to axis coinciding with center of gravity of cross-section	Inches	Millimeters

Rankine's Formulas			
Material	Both Ends of Column Fixed	One End Fixed and One End Rounded	Both Ends Rounded
Steel	$p = \dfrac{S}{1 + \dfrac{l^2}{25{,}000 r^2}}$	$p = \dfrac{S}{1 + \dfrac{l^2}{12{,}500 r^2}}$	$p = \dfrac{S}{1 + \dfrac{l^2}{6250 r^2}}$
Cast Iron	$p = \dfrac{S}{1 + \dfrac{l^2}{5000 r^2}}$	$p = \dfrac{S}{1 + \dfrac{l^2}{2500 r^2}}$	$p = \dfrac{S}{1 + \dfrac{l^2}{1250 r^2}}$
Wrought Iron	$p = \dfrac{S}{1 + \dfrac{l^2}{35{,}000 r^2}}$	$p = \dfrac{S}{1 + \dfrac{l^2}{17{,}500 r^2}}$	$p = \dfrac{S}{1 + \dfrac{l^2}{8750 r^2}}$
Timber	$p = \dfrac{S}{1 + \dfrac{l^2}{3000 r^2}}$	$p = \dfrac{S}{1 + \dfrac{l^2}{1500 r^2}}$	$p = \dfrac{S}{1 + \dfrac{l^2}{750 r^2}}$

Formulas Modified for Eccentrically Loaded Columns			
Material	Both Ends of Column Fixed	One End Fixed and One End Rounded	Both Ends Rounded
Steel	$p = \dfrac{S}{1 + \dfrac{l^2}{25{,}000 r^2} + \dfrac{cz}{r^2}}$	$p = \dfrac{S}{1 + \dfrac{l^2}{12{,}500 r^2} + \dfrac{cz}{r^2}}$	$p = \dfrac{S}{1 + \dfrac{l^2}{6250 r^2} + \dfrac{cz}{r^2}}$
For materials other than steel, such as cast iron, use the Rankine formulas given in the upper table and add to the denominator the quantity cz/r^2			

Euler's Formulas for Slender Columns			
Both Ends of Column Fixed	One End Fixed and One End Rounded	Both Ends Rounded	One End Fixed and One End Free
$P = \dfrac{4\pi^2 IE}{l^2}$	$P = \dfrac{2\pi^2 IE}{l^2}$	$P = \dfrac{\pi^2 IE}{l^2}$	$P = \dfrac{\pi^2 IE}{4l^2}$

Allowable Working Loads for Columns: To find the total allowable working load for a given section, divide the total ultimate load P (or $p \times$ area), as found by the appropriate formula above, by a suitable factor of safety.

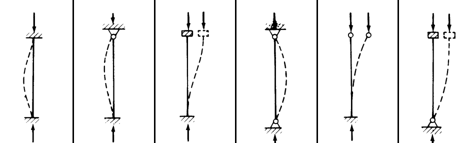
Allowable Concentric Loads for Steel Pipe Columns

STANDARD STEEL PIPE								
Effective Length (*KL*), Feet[a]	Nominal Diameter of Pipe, Inches							
	12	10	8	6	5	4	3½	3
	Wall Thickness of Pipe, Inch							
	0.375	0.365	0.322	0.280	0.258	0.237	0.226	.216
	Weight per Foot of Pipe, Pounds							
	49.56	40.48	28.55	18.97	14.62	10.79	9.11	7.58
Allowable Concentric Loads in Thousands of Pounds								
6	303	246	171	110	83	59	48	38
7	301	243	168	108	81	57	46	36
8	299	241	166	106	78	54	44	34
9	296	238	163	103	76	52	41	31
10	293	235	161	101	73	49	38	28
11	291	232	158	98	71	46	35	25
12	288	229	155	95	68	43	32	22
13	285	226	152	92	65	40	29	19
14	282	223	149	89	61	36	25	16
15	278	220	145	86	58	33	22	14
16	275	216	142	82	55	29	19	12
17	272	213	138	79	51	26	17	11
18	268	209	135	75	47	23	15	10
19	265	205	131	71	43	21	14	9
20	261	201	127	67	39	19	12	
22	254	193	119	59	32	15	10	
24	246	185	111	51	27	13		
25	242	180	106	47	25	12		
26	238	176	102	43	23			
EXTRA STRONG STEEL PIPE								
Effective Length (*KL*), Feet[a]	Nominal Diameter of Pipe, Inches							
	12	10	8	6	5	4	3½	3
	Wall Thickness of Pipe, Inch							
	0.500	0.500	0.500	0.432	0.375	0.337	0.318	.300
	Weight per Foot of Pipe, Pounds							
	65.42	54.74	43.39	28.57	20.78	14.98	12.50	10.25
Allowable Concentric Loads in Thousands of Pounds								
6	400	332	259	166	118	81	66	52
7	397	328	255	162	114	78	63	48
8	394	325	251	159	111	75	59	45
9	390	321	247	155	107	71	55	41
10	387	318	243	151	103	67	51	37
11	383	314	239	146	99	63	47	33
12	379	309	234	142	95	59	43	28
13	375	305	229	137	91	54	38	24
14	371	301	224	132	86	49	33	21
15	367	296	219	127	81	44	29	18
16	363	291	214	122	76	39	25	16
18	353	281	203	111	65	31	20	12
19	349	276	197	105	59	28	18	11
20	344	271	191	99	54	25	16	
21	337	265	185	92	48	22	14	
22	334	260	179	86	44	21		
24	323	248	166	73	37	17		
26	312	236	152	62	32			
28	301	224	137	54	27			

[a] With respect to radius of gyration. The effective length (*KL*) is the actual unbraced length, *L*, in feet, multiplied by the effective length factor (*K*) which is dependent upon the restraint at the ends of the unbraced length and the means available to resist lateral movements. *K* may be determined by referring to the last portion of this table.

Allowable Concentric Loads for Steel Pipe Columns *(Continued)*

DOUBLE-EXTRA STRONG STEEL PIPE					
Effective Length (*KL*), Feet[a]	Nominal Diameter of Pipe, Inches				
	8	6	5	4	3
	Wall Thickness of Pipe, Inch				
	0.875	0.864	0.750	0.674	0.600
	Weight per Foot of Pipe, Pounds				
	72.42	53.16	38.55	27.54	18.58
	Allowable Concentric Loads in Thousands of Pounds				
6	431	306	216	147	91
7	424	299	209	140	84
8	417	292	202	133	77
9	410	284	195	126	69
10	403	275	187	118	60
11	395	266	178	109	51
12	387	257	170	100	43
13	378	247	160	91	37
14	369	237	151	81	32
15	360	227	141	70	28
16	351	216	130	62	24
17	341	205	119	55	22
18	331	193	108	49	
19	321	181	97	44	
20	310	168	87	40	
22	288	142	72	33	
24	264	119	61		
26	240	102	52		
28	213	88	44		

EFFECTIVE LENGTH FACTORS (*K*) FOR VARIOUS COLUMN CONFIGURATIONS						
	(a)	(b)	(c)	(d)	(e)	(f)
Buckled shape of column is shown by dashed line						
Theoretical *K* value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value when ideal conditions are approximated	0.65	0.80	1.2	1.0	2.10	2.0
End condition code		Rotation fixed and translation fixed				
		Rotation free and translation fixed				
		Rotation fixed and translation free				
		Rotation free and translation free				

Load tables are given for 36 ksi yield stress steel. No load values are given below the heavy horizontal lines, because the *Kl*/*r* ratios (where *l* is the actual unbraced length in inches and *r* is the governing radius of gyration in inches) would exceed 200.

Data from "Manual of Steel Construction," 8th ed., 1980, with permission of the American Institute of Steel Construction.

Factor of Safety for Machine Columns: When the conditions of loading and the physical qualities of the material used are accurately known, a factor of safety as low as 1.25 is sometimes used when minimum weight is important. Usually, however, a factor of safety of 2 to 2.5 is applied for steady loads. The factor of safety represents the ratio of the critical load P_{cr} to the working load.

Application of Euler and Johnson Formulas: To determine whether the Euler or Johnson formula is applicable in any particular case, it is necessary to determine the value of the quantity $Q \div r^2$. If $Q \div r^2$ is greater than 2, then the Euler Formula (1) should be used; if $Q \div r^2$ is less than 2, then the J. B. Johnson formula is applicable. Most compression members in machine design are in the range of proportions covered by the Johnson formula. For this reason a good procedure is to design machine elements on the basis of the Johnson formula and then as a check calculate $Q \div r^2$ to determine whether the Johnson formula applies or the Euler formula should have been used.

Example 1, Compression Member Design: A rectangular machine member 24 inches long and $\frac{1}{2} \times 1$ inch in cross-section is to carry a compressive load of 4000 pounds along its axis. What is the factor of safety for this load if the material is machinery steel having a yield point of 40,000 pounds per square inch, the load is steady, and each end of the rod has a ball connection so that $n = 1$?

From Formula (3)

$$Q = \frac{40{,}000 \times 24 \times 24}{1 \times 3.1416 \times 3.1416 \times 30{,}000{,}000} = 0.0778$$

(The values 40,000 and 30,000,000 were obtained from the table *Strength Data for Iron and Steel* on page 476.)

The radius of gyration r for a rectangular section (page 219) is $0.289 \times$ the dimension in the direction of bending. In columns, bending is most apt to occur in the direction in which the section is the weakest, the $\frac{1}{2}$-inch dimension in this example. Hence, least radius of gyration $r = 0.289 \times \frac{1}{2} = 0.145$ inch.

$$\frac{Q}{r^2} = \frac{0.0778}{(0.145)^2} = 3.70$$

which is more than 2 so that the Euler formula will be used.

$$P_{cr} = \frac{s_y A r^2}{Q} = \frac{40{,}000 \times \frac{1}{2} \times 1}{3.70}$$

$$= 5400 \text{ pounds so that the factor of safety is } 5400 \div 4000 = 1.35$$

Example 2, Compression Member Design: In the preceding example, the column formulas were used to check the adequacy of a column of known dimensions. The more usual problem involves determining what the dimensions should be to resist a specified load. For example,:

A 24-inch long bar of rectangular cross-section with width w twice its depth d is to carry a load of 4000 pounds. What must the width and depth be if a factor of safety of 1.35 is to be used?

First determine the critical load P_{cr}:

$$P_{cr} = \text{working load} \times \text{factor of safety}$$

$$= 4000 \times 1.35 = 5400 \text{ pounds}$$

Next determine Q which, as before, will be 0.0778.

Assume Formula (2) applies:

$$P_{cr} = A s_y \left(1 - \frac{Q}{4r^2}\right)$$

$$5400 = w \times d \times 40{,}000\left(1 - \frac{0.0778}{4r^2}\right)$$

$$= 2d^2 \times 40{,}000\left(1 - \frac{0.01945}{r^2}\right)$$

$$\frac{5400}{40{,}000 \times 2} = d^2\left(1 - \frac{0.01945}{r^2}\right)$$

As mentioned in Example 1 the least radius of gyration r of a rectangle is equal to 0.289 times the least dimension, d, in this case. Therefore, substituting for d the value $r \div 0.289$,

$$\frac{5400}{40{,}000 \times 2} = \left(\frac{r}{0.289}\right)^2\left(1 - \frac{0.01945}{r^2}\right)$$

$$\frac{5400 \times 0.289 \times 0.289}{40{,}000 \times 2} = r^2 - 0.01945$$

$$0.005638 = r^2 - 0.01945$$

$$r^2 = 0.0251$$

Checking to determine if $Q \div r^2$ is greater or less than 2,

$$\frac{Q}{r^2} = \frac{0.0778}{0.0251} = 3.1$$

therefore Formula (1) should have been used to determine r and dimensions w and d. Using Formula (1),

$$5400 = \frac{40{,}000 \times 2d^2 \times r^2}{Q} = \frac{40{,}000 \times 2 \times \left(\frac{r}{0.289}\right)^2 r^2}{0.0778}$$

$$r^4 = \frac{5400 \times 0.0778 \times 0.289 \times 0.289}{40{,}000 \times 2}$$

$$d = \frac{0.145}{0.289} = 0.50 \text{ inch}$$

and $w = 2d = 1$ inch as in the previous example.

American Institute of Steel Construction.—For main or secondary compression members with l/r ratios up to 120, safe unit stress = $17{,}000 - 0.485l^2/r^2$. For columns and bracing or other secondary members with l/r ratios above 120,

$$\text{Safe unit stress, psi} = \frac{18{,}000}{1 + l^2/18{,}000r^2}$$ for bracing and secondary members. For main members, safe unit stress, psi $= \dfrac{18{,}000}{1 + l^2/18{,}000r^2} \times \left(1.6 - \dfrac{l/r}{200}\right)$

Pipe Columns: Allowable concentric loads for steel pipe columns based on the above formulas are given in the table on page 264.

PLATES, SHELLS, AND CYLINDERS

Flat Stayed Surfaces.—Large flat areas are often held against pressure by stays distributed at regular intervals over the surface. In boiler work, these stays are usually screwed into the plate and the projecting end riveted over to insure steam tightness. The U.S. Board of Supervising Inspectors and the American Boiler Makers Association rules give the following formula for flat stayed surfaces:

$$P = \frac{C \times t^2}{S^2}$$

in which P = pressure in pounds per square inch

C = a constant, which equals 112 for plates $^7/_{16}$ inch and under; 120, for plates over $^7/_{16}$ inch thick; 140, for plates with stays having a nut and bolt on the inside and outside; and 160, for plates with stays having washers of at least one-half the thickness of the plate, and with a diameter at least one-half of the greatest pitch.

t = thickness of plate in 16ths of an inch (thickness = $^7/_{16}$, t = 7)

S = greatest pitch of stays in inches

Strength and Deflection of Flat Plates.—Generally, the formulas used to determine stresses and deflections in flat plates are based on certain assumptions that can be closely approximated in practice. These assumptions are:

1) the thickness of the plate is not greater than one-quarter the least width of the plate;

2) the greatest deflection when the plate is loaded is less than one-half the plate thickness;

3) the maximum tensile stress resulting from the load does not exceed the elastic limit of the material; and

4) all loads are perpendicular to the plane of the plate.

Plates of ductile materials fail when the maximum stress resulting from deflection under load exceeds the yield strength; for brittle materials, failure occurs when the maximum stress reaches the ultimate tensile strength of the material involved.

Square and Rectangular Flat Plates.—The formulas that follow give the maximum stress and deflection of flat steel plates supported in various ways and subjected to the loading indicated. These formulas are based upon a modulus of elasticity for steel of 30,000,000 pounds per square inch and a value of Poisson's ratio of 0.3. If the formulas for maximum stress, S, are applied without modification to other materials such as cast iron, aluminum, and brass for which the range of Poisson's ratio is about 0.26 to 0.34, the maximum stress calculations will be in error by not more than about 3 per cent. The deflection formulas may also be applied to materials other than steel by substituting in these formulas the appropriate value for E, the modulus of elasticity of the material (see pages 476 and 477). The deflections thus obtained will not be in error by more than about 3 per cent.

In the stress and deflection formulas that follow,

p = uniformly distributed load acting on plate, pounds per square inch

W = total load on plate, pounds; $W = p \times$ area of plate

L = distance between supports (length of plate), inches. For rectangular plates, L = long side, l = short side

t = thickness of plate, inches

S = maximum tensile stress in plate, pounds per square inch

d = maximum deflection of plate, inches

E = modulus of elasticity in tension. E = 30,000,000 pounds per square inch for steel

If metric SI units are used in the formulas, then,

W = total load on plate, newtons

L = **distance between supports (length of plate), millimeters. For rectangular plates, L = long side, l = short side**

t = **thickness of plate, millimeters**

S = **maximum tensile stress in plate, newtons per mm squared**

d = **maximum deflection of plate, mm**

E = **modulus of elasticity, newtons per mm squared**

A) Square flat plate supported at top and bottom of all four edges and a uniformly distributed load over the surface of the plate.

$$S = \frac{0.29W}{t^2} \quad (1) \qquad d = \frac{0.0443WL^2}{Et^3} \quad (2)$$

B) Square flat plate supported at the bottom only of all four edges and a uniformly distributed load over the surface of the plate.

$$S = \frac{0.28W}{t^2} \quad (3) \qquad d = \frac{0.0443WL^2}{Et^3} \quad (4)$$

C) Square flat plate with all edges firmly fixed and a uniformly distributed load over the surface of the plate.

$$S = \frac{0.31W}{t^2} \quad (5) \qquad d = \frac{0.0138WL^2}{Et^3} \quad (6)$$

D) Square flat plate with all edges firmly fixed and a uniform load over small circular area at the center. In Equations (7) and (9), r_0 = radius of area to which load is applied. If $r_0 < 1.7t$, use r_s where $r_s = \sqrt{1.6r_0{}^2 + t^2} - 0.675t$.

$$S = \frac{0.62W}{t^2}\log_e\left(\frac{L}{2r_0}\right) \quad (7) \qquad d = \frac{0.0568WL^2}{Et^3} \quad (8)$$

E) Square flat plate with all edges supported above and below, or below only, and a concentrated load at the center. (See Case 4, above, for definition of r_0).

$$S = \frac{0.62W}{t^2}\left[\log_e\left(\frac{L}{2r_0}\right) + 0.577\right] \quad (9) \qquad d = \frac{0.1266WL^2}{Et^3} \quad (10)$$

F) Rectangular plate with all edges supported at top and bottom and a uniformly distributed load over the surface of the plate.

$$S = \frac{0.75W}{t^2\left(\frac{L}{l} + 1.61\frac{l^2}{L^2}\right)} \quad (11) \qquad d = \frac{0.1422W}{Et^3\left(\frac{L}{l^3} + \frac{2.21}{L^2}\right)} \quad (12)$$

G) Rectangular plate with all edges fixed and a uniformly distributed load over the surface of the plate.

$$S = \frac{0.5W}{t^2\left(\frac{L}{l} + \frac{0.623l^5}{L^5}\right)} \quad (13) \qquad d = \frac{0.0284W}{Et^3\left(\frac{L}{l^3} + \frac{1.056l^2}{L^4}\right)} \quad (14)$$

Circular Flat Plates.—In the following formulas, R = radius of plate to supporting edge in inches; W = total load in pounds; and other symbols are the same as used for square and rectangular plates.

If metric SI units are used, R = radius of plate to supporting edge in millimeters, and the values of other symbols are the same as those used for square and rectangular plates.

A) Edge supported around the circumference and a uniformly distributed load over the surface of the plate.

$$S = \frac{0.39W}{t^2} \quad (1) \qquad d = \frac{0.221WR^2}{Et^3} \quad (2)$$

B) Edge fixed around circumference and a uniformly distributed load over the surface of the plate.

$$S = \frac{0.24W}{t^2} \quad (3) \qquad d = \frac{0.0543WR^2}{Et^3} \quad (4)$$

C) Edge supported around the circumference and a concentrated load at the center.

$$S = \frac{0.48W}{t^2}\left[1 + 1.3\log_e\frac{R}{0.325t} - 0.0185\frac{t^2}{R^2}\right] \quad (5) \qquad d = \frac{0.55WR^2}{Et^3} \quad (6)$$

D) Edge fixed around circumference and a concentrated load at the center.

$$S = \frac{0.62W}{t^2}\left[\log_e\frac{R}{0.325t} + 0.0264\frac{t^2}{R^2}\right] \quad (7) \qquad d = \frac{0.22WR^2}{Et^3} \quad (8)$$

Strength of Cylinders Subjected to Internal Pressure.—In designing a cylinder to withstand internal pressure, the choice of formula to be used depends on 1) the kind of material of which the cylinder is made (whether brittle or ductile); 2) the construction of the cylinder ends (whether open or closed); and 3) whether the cylinder is classed as a thin- or a thick-walled cylinder.

A cylinder is considered to be thin-walled when the ratio of wall thickness to inside diameter is 0.1 or less and thick-walled when this ratio is greater than 0.1. Materials such as cast iron, hard steel, cast aluminum are considered to be brittle materials; low-carbon steel, brass, bronze, etc. are considered to be ductile.

In the formulas that follow, p = internal pressure, pounds per square inch; D = inside diameter of cylinder, inches; t = wall thickness of cylinder, inches; μ = Poisson's ratio, = 0.3 for steel, 0.26 for cast iron, 0.34 for aluminum and brass; and S = allowable tensile stress, pounds per square inch.

Metric SI units can be used in Formulas (1), (3), (4), and (5), where p = internal pressure in newtons per square millimeter; D = inside diameter of cylinder, millimeters; t = wall thickness, mm; μ = Poisson's ratio, = 0.3 for steel, 0.26 for cast iron, and 0.34 for aluminum and brass; and S = allowable tensile stress, N/mm². For the use of metric SI units in Formula (2), see below.

Thin-walled cylinders:

$$t = \frac{Dp}{2S} \quad (1)$$

For low-pressure cylinders of cast iron such as are used for certain engine and press applications, a formula in common use is

$$t = \frac{Dp}{2500} + 0.3 \quad (2)$$

This formula is based on allowable stress of 1250 pounds per square inch and will give a wall thickness 0.3 inch greater than Formula (1) to allow for variations in metal thickness that may result from the casting process.

If metric SI units are used in Formula (2), t = cylinder wall thickness in millimeters; D = inside diameter of cylinder, mm; and the allowable stress is in newtons per square

millimeter. The value of 0.3 inches additional wall thickness is 7.62 mm, and the next highest number in preferred metric basic sizes is 8 mm.

Thick-walled cylinders of brittle material, ends open or closed: Lamé's equation is used when cylinders of this type are subjected to internal pressure.

$$t = \frac{D}{2}\left[\sqrt{\frac{S+p}{S-p}} - 1\right] \quad (3)$$

The table *Ratio of Outside Radius to Inside Radius, Thick CylindersAllowable Stress in Metal per Sq. In. of Section* on page 272 is for convenience in calculating the dimensions of cylinders under high internal pressure without the use of Formula (3).

Example, Use of the Table: Assume that a cylinder of 10 inches inside diameter is to withstand a pressure of 2500 pounds per square inch; the material is cast iron and the allowable stress is 6000 pounds per square inch. To solve the problem, locate the allowable stress per square inch in the left-hand column of the table and the working pressure at the top of the columns. Then find the ratio between the outside and inside radii in the body of the table. In this example, the ratio is 1.558, and hence the outside diameter of the cylinder should be 10 × 1.558, or about 15⅝ inches. The thickness of the cylinder wall will therefore be (15.558 − 10)/2 = 2.779 inches.

Unless very high-grade material is used and sound castings assured, cast iron should not be used for pressures exceeding 2000 pounds per square inch. It is well to leave more metal in the bottom of a hydraulic cylinder than is indicated by the results of calculations, because a hole of some size must be cored in the bottom to permit the entrance of a boring bar when finishing the cylinder, and when this hole is subsequently tapped and plugged it often gives trouble if there is too little thickness.

For steady or gradually applied stresses, the maximum allowable fiber stress S may be assumed to be from 3500 to 4000 pounds per square inch for cast iron; from 6000 to 7000 pounds per square inch for brass; and 12,000 pounds per square inch for steel castings. For intermittent stresses, such as in cylinders for steam and hydraulic work, 3000 pounds per square inch for cast iron; 5000 pounds per square inch for brass; and 10,000 pounds per square inch for steel castings, is ordinarily used. These values give ample factors of safety.

***Note:* In metric SI units, 1000 pounds per square inch equals 6.895 newtons per square millimeter.**

Thick-walled cylinders of ductile material, closed ends: Clavarino's equation is used:

$$t = \frac{D}{2}\left[\sqrt{\frac{S+(1-2\mu)p}{S-(1+\mu)p}} - 1\right] \quad (4)$$

Spherical Shells Subjected to Internal Pressure.—Let:

D = internal diameter of shell in inches
p = internal pressure in pounds per square inch
S = safe tensile stress per square inch
t = thickness of metal in the shell, in inches. Then: $t = \frac{pD}{4S}$

This formula also applies to hemi-spherical shells, such as the hemi-spherical head of a cylindrical container subjected to internal pressure, etc.

If metric SI units are used, then:

D = **internal diameter of shell in millimeters**
p = **internal pressure in newtons per square millimeter**
S = **safe tensile stress in newtons per square millimeter**
t = **thickness of metal in the shell in millimeters**

Meters can be used in the formula in place of millimeters, providing the treatment is consistent throughout.

Ratio of Outside Radius to Inside Radius, Thick Cylinders

Allowable Stress in Metal per Sq. In. of Section	Working Pressure in Cylinder, Pounds per Square Inch						
	1000	2000	3000	4000	5000	6000	7000
2,000	1.732	…	…	…	…	…	…
2,500	1.527	…	…	…	…	…	…
3,000	1.414	2.236	…	…	…	…	…
3,500	1.341	1.915	…	…	…	…	…
4,000	1.291	1.732	2.645	…	…	…	…
4,500	1.253	1.612	2.236	…	…	…	…
5000	1.224	1.527	2.000	3.000	…	…	…
5,500	1.201	1.464	1.844	2.516	…	…	…
6,000	1.183	1.414	1.732	2.236	3.316	…	…
6,500	…	1.374	1.647	2.049	2.768	…	…
7,000	…	1.341	1.581	1.914	2.449	3.605	…
7,500	…	1.314	1.527	1.813	2.236	3.000	…
8,000	…	1.291	1.483	1.732	2.081	2.645	3.872
8,500	…	1.271	1.446	1.666	1.963	2.408	3.214
9,000	…	1.253	1.414	1.612	1.871	2.236	2.828
9,500	…	1.235	1.386	1.566	1.795	2.104	2.569
10,000	…	1.224	1.362	1.527	1.732	2.000	2.380
10,500	…	1.212	1.341	1.493	1.678	1.915	2.236
11,000	…	1.201	1.322	1.464	1.633	1.844	2.121
11,500	…	1.193	1.306	1.437	1.593	1.784	2.027
12,000	…	1.183	1.291	1.414	1.558	1.732	1.949
12,500	…	…	1.277	1.393	1.527	1.687	1.878
13,000	…	…	1.264	1.374	1.500	1.647	1.825
13,500	…	…	1.253	1.357	1.475	1.612	1.775
14,000	…	…	1.243	1.341	1.453	1.581	1.732
14,500	…	…	1.233	1.327	1.432	1.553	1.693
15,000	…	…	1.224	1.314	1.414	1.527	1.658
16,000	…	…	1.209	1.291	1.381	1.483	1.599

Thick-walled cylinders of ductile material; open ends: Birnie's equation is used:

$$t = \frac{D}{2}\left[\sqrt{\frac{S + (1 - \mu)p}{S - (1 + \mu)p}} - 1\right] \qquad (5)$$

Example: Find the thickness of metal required in the hemi-spherical end of a cylindrical vessel, 2 feet in diameter, subjected to an internal pressure of 500 pounds per square inch. The material is mild steel and a tensile stress of 10,000 pounds per square inch is allowable.

$$t = \frac{500 \times 2 \times 12}{4 \times 10{,}000} = 0.3 \text{ inch}$$

A similar example using metric SI units is as follows: find the thickness of metal required in the hemi-spherical end of a cylindrical vessel, 750 mm in diameter, subjected to an internal pressure of 3 newtons/mm². The material is mild steel and a tensile stress of 70 newtons/mm² is allowable.

$$t = \frac{3 \times 750}{4 \times 70} = 8.04 \text{ mm}$$

If the radius of curvature of the domed head of a boiler or container subjected to internal pressure is made equal to the diameter of the boiler, the thickness of the cylindrical shell and of the spherical head should be made the same. For example, if a boiler is 3 feet in diameter, the radius of curvature of its head should also be 3 feet, if material of the same thickness is to be used and the stresses are to be equal in both the head and cylindrical portion.

Collapsing Pressure of Cylinders and Tubes Subjected to External Pressures.—The following formulas may be used for finding the collapsing pressures of lap-welded Bessemer steel tubes:

$$P = 86{,}670\frac{t}{D} - 1386 \quad (1)$$

$$P = 50{,}210{,}000\left(\frac{t}{D}\right)^3 \quad (2)$$

in which P = collapsing pressure in pounds per square inch; D = outside diameter of tube or cylinder in inches; t = thickness of wall in inches.

Formula (1) is for values of P greater than 580 pounds per square inch, and Formula (2) is for values of P less than 580 pounds per square inch. These formulas are substantially correct for all lengths of pipe greater than six diameters between transverse joints that tend to hold the pipe to a circular form. The pressure P found is the actual collapsing pressure, and a suitable factor of safety must be used. Ordinarily, a factor of safety of 5 is sufficient. In cases where there are repeated fluctuations of the pressure, vibration, shocks and other stresses, a factor of safety of from 6 to 12 should be used.

If metric SI units are used the formulas are:

$$P = 597.6\frac{t}{D} - 9.556 \quad (3)$$

$$P = 346{,}200\left(\frac{t}{D}\right)^3 \quad (4)$$

where P = collapsing pressure in newtons per square millimeter; D = outside diameter of tube or cylinder in millimeters; and t = thickness of wall in millimeters. Formula (3) is for values of P greater than 4 N/mm², and Formula (4) is for values of P less than 4 N/mm².

The table "Tubes Subjected to External Pressure" is based upon the requirements of the Steam Boat Inspection Service of the Department of Commerce and Labor and gives the permissible working pressures and corresponding minimum wall thickness for long, plain, lap-welded and seamless steel flues subjected to external pressure only. The table thicknesses have been calculated from the formula:

$$t = \frac{[(F \times p) + 1386]D}{86{,}670}$$

in which D = outside diameter of flue or tube in inches; t = thickness of wall in inches; p = working pressure in pounds per square inch; F = factor of safety. The formula is applicable to working pressures greater than 100 pounds per square inch, to outside diameters from 7 to 18 inches, and to temperatures less than 650°F.

The preceding Formulas (1) and (2) were determined by Prof. R. T. Stewart, Dean of the Mechanical Engineering Department of the University of Pittsburgh, in a series of experiments carried out at the plant of the National Tube Co., McKeesport, Pa.

The apparent fiber stress under which the different tubes failed varied from about 7000 pounds per square inch for the relatively thinnest to 35,000 pounds per square inch for the relatively thickest walls. The average yield point of the material tested was 37,000 pounds and the tensile strength 58,000 pounds per square inch, so it is evident that the strength of a tube subjected to external fluid collapsing pressure is not dependent alone upon the elastic limit or ultimate strength of the material from which it is made.

Tubes Subjected to External Pressure

Outside Diameter of Tube, Inches	Working Pressure in Pounds per Square Inch						
	100	120	140	160	180	200	220
	Thickness of Tube in Inches. Safety Factor, 5						
7	0.152	0.160	0.168	0.177	0.185	0.193	0.201
8	0.174	0.183	0.193	0.202	0.211	0.220	0.229
9	0.196	0.206	0.217	0.227	0.237	0.248	0.258
10	0.218	0.229	0.241	0.252	0.264	0.275	0.287
11	0.239	0.252	0.265	0.277	0.290	0.303	0.316
12	0.261	0.275	0.289	0.303	0.317	0.330	0.344
13	0.283	0.298	0.313	0.328	0.343	0.358	0.373
14	0.301	0.320	0.337	0.353	0.369	0.385	0.402
15	0.323	0.343	0.361	0.378	0.396	0.413	0.430
16	0.344	0.366	0.385	0.404	0.422	0.440	0.459
16	0.366	0.389	0.409	0.429	0.448	0.468	0.488
18	0.387	0.412	0.433	0.454	0.475	0.496	0.516

Dimensions and Maximum Allowable Pressure of Tubes Subjected to External Pressure

Outside Diam., Inches	Thickness of Material, Inches	Maximum Pressure Allowed, psi	Outside Diam., Inches	Thickness of Material, Inches	Maximum Pressure Allowed, psi	Outside Diam., Inches	Thickness of Material, Inches	Maximum Pressure Allowed, psi
2	0.095	427	3	0.109	327	4	0.134	303
2¼	0.095	380	3¼	0.120	332	4½	0.134	238
2½	0.109	392	3½	0.120	308	5	0.148	235
2¾	0.109	356	3¾	0.120	282	6	0.165	199

SHAFTS

Shaft Calculations

Torsional Strength of Shafting.—In the formulas that follow,

α = angular deflection of shaft in degrees

c = distance from center of gravity to extreme fiber

D = diameter of shaft in inches

G = torsional modulus of elasticity = 11,500,000 pounds per square inch for steel

J = polar moment of inertia of shaft cross-section (see table)

l = length of shaft in inches

N = angular velocity of shaft in revolutions per minute

P = power transmitted in horsepower

S_s = allowable torsional shearing stress in pounds per square inch

T = torsional or twisting moment in inch-pounds

Z_p = polar section modulus (see table page 278)

The allowable twisting moment for a shaft of any cross-section such as circular, square, etc., is:

$$T = S_s \times Z_p \tag{1}$$

For a shaft delivering P horsepower at N revolutions per minute the twisting moment T being transmitted is:

$$T = \frac{63{,}000P}{N} \tag{2}$$

The twisting moment T as determined by this formula should be less than the value determined by using Formula (1) if the maximum allowable stress S_s is not to be exceeded.

The diameter of a solid circular shaft required to transmit a given torque T is:

$$D = \sqrt[3]{\frac{5.1T}{S_s}} \quad (3a) \qquad \text{or} \qquad D = \sqrt[3]{\frac{321{,}000P}{NS_s}} \quad (3b)$$

The allowable stresses that are generally used in practice are: 4000 pounds per square inch for main power-transmitting shafts; 6000 pounds per square inch for lineshafts carrying pulleys; and 8500 pounds per square inch for small, short shafts, countershafts, etc. Using these allowable stresses, the horsepower P transmitted by a shaft of diameter D, or the diameter D of a shaft to transmit a given horsepower P may be determined from the following formulas:

For main power-transmitting shafts:

$$P = \frac{D^3N}{80} \quad (4a) \qquad \text{or} \qquad D = \sqrt[3]{\frac{80P}{N}} \quad (4b)$$

For lineshafts carrying pulleys:

$$P = \frac{D^3N}{53.5} \quad (5a) \qquad \text{or} \qquad D = \sqrt[3]{\frac{53.5P}{N}} \quad (5b)$$

For small, short shafts:

$$P = \frac{D^3 N}{38} \quad (6a) \qquad \text{or} \qquad D = \sqrt[3]{\frac{38P}{N}} \quad (6b)$$

Shafts that are subjected to shocks, such as sudden starting and stopping, should be given a greater factor of safety resulting in the use of lower allowable stresses than those just mentioned.

Example: What should be the diameter of a lineshaft to transmit 10 horsepower if the shaft is to make 150 revolutions per minute? Using Formula (5b),

$$D = \sqrt[3]{\frac{53.5 \times 10}{150}} = 1.53 \text{ or, say, } 1\tfrac{9}{16} \text{ inches}$$

Example: What horsepower would be transmitted by a short shaft, 2 inches in diameter, carrying two pulleys close to the bearings, if the shaft makes 300 revolutions per minute? Using Formula (6a),

$$P = \frac{2^3 \times 300}{38} = 63 \text{ horsepower}$$

Torsional Strength of Shafting, Calculations in Metric SI Units.—The allowable twisting moment for a shaft of any cross-section such as circular, square, etc., can be calculated from:

$$T = S_s \times Z_p \quad (1)$$

where T = torsional or twisting moment in newton-millimeters; S_s = allowable torsional shearing stress in newtons per square millimeter; and Z_p = polar section modulus in millimeters^3.

For a shaft delivering power of P kilowatts at N revolutions per minute, the twisting moment T being transmitted is:

$$T = \frac{9.55 \times 10^6 P}{N} \quad (2) \qquad \text{or} \qquad T = \frac{10^6 P}{\omega} \quad (2a)$$

where T is in newton-millimeters, and ω = angular velocity in radians per second.

The diameter D of a solid circular shaft required to transmit a given torque T is:

$$D = \sqrt[3]{\frac{5.1T}{S_s}} \quad (3a) \qquad \text{or} \qquad D = \sqrt[3]{\frac{48.7 \times 10^6 P}{NS_s}} \quad (3b)$$

or

$$D = \sqrt[3]{\frac{5.1 \times 10^6 P}{\omega S_s}} \quad (3c)$$

where D is in millimeters; T is in newton-millimeters; P is power in kilowatts; N = revolutions per minute; S_s = allowable torsional shearing stress in newtons per square millimeter, and ω = angular velocity in radians per second.

If 28 newtons/mm^2 and 59 newtons/mm^2 are taken as the generally allowed stresses for main power-transmitting shafts and small short shafts, respectively, then using these allowable stresses, the power P transmitted by a shaft of diameter D, or the diameter D of a shaft to transmit a given power P may be determined from the following formulas:

For main power-transmitting shafts:

$$P = \frac{D^3 N}{1.77 \times 10^6} \quad (4a) \qquad \text{or} \qquad D = \sqrt[3]{\frac{1.77 \times 10^6 P}{N}} \quad (4b)$$

For small, short shafts:

$$P = \frac{D^3 N}{0.83 \times 10^6} \quad (5a) \qquad \text{or} \qquad D = \sqrt[3]{\frac{0.83 \times 10^6 P}{N}} \quad (5b)$$

where P is in kilowatts, D is in millimeters, and N = revolutions per minute.

Example: **What should be the diameter of a power-transmitting shaft to transmit 150 kW at 500 rpm?**

$$D = \sqrt[3]{\frac{1.77 \times 10^6 \times 150}{500}} = 81 \text{ millimeters}$$

Example: **What power would a short shaft, 50 millimeters in diameter, transmit at 400 rpm?**

$$P = \frac{50^3 \times 400}{0.83 \times 10^6} = 60 \text{ kilowatts}$$

Polar Moment of Inertia and Section Modulus.—The *polar moment of inertia*, J, of a cross-section with respect to a polar axis, that is, an axis at right angles to the plane of the cross-section, is defined as the moment of inertia of the cross-section with respect to the point of intersection of the axis and the plane. The polar moment of inertia may be found by taking the sum of the moments of inertia about two perpendicular axes lying in the plane of the cross-section and passing through this point. Thus, for example, the polar moment of inertia of a circular or a square area with respect to a polar axis through the center of gravity is equal to two times the moment of inertia with respect to an axis lying in the plane of the cross-section and passing through the center of gravity.

The polar moment of inertia with respect to a polar axis through the center of gravity is required for problems involving the torsional strength of shafts since this axis is usually the axis about which twisting of the shaft takes place.

The *polar section modulus* (also called section modulus of torsion), Z_p, for *circular* sections may be found by dividing the polar moment of inertia, J, by the distance c from the center of gravity to the most remote fiber. This method may be used to find the *approximate* value of the polar section modulus of sections that are *nearly* round. For other than circular cross-sections, however, the polar section modulus *does not* equal the polar moment of inertia divided by the distance c.

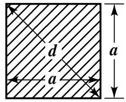
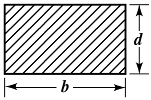
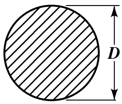
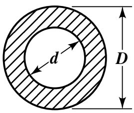
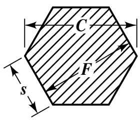
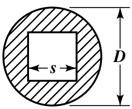
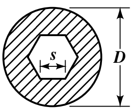
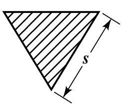
The accompanying table gives formulas for the polar section modulus for several different cross-sections. The polar section modulus multiplied by the allowable torsional shearing stress gives the allowable twisting moment to which a shaft may be subjected, see Formula (1).

Torsional Deflection of Circular Shafts.—Shafting must often be proportioned not only to provide the strength required to transmit a given torque, but also to prevent torsional deflection (twisting) through a greater angle than has been found satisfactory for a given type of service.

For a solid circular shaft the torsional deflection in degrees is given by:

$$\alpha = \frac{584 Tl}{D^4 G} \quad (6)$$

Polar Moment of Inertia and Polar Section Modulus

Section	Polar Moment of Inertia J	Polar Section Modulus Z_p
	$\frac{a^4}{6} = 0.1667a^4$	$0.208a^3 = 0.074d^3$
	$\frac{bd(b^2 + d^2)}{12}$	$\frac{bd^2}{3 + 1.8\frac{d}{b}}$ (*d* is the shorter side)
	$\frac{\pi D^4}{32} = 0.098D^4$ (see also footnote, page 229)	$\frac{\pi D^3}{16} = 0.196D^3$ (see also footnote, page 229)
	$\frac{\pi}{32}(D^4 - d^4) = 0.098(D^4 - d^4)$	$\frac{\pi}{16}\left(\frac{D^4 - d^4}{D}\right) = 0.196\left(\frac{D^4 - d^4}{D}\right)$
	$\frac{5\sqrt{3}}{8}s^4 = 1.0825s^4 = 0.12F^4$	$0.20F^3$
	$\frac{\pi D^4}{32} - \frac{s^4}{6} = 0.098D^4 - 0.167s^4$	$\frac{\pi D^3}{16} - \frac{s^4}{3D} = 0.196D^3 - 0.333\frac{s^4}{D}$
	$\frac{\pi D^4}{32} - \frac{5\sqrt{3}}{8}s^4 = 0.098D^4 - 1.0825s^4$	$\frac{\pi D^3}{16} - \frac{5\sqrt{3}}{4D}s^4 = 0.196D^3 - 2.165\frac{s^4}{D}$
	$\frac{\sqrt{3}}{48}s^4 = 0.036s^4$	$\frac{s^3}{20} = 0.05s^3$

Example: Find the torsional deflection for a solid steel shaft 4 inches in diameter and 48 inches long, subjected to a twisting moment of 24,000 inch-pounds. By Formula (6),

$$\alpha = \frac{584 \times 24{,}000 \times 48}{4^4 \times 11{,}500{,}000} = 0.23 \text{ degree}$$

Formula (6) can be used with metric SI units, where α = angular deflection of shaft in degrees; T = torsional moment in newton-millimeters; l = length of shaft in millimeters; D = diameter of shaft in millimeters; and G = torsional modulus of elasticity in newtons per square millimeter.

Example: **Find the torsional deflection of a solid steel shaft, 100 mm in diameter and 1300 mm long, subjected to a twisting moment of 3×10^6 newton-millimeters. The torsional modulus of elasticity is 80,000 newtons/mm ^{2}. By Formula (6)**

$$\alpha = \frac{584 \times 3 \times 10^6 \times 1300}{100^4 \times 80{,}000} = 0.285 \text{ degree}$$

The diameter of a shaft that is to have a maximum torsional deflection α is given by:

$$D = 4.9 \times \sqrt[4]{\frac{Tl}{G\alpha}} \qquad (7)$$

Formula (7) can be used with metric SI units, where D = diameter of shaft in millimeters; T = torsional moment in newton-millimeters; l = length of shaft in millimeters; G = torsional modulus of elasticity in newtons per square millimeter; and α = angular deflection of shaft in degrees.

According to some authorities, the allowable twist in steel transmission shafting should not exceed 0.08 degree per foot length of the shaft. The diameter D of a shaft that will permit a maximum angular deflection of 0.08 degree per foot of length for a given torque T or for a given horsepower P can be determined from the formulas:

$$D = 0.29\sqrt[4]{T} \qquad (8a) \qquad \text{or} \qquad D = 4.6 \times \sqrt[4]{\frac{P}{N}} \qquad (8b)$$

Using metric SI units and assuming an allowable twist in steel transmission shafting of 0.26 degree per meter length, Formulas (8a) and (8b) become:

$$D = 2.26\sqrt[4]{T} \qquad \text{or} \qquad D = 125.7 \times \sqrt[4]{\frac{P}{N}}$$

where D = diameter of shaft in millimeters; T = torsional moment in newton-millimeters; P = power in kilowatts; and N = revolutions per minute.

Another rule that has been generally used in mill practice limits the deflection to 1 degree in a length equal to 20 times the shaft diameter. For a given torque or horsepower, the diameter of a shaft having this maximum deflection is given by:

$$D = 0.1\sqrt[3]{T} \qquad (9a) \qquad \text{or} \qquad D = 4.0 \times \sqrt[3]{\frac{P}{N}} \qquad (9b)$$

Example: Find the diameter of a steel lineshaft to transmit 10 horsepower at 150 revolutions per minute with a torsional deflection not exceeding 0.08 degree per foot of length. By Formula (8b),

$$D = 4.6 \times \sqrt[4]{\frac{10}{150}} = 2.35 \text{ inches}$$

This diameter is larger than that obtained for the same horsepower and rpm in the example given for Formula (5b) in which the diameter was calculated for strength considerations only. The usual procedure in the design of shafting which is to have a specified maximum angular deflection is to compute the diameter first by means of Formulas (7), (8a), (8b), (9a), or (9b) and then by means of Formulas (3a), (3b), (4b), (5b), or (6b), using the larger of the two diameters thus found.

Linear Deflection of Shafting.—For steel lineshafting, it is considered good practice to limit the linear deflection to a maximum of 0.010 inch per foot of length. The maximum distance in feet between bearings, for average conditions, in order to avoid excessive linear deflection, is determined by the formulas:

$$L = 8.95\sqrt[3]{D^2} \quad \text{for shafting subject to no bending action except it's own weight}$$

$$L = 5.2\sqrt[3]{D^2} \quad \text{for shafting subject to bending action of pulleys, etc.}$$

in which D = diameter of shaft in inches and L = maximum distance between bearings in feet. Pulleys should be placed as close to the bearings as possible.

In general, shafting up to three inches in diameter is almost always made from cold-rolled steel. This shafting is true and straight and needs no turning, but if keyways are cut in the shaft, it must usually be straightened afterwards, as the cutting of the keyways relieves the tension on the surface of the shaft produced by the cold-rolling process. Sizes of shafting from three to five inches in diameter may be either cold-rolled or turned, more frequently the latter, and all larger sizes of shafting must be turned because cold-rolled shafting is not available in diameters larger than 5 in.

Diameters of Finished Shafting (former American Standard ASA B17.1)

Diameters, Inches		Minus Tolerances, Inches[a]	Diameters, Inches		Minus Tolerances, Inches[a]	Diameters, Inches		Minus Tolerances, Inches[a]
Transmission Shafting	Machinery Shafting		Transmission Shafting	Machinery Shafting		Transmission Shafting	Machinery Shafting	
	1/2	0.002		1 13/16	0.003		3 3/4	0.004
	9/16	0.002		1 7/8	0.003		3 7/8	0.004
	5/8	0.002	1 15/16	1 15/16	0.003	3 15/16	4	0.004
	11/16	0.002		2	0.003		4 1/4	0.005
	3/4	0.002		2 1/16	0.004	4 7/16	4 1/2	0.005
	13/16	0.002		2 1/8	0.004		4 3/4	0.005
	7/8	0.002	2 3/16	2 3/16	0.004	4 15/16	5	0.005
15/16	15/16	0.002		2 1/4	0.004		5 1/4	0.005
	1	0.002		2 5/16	0.004	5 7/16	5 1/2	0.005
	1 1/16	0.003		2 3/8	0.004		5 3/4	0.005
	1 1/8	0.003	2 7/16	2 7/16	0.004	5 15/16	6	0.005
1 3/16	1 3/16	0.003		2 1/2	0.004		6 1/4	0.006
	1 1/4	0.003		2 5/8	0.004	6 1/2	6 1/2	0.006
	1 5/16	0.003		2 3/4	0.004		6 3/4	0.006
	1 3/8	0.003	2 15/16	2 7/8	0.004	7	7	0.006
1 7/16	1 7/16	0.003		3	0.004		7 1/4	0.006
	1 1/2	0.003		3 1/8	0.004	7 1/2	7 1/2	0.006
	1 9/16	0.003		3 1/4	0.004		7 3/4	0.006
	1 5/8	0.003		3 3/8	0.004	8	8	0.006
1 11/16	1 11/16	0.003	3 7/16	3 1/2	0.004	…	…	…
	1 3/4	0.003		3 5/8	0.004	…	…	…

[a] *Note*:—These tolerances are *negative* or minus and represent the maximum allowable variation *below* the exact nominal size. For instance the maximum diameter of the 1 15/16 inch shaft is 1.938 inch and its minimum allowable diameter is 1.935 inch. Stock lengths of finished transmission shafting shall be: 16, 20 and 24 feet.

Design of Transmission Shafting.—The following guidelines for the design of shafting for transmitting a given amount of power under various conditions of loading are based upon formulas given in the former American Standard ASA B17c Code for the Design of Transmission Shafting. These formulas are based on the *maximum-shear theory* of failure which assumes that the elastic limit of a *ductile* ferrous material in shear is practically one-half its elastic limit in tension. This theory agrees, very nearly, with the results of tests on ductile materials and has gained wide acceptance in practice.

The formulas given apply in all shaft designs including shafts for special machinery. The limitation of these formulas is that they provide only for the strength of shafting and are not concerned with the torsional or lineal deformations which may, in shafts used in machine design, be the controlling factor (see *Torsional Deflection of Circular Shafts* and *Linear Deflection of Shafting* for deflection considerations). In the formulas that follow,

$B = \sqrt[3]{1 \div (1 - K^4)}$ (see Table 3)
D = outside diameter of shaft in inches
D_1 = inside diameter of a hollow shaft in inches
K_m = shock and fatigue factor to be applied in every case to the computed bending moment (see Table 1)
K_t = combined shock and fatigue factor to be applied in every case to the computed torsional moment (see Table 1)
M = maximum bending moment in inch-pounds
N = revolutions per minute
P = maximum power to be transmitted by the shaft in horsepower
p_t = maximum allowable shearing stress under combined loading conditions in pounds per square inch (see Table 2)
S = maximum allowable flexural (bending) stress, in either tension or compression in pounds per square inch (see Table 2)
S_s = maximum allowable torsional shearing stress in pounds per square inch (see Table 2)
T = maximum torsional moment in inch-pounds
V = maximum transverse shearing load in pounds

For shafts subjected to pure torsional loads only,

$$D = B\sqrt[3]{\frac{5.1K_tT}{S_s}} \quad (1a) \qquad \text{or} \qquad D = B\sqrt[3]{\frac{321{,}000K_tP}{S_sN}} \quad (1b)$$

For stationary shafts subjected to bending only,

$$D = B\sqrt[3]{\frac{10.2K_mM}{S}} \quad (2)$$

For shafts subjected to combined torsion and bending,

$$D = B\sqrt[3]{\frac{5.1}{p_t}\sqrt{(K_mM)^2 + (K_tT)^2}} \quad (3a)$$

or

$$D = B\sqrt[3]{\frac{5.1}{p_t}\sqrt{(K_mM)^2 + \left(\frac{63{,}000K_tP}{N}\right)^2}} \quad (3b)$$

Formulas (1a) to (3b) may be used for solid shafts or for hollow shafts. For solid shafts the factor B is equal to 1, whereas for hollow shafts the value of B depends on the value of K which, in turn, depends on the ratio of the inside diameter of the shaft to the outside diameter ($D_1 \div D = K$). Table 3 gives values of B corresponding to various values of K.

For short solid shafts subjected only to heavy transverse shear, the diameter of shaft required is:

$$D = \sqrt{\frac{1.7V}{S_s}} \qquad (4)$$

Formulas (1a), (2), (3a) and (4), can be used unchanged with metric SI units. Formula (1b) becomes:

$$D = B\sqrt[3]{\frac{48.7K_tP}{S_sN}}$$ **and Formula (3b) becomes:**

$$D = B\sqrt[3]{\frac{5.1}{p_t}\sqrt{(K_mM)^2 + \left(\frac{9.55K_tP}{N}\right)^2}}$$

Throughout the formulas, D = outside diameter of shaft in millimeters; T = maximum torsional moment in newton-millimeters; S_s = maximum allowable torsional shearing stress in newtons per millimeter squared (see Table 2); P = maximum power to be transmitted in milliwatts; N = revolutions per minute; M = maximum bending moment in newton-millimeters; S = maximum allowable flexural (bending) stress, either in tension or compression in newtons per millimeter squared (see Table 2); p_t = maximum allowable shearing stress under combined loading conditions in newtons per millimeter squared; and V = maximum transverse shearing load in kilograms. The factors K_m, K_t, and B are unchanged, and D_1 = the inside diameter of a hollow shaft in millimeters.

Table 1. Recommended Values of the Combined Shock and Fatigue Factors for Various Types of Load

Type of Load	Stationary Shafts		Rotating Shafts	
	K_m	K_t	K_m	K_t
Gradually applied and steady	1.0	1.0	1.5	1.0
Suddenly applied, minor shocks only	1.5–2.0	1.5–2.0	1.5–2.0	1.0–1.5
Suddenly applied, heavy shocks	…	…	2.0–3.0	1.5–3.0

Table 2. Recommended Maximum Allowable Working Stresses for Shafts Under Various Types of Load

Material	Type of Load		
	Simple Bending	Pure Torsion	Combined Stress
"Commercial Steel" shafting without keyways	S = 16,000	S_s = 8000	p_t = 8000
"Commercial Steel" shafting with keyways	S = 12,000	S_s = 6000	p_t = 6000
Steel purchased under definite physical specs.	(See note [a])	(See note [b])	(See note [b])

[a] S = 60 per cent of the elastic limit in tension but not more than 36 per cent of the ultimate tensile strength.

[b] S_s and p_t = 30 per cent of the elastic limit in tension but not more than 18 per cent of the ultimate tensile strength.

If the values in the Table are converted to metric SI units, note that 1000 pounds per square inch = 6.895 newtons per square millimeter.

Table 3. Values of the Factor *B* Corresponding to Various Values of *K* for Hollow Shafts

$K = \frac{D_1}{D} =$	0.95	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.50
$B = \sqrt[3]{1 \div (1 - K^4)}$	1.75	1.43	1.28	1.19	1.14	1.10	1.07	1.05	1.03	1.02

For solid shafts, $B = 1$ since $K = 0$. $[B = \sqrt[3]{1 \div (1 - K^4)} = \sqrt[3]{1 \div (1 - 0)} = 1]$

Effect of Keyways on Shaft Strength.—Keyways cut into a shaft reduce its load carrying ability, particularly when impact loads or stress reversals are involved. To ensure an adequate factor of safety in the design of a shaft with standard keyway (width, one-quarter, and depth, one-eighth of shaft diameter), the former Code for Transmission Shafting tentatively recommended that shafts with keyways be designed on the basis of a solid circular shaft using not more than 75 per cent of the working stress recommended for the solid shaft. See also page 2342.

Formula for Shafts of Brittle Materials.—The preceding formulas are applicable to ductile materials and are based on the maximum-shear theory of failure which assumes that the elastic limit of a *ductile* material in shear is one-half its elastic limit in tension.

Brittle materials are generally stronger in shear than in tension; therefore, the maximum-shear theory is not applicable. The *maximum-normal-stress theory* of failure is now generally accepted for the design of shafts made from brittle materials. A material may be considered to be brittle if its elongation in a 2-inch gage length is less than 5 per cent. Materials such as cast iron, hardened tool steel, hard bronze, etc., conform to this rule. The diameter of a shaft made of a brittle material may be determined from the following formula which is based on the maximum-normal-stress theory of failure:

$$D = B\sqrt[3]{\frac{5.1}{S_t}[(K_m M) + \sqrt{(K_m M)^2 + (K_t T)^2}]}$$

where S_t is the maximum allowable tensile stress in pounds per square inch and the other quantities are as previously defined.

The formula can be used unchanged with metric SI units, where D = outside diameter of shaft in millimeters; S_t = the maximum allowable tensile stress in newtons per millimeter squared; M = maximum bending moment in newton-millimeters; and T = maximum torsional moment in newton-millimeters. The factors K_m, K_t, and B are unchanged.

Critical Speed of Rotating Shafts.—At certain speeds, a rotating shaft will become dynamically unstable and the resulting vibrations and deflections can result in damage not only to the shaft but to the machine of which it is a part. The speeds at which such dynamic instability occurs are called the critical speeds of the shaft. On page 186 are given formulas for the critical speeds of shafts subject to various conditions of loading and support. A shaft may be safely operated either above or below its critical speed, good practice indicating that the operating speed be at least 20 per cent above or below the critical.

The formulas commonly used to determine critical speeds are sufficiently accurate for general purposes. However, the torque applied to a shaft has an important effect on its critical speed. Investigations have shown that the critical speeds of a uniform shaft are decreased as the applied torque is increased, and that there exist critical torques which will reduce the corresponding critical speed of the shaft to zero. A detailed analysis of the effects of applied torques on critical speeds may be found in a paper. "Critical Speeds of Uniform Shafts under Axial Torque," by Golomb and Rosenberg presented at the First U.S. National Congress of Applied Mechanics in 1951.

Comparison of Hollow and Solid Shafting with Same Outside Diameter.—The table that follows gives the per cent decrease in strength and weight of a hollow shaft relative to the strength and weight of a solid shaft of the same diameter. The upper figures in each line give the per cent decrease in strength and the lower figures give the per cent decrease in weight.

Example: A 4-inch shaft, with a 2-inch hole through it, has a weight 25 per cent less than a solid 4-inch shaft, but its strength is decreased only 6.25 per cent.

Comparative Torsional Strengths and Weights of Hollow and Solid Shafting with Same Outside Diameter

Diam. of Solid and Hollow Shaft, Inches	Diameter of Axial Hole in Hollow Shaft, Inches									
	1	1¼	1½	1¾	2	2½	3	3½	4	4½
1½	19.76	48.23	…	…	…	…	…	…	…	…
	44.44	69.44	…	…	…	…	…	…	…	…
1¾	10.67	26.04	53.98	…	…	…	…	…	…	…
	32.66	51.02	73.49	…	…	…	…	…	…	…
2	6.25	15.26	31.65	58.62	…	…	…	…	…	…
	25.00	39.07	56.25	76.54	…	…	…	…	…	…
2¼	3.91	9.53	19.76	36.60	62.43	…	…	…	…	…
	19.75	30.87	44.44	60.49	79.00	…	…	…	…	…
2½	2.56	6.25	12.96	24.01	40.96	…	…	…	…	…
	16.00	25.00	36.00	49.00	64.00	…	…	…	…	…
2¾	1.75	4.28	8.86	16.40	27.98	68.30	…	…	…	…
	13.22	20.66	29.74	40.48	52.89	82.63	…	…	…	…
3	1.24	3.01	6.25	11.58	19.76	48.23	…	…	…	…
	11.11	17.36	25.00	34.01	44.44	69.44	…	…	…	…
3¼	0.87	2.19	4.54	8.41	14.35	35.02	72.61	…	…	…
	9.46	14.80	21.30	29.00	37.87	59.17	85.22	…	…	…
3½	0.67	1.63	3.38	6.25	10.67	26.04	53.98	…	…	…
	8.16	12.76	18.36	25.00	32.66	51.02	73.49	…	…	…
3¾	0.51	1.24	2.56	4.75	8.09	19.76	40.96	75.89	…	…
	7.11	11.11	16.00	21.77	28.45	44.44	64.00	87.10	…	…
4	0.40	0.96	1.98	3.68	6.25	15.26	31.65	58.62	…	…
	6.25	9.77	14.06	19.14	25.00	39.07	56.25	76.56	…	…
4¼	0.31	0.74	1.56	2.89	4.91	11.99	24.83	46.00	78.47	…
	5.54	8.65	12.45	16.95	22.15	34.61	49.85	67.83	88.59	…
4½	0.25	0.70	1.24	2.29	3.91	9.53	19.76	36.60	62.43	…
	4.94	7.72	11.11	15.12	19.75	30.87	44.44	60.49	79.00	…
4¾	0.20	0.50	1.00	1.85	3.15	7.68	15.92	29.48	50.29	80.56
	4.43	6.93	9.97	13.57	17.73	27.70	39.90	54.29	70.91	89.75
5	0.16	0.40	0.81	1.51	2.56	6.25	12.96	24.01	40.96	65.61
	4.00	6.25	8.10	12.25	16.00	25.00	36.00	49.00	64.00	81.00
5½	0.11	0.27	0.55	1.03	1.75	4.27	8.86	16.40	27.98	44.82
	3.30	5.17	7.43	10.12	13.22	20.66	29.76	40.48	52.89	66.94
6	0.09	0.19	0.40	0.73	1.24	3.02	6.25	11.58	19.76	31.65
	2.77	4.34	6.25	8.50	11.11	17.36	25.00	34.02	44.44	56.25
6½	0.06	0.14	0.29	0.59	0.90	2.19	4.54	8.41	14.35	23.98
	2.36	3.70	5.32	7.24	9.47	14.79	21.30	28.99	37.87	47.93
7	0.05	0.11	0.22	0.40	0.67	1.63	3.38	6.25	10.67	17.08
	2.04	3.19	4.59	6.25	8.16	12.76	18.36	25.00	32.66	41.33
7½	0.04	0.08	0.16	0.30	0.51	1.24	2.56	4.75	8.09	12.96
	1.77	2.77	4.00	5.44	7.11	11.11	16.00	21.77	28.45	36.00
8	0.03	0.06	0.13	0.23	0.40	0.96	1.98	3.68	6.25	10.02
	1.56	2.44	3.51	4.78	6.25	9.77	14.06	19.14	25.00	31.64

The upper figures in each line give number of per cent decrease in strength; the lower figures give per cent decrease in weight.

SPRINGS*

Springs

Introduction.—Many advances have been made in the spring industry in recent years. For example: developments in materials permit longer fatigue life at higher stresses; simplified design procedures reduce the complexities of design, and improved methods of manufacture help to speed up some of the complicated fabricating procedures and increase production. New types of testing instruments and revised tolerances also permit higher standards of accuracy. Designers should also consider the possibility of using standard springs now available from stock. They can be obtained from spring manufacturing companies located in different areas, and small shipments usually can be made quickly.

Designers of springs require information in the following order of precedence to simplify design procedures.

1) Spring materials and their applications
2) Allowable spring stresses
3) Spring design data with tables of spring characteristics, tables of formulas, and tolerances.

Only the more commonly used types of springs are covered in detail here. Special types and designs rarely used such as torsion bars, volute springs, Belleville washers, constant force, ring and spiral springs and those made from rectangular wire are only described briefly.

Notation.—The following symbols are used in spring equations:

AC = Active coils
b = Widest width of rectangular wire, inches
CL = Compressed length, inches
D = Mean coil diameter, inches $= OD - d$
d = Diameter of wire or side of square, inches
E = Modulus of elasticity in tension, pounds per square inch
F = Deflection, for N coils, inches
$F°$ = Deflection, for N coils, rotary, degrees
f = Deflection, for one active coil
FL = Free length, unloaded spring, inches
G = Modulus of elasticity in torsion, pounds per square inch
IT = Initial tension, pounds
K = Curvature stress correction factor
L = Active length subject to deflection, inches
N = Number of active coils, total
P = Load, pounds
p = pitch, inches
R = Distance from load to central axis, inches
S or S_t = Stress, torsional, pounds per square inch
S_b = Stress, bending, pounds per square inch
SH = Solid height
S_{it} = Stress, torsional, due to initial tension, pounds per square inch
T = Torque $= P \times R$, pound-inches
TC = Total coils
t = Thickness, inches
U = Number of revolutions $= F°/360°$

* This section was compiled by Harold Carlson, P. E., Consulting Engineer, Lakewood, N.J.

Spring Materials

The spring materials most commonly used include high-carbon spring steels, alloy spring steels, stainless spring steels, copper-base spring alloys, and nickel-base spring alloys.

High-Carbon Spring Steels in Wire Form.—These spring steels are the most commonly used of all spring materials because they are the least expensive, are easily worked, and are readily available. However, they are not satisfactory for springs operating at high or low temperatures or for shock or impact loading. The following wire forms are available:

Music Wire, ASTM A228 (0.80–0.95 per cent carbon): This is the most widely used of all spring materials for small springs operating at temperatures up to about 250 degrees F. It is tough, has a high tensile strength, and can withstand high stresses under repeated loading. The material is readily available in round form in diameters ranging from 0.005 to 0.125 inch and in some larger sizes up to $\frac{3}{16}$ inch. It is not available with high tensile strengths in square or rectangular sections. Music wire can be plated easily and is obtainable pretinned or preplated with cadmium, but plating after spring manufacture is usually preferred for maximum corrosion resistance.

Oil-Tempered MB Grade, ASTM A229 (0.60–0.70 per cent carbon): This general-purpose spring steel is commonly used for many types of coil springs where the cost of music wire is prohibitive and in sizes larger than are available in music wire. It is readily available in diameters ranging from 0.125 to 0.500 inch, but both smaller and larger sizes may be obtained. The material should not be used under shock and impact loading conditions, at temperatures above 350 degrees F., or at temperatures in the sub-zero range. Square and rectangular sections of wire are obtainable in fractional sizes. Annealed stock also can be obtained for hardening and tempering after coiling. This material has a heat-treating scale that must be removed before plating.

Oil-Tempered HB Grade, SAE 1080 (0.75–0.85 per cent carbon): This material is similar to the MB Grade except that it has a higher carbon content and a higher tensile strength. It is obtainable in the same sizes and is used for more accurate requirements than the MB Grade, but is not so readily available. In lieu of using this material it may be better to use an alloy spring steel, particularly if a long fatigue life or high endurance properties are needed. Round and square sections are obtainable in the oil-tempered or annealed conditions.

Hard-Drawn MB Grade, ASTM A227 (0.60–0.70 per cent carbon): This grade is used for general-purpose springs where cost is the most important factor. Although increased use in recent years has resulted in improved quality, it is best not to use it where long life and accuracy of loads and deflections are important. It is available in diameters ranging from 0.031 to 0.500 inch and in some smaller and larger sizes also. The material is available in square sections but at reduced tensile strengths. It is readily plated. Applications should be limited to those in the temperature range of 0 to 250 degrees F.

High-Carbon Spring Steels in Flat Strip Form.—Two types of thin, flat, high-carbon spring steel strip are most widely used although several other types are obtainable for specific applications in watches, clocks, and certain instruments. These two compositions are used for over 95 per cent of all such applications. Thin sections of these materials under 0.015 inch having a carbon content of over 0.85 per cent and a hardness of over 47 on the Rockwell C scale are susceptible to hydrogen-embrittlement even though special plating and heating operations are employed. The two types are described as follows:

Cold-Rolled Spring Steel, Blue-Tempered or Annealed, SAE 1074, also 1064, and 1070 (0.60 to 0.80 per cent carbon): This very popular spring steel is available in thicknesses ranging from 0.005 to 0.062 inch and in some thinner and thicker sections. The material is available in the annealed condition for forming in 4-slide machines and in presses, and can readily be hardened and tempered after forming. It is also available in the heat-treated or blue-tempered condition. The steel is obtainable in several finishes such as straw color, blue color, black, or plain. Hardnesses ranging from 42 to 46 Rockwell C are recommended for spring applications. Uses include spring clips, flat springs, clock springs, and motor, power, and spiral springs.

Cold-Rolled Spring Steel, Blue-Tempered Clock Steel, SAE 1095 (0.90 to 1.05 per cent carbon): This popular type should be used principally in the blue-tempered condition. Although obtainable in the annealed condition, it does not always harden properly during heat-treatment as it is a “shallow” hardening type. It is used principally in clocks and motor springs. End sections of springs made from this steel are annealed for bending or piercing operations. Hardnesses usually range from 47 to 51 Rockwell C.

Other materials available in strip form and used for flat springs are brass, phosphor-bronze, beryllium-copper, stainless steels, and nickel alloys.

Alloy Spring Steels.—These spring steels are used for conditions of high stress, and shock or impact loadings. They can withstand both higher and lower temperatures than the high-carbon steels and are obtainable in either the annealed or pretempered conditions.

Chromium Vanadium, ASTM A231: This very popular spring steel is used under conditions involving higher stresses than those for which the high-carbon spring steels are recommended and is also used where good fatigue strength and endurance are needed. It behaves well under shock and impact loading. The material is available in diameters ranging from 0.031 to 0.500 inch and in some larger sizes also. In square sections it is available in fractional sizes. Both the annealed and pretempered types are available in round, square, and rectangular sections. It is used extensively in aircraft-engine valve springs and for springs operating at temperatures up to 425 degrees F.

Silicon Manganese: This alloy steel is quite popular in Great Britain. It is less expensive than chromium-vanadium steel and is available in round, square, and rectangular sections in both annealed and pretempered conditions in sizes ranging from 0.031 to 0.500 inch. It was formerly used for knee-action springs in automobiles. It is used in flat leaf springs for trucks and as a substitute for more expensive spring steels.

Chromium Silicon, ASTM A401: This alloy is used for highly stressed springs that require long life and are subjected to shock loading. It can be heat-treated to higher hardnesses than other spring steels so that high tensile strengths are obtainable. The most popular sizes range from 0.031 to 0.500 inch in diameter. Very rarely are square, flat, or rectangular sections used. Hardnesses ranging from 50 to 53 Rockwell C are quite common and the alloy may be used at temperatures up to 475 degrees F. This material is usually ordered specially for each job.

Stainless Spring Steels.—The use of stainless spring steels has increased and several compositions are available all of which may be used for temperatures up to 550 degrees F. They are all corrosion resistant. Only the stainless 18-8 compositions should be used at sub-zero temperatures.

Stainless Type 302, ASTM A313 (18 per cent chromium, 8 per cent nickel): This stainless spring steel is very popular because it has the highest tensile strength and quite uniform properties. It is cold-drawn to obtain its mechanical properties and cannot be hardened by heat treatment. This material is nonmagnetic only when fully annealed and becomes slightly magnetic due to the cold-working performed to produce spring properties. It is suitable for use at temperatures up to 550 degrees F. and for sub-zero temperatures. It is very corrosion resistant. The material best exhibits its desirable mechanical properties in diameters ranging from 0.005 to 0.1875 inch although some larger diameters are available. It is also available as hard-rolled flat strip. Square and rectangular sections are available but are infrequently used.

Stainless Type 304, ASTM A313 (18 per cent chromium, 8 per cent nickel): This material is quite similar to Type 302, but has better bending properties and about 5 per cent lower tensile strength. It is a little easier to draw, due to the slightly lower carbon content.

Stainless Type 316, ASTM A313 (18 per cent chromium, 12 per cent nickel, 2 per cent molybdenum): This material is quite similar to Type 302 but is slightly more corrosion resistant because of its higher nickel content. Its tensile strength is 10 to 15 per cent lower than Type 302. It is used for aircraft springs.

Stainless Type 17-7 PH ASTM A313 (17 per cent chromium, 7 per cent nickel): This alloy, which also contains small amounts of aluminum and titanium, is formed in a moderately hard state and then precipitation hardened at relatively low temperatures for several hours to produce tensile strengths nearly comparable to music wire. This material is not readily available in all sizes, and has limited applications due to its high manufacturing cost.

Stainless Type 414, SAE 51414 (12 per cent chromium, 2 per cent nickel): This alloy has tensile strengths about 15 per cent lower than Type 302 and can be hardened by heat-treatment. For best corrosion resistance it should be highly polished or kept clean. It can be obtained hard drawn in diameters up to 0.1875 inch and is commonly used in flat cold-rolled strip for stampings. The material is not satisfactory for use at low temperatures.

Stainless Type 420, SAE 51420 (13 per cent chromium): This is the best stainless steel for use in large diameters above 0.1875 inch and is frequently used in smaller sizes. It is formed in the annealed condition and then hardened and tempered. It does not exhibit its stainless properties until after it is hardened. Clean bright surfaces provide the best corrosion resistance, therefore the heat-treating scale must be removed. Bright hardening methods are preferred.

Stainless Type 431, SAE 51431 (16 per cent chromium, 2 per cent nickel): This spring alloy acquires high tensile properties (nearly the same as music wire) by a combination of heat-treatment to harden the wire plus cold-drawing after heat-treatment. Its corrosion resistance is not equal to Type 302.

Copper-Base Spring Alloys.—Copper-base alloys are important spring materials because of their good electrical properties combined with their good resistance to corrosion. Although these materials are more expensive than the high-carbon and the alloy steels, they nevertheless are frequently used in electrical components and in sub-zero temperatures.

Spring Brass, ASTM B 134 (70 per cent copper, 30 per cent zinc): This material is the least expensive and has the highest electrical conductivity of the copper-base alloys. It has a low tensile strength and poor spring qualities, but is extensively used in flat stampings and where sharp bends are needed. It cannot be hardened by heat-treatment and should not be used at temperatures above 150 degrees F., but is especially good at sub-zero temperatures. Available in round sections and flat strips, this hard-drawn material is usually used in the “spring hard” temper.

Phosphor Bronze, ASTM B 159 (95 per cent copper, 5 per cent tin): This alloy is the most popular of this group because it combines the best qualities of tensile strength, hardness, electrical conductivity, and corrosion resistance with the least cost. It is more expensive than brass, but can withstand stresses 50 per cent higher.The material cannot be hardened by heat-treatment. It can be used at temperatures up to 212 degrees F. and at sub-zero temperatures. It is available in round sections and flat strip, usually in the “extra-hard” or “spring hard” tempers. It is frequently used for contact fingers in switches because of its low arcing properties. An 8 per cent tin composition is used for flat springs and a superfine grain composition called “Duraflex,” has good endurance properties.

Beryllium Copper, ASTM B 197 (98 per cent copper, 2 per cent beryllium): This alloy can be formed in the annealed condition and then precipitation hardened after forming at temperatures around 600 degrees F, for 2 to 3 hours. This treatment produces a high hardness combined with a high tensile strength. After hardening, the material becomes quite brittle and can withstand very little or no forming. It is the most expensive alloy in the group and heat-treating is expensive due to the need for holding the parts in fixtures to prevent distortion. The principal use of this alloy is for carrying electric current in switches and in electrical components. Flat strip is frequently used for contact fingers.

Nickel-Base Spring Alloys.—Nickel-base alloys are corrosion resistant, withstand both elevated and sub-zero temperatures, and their non-magnetic characteristic makes them useful for such applications as gyroscopes, chronoscopes, and indicating instruments. These materials have a high electrical resistance and therefore should not be used for conductors of electrical current.

Monel (67 per cent nickel, 30 per cent copper):* This material is the least expensive of the nickel-base alloys. It also has the lowest tensile strength but is useful due to its resistance to the corrosive effects of sea water and because it is nearly non-magnetic. The alloy can be subjected to stresses slightly higher than phosphor bronze and nearly as high as beryllium copper. Its high tensile strength and hardness are obtained as a result of cold-drawing and cold-rolling only, since it can not be hardened by heat-treatment. It can be used at temperatures ranging from −100 to +425 degrees F. at normal operating stresses and is available in round wires up to $\frac{3}{16}$ inch in diameter with quite high tensile strengths. Larger diameters and flat strip are available with lower tensile strengths.

*"K" Monel * (66 per cent nickel, 29 per cent copper, 3 per cent aluminum):* This material is quite similar to Monel except that the addition of the aluminum makes it a precipitation-hardening alloy. It may be formed in the soft or fairly hard condition and then hardened by a long-time age-hardening heat-treatment to obtain a tensile strength and hardness above Monel and nearly as high as stainless steel. It is used in sizes larger than those usually used with Monel, is non-magnetic and can be used in temperatures ranging from − 100 to + 450 degrees F. at normal working stresses under 45,000 pounds per square inch.

Inconel (78 per cent nickel, 14 per cent chromium, 7 per cent iron):* This is one of the most popular of the non-magnetic nickel-base alloys because of its corrosion resistance and because it can be used at temperatures up to 700 degrees F. It is more expensive than stainless steel but less expensive than beryllium copper. Its hardness and tensile strength is higher than that of "K" Monel and is obtained as a result of cold-drawing and cold-rolling only. It cannot be hardened by heat treatment. Wire diameters up to $\frac{1}{4}$ inch have the best tensile properties. It is often used in steam valves, regulating valves, and for springs in boilers, compressors, turbines, and jet engines.

Inconel "X" (70 per cent nickel, 16 per cent chromium, 7 per cent iron):* This material is quite similar to Inconel but the small amounts of titanium, columbium and aluminum in its composition make it a precipitation-hardening alloy. It can be formed in the soft or partially hard condition and then hardened by holding it at 1200 degrees F. for 4 hours. It is non-magnetic and is used in larger sections than Inconel. This alloy is used at temperatures up to 850 degrees F. and at stresses up to 55,000 pounds per square inch.

Duranickel ("Z" Nickel) (98 per cent nickel):* This alloy is non-magnetic, corrosion resistant, has a high tensile strength and is hardenable by precipitation hardening at 900 degrees F. for 6 hours. It may be used at the same stresses as Inconel but should not be used at temperatures above 500 degrees F.

Nickel-Base Spring Alloys with Constant Moduli of Elasticity.—Some special nickel alloys have a constant modulus of elasticity over a wide temperature range. These materials are especially useful where springs undergo temperature changes and must exhibit uniform spring characteristics. These materials have a low or zero thermo-elastic coefficient

* Trade name of the International Nickel Company.

and therefore do not undergo variations in spring stiffness because of modulus changes due to temperature differentials. They also have low hysteresis and creep values which makes them preferred for use in food-weighing scales, precision instruments, gyroscopes, measuring devices, recording instruments and computing scales where the temperature ranges from − 50 to + 150 degrees F. These materials are expensive, none being regularly stocked in a wide variety of sizes. They should not be specified without prior discussion with spring manufacturers because some suppliers may not fabricate springs from these alloys due to the special manufacturing processes required. All of these alloys are used in small wire diameters and in thin strip only and are covered by U.S. patents. They are more specifically described as follows:

Elinvar[*] *(nickel, iron, chromium):* This alloy, the first constant-modulus alloy used for hairsprings in watches, is an austenitic alloy hardened only by cold-drawing and cold-rolling. Additions of titanium, tungsten, molybdenum and other alloying elements have brought about improved characteristics and precipitation-hardening abilities. These improved alloys are known by the following trade names: Elinvar Extra, Durinval, Modulvar and Nivarox.

Ni-Span C[*] *(nickel, iron, chromium, titanium):* This very popular constant-modulus alloy is usually formed in the 50 per cent cold-worked condition and precipitation-hardened at 900 degrees F. for 8 hours, although heating up to 1250 degrees F. for 3 hours produces hardnesses of 40 to 44 Rockwell C, permitting safe torsional stresses of 60,000 to 80,000 pounds per square inch. This material is ferromagnetic up to 400 degrees F; above that temperature it becomes non-magnetic.

Iso-Elastic[†] *(nickel, iron, chromium, molybdenum):* This popular alloy is relatively easy to fabricate and is used at safe torsional stresses of 40,000 to 60,000 pounds per square inch and hardnesses of 30 to 36 Rockwell C. It is used principally in dynamometers, instruments, and food-weighing scales.

Elgiloy[‡] *(nickel, iron, chromium, cobalt):* This alloy, also known by the trade names 8J Alloy, Durapower, and Cobenium, is a non-magnetic alloy suitable for sub-zero temperatures and temperatures up to about 1000 degrees F., provided that torsional stresses are kept under 75,000 pounds per square inch. It is precipitation-hardened at 900 degrees F. for 8 hours to produce hardnesses of 48 to 50 Rockwell C. The alloy is used in watch and instrument springs.

Dynavar[**] *(nickel, iron, chromium, cobalt):* This alloy is a non-magnetic, corrosion-resistant material suitable for sub-zero temperatures and temperatures up to about 750 degrees F., provided that torsional stresses are kept below 75,000 pounds per square inch. It is precipitation-hardened to produce hardnesses of 48 to 50 Rockwell C and is used in watch and instrument springs.

Spring Stresses

Allowable Working Stresses for Springs.—The safe working stress for any particular spring depends to a large extent on the following items:

1) Type of spring — whether compression, extension, torsion, etc.;
2) Size of spring — small or large, long or short;
3) Spring material;
4) Size of spring material;
5) Type of service — light, average, or severe;
6) Stress range — low, average, or high;

[*] Trade name of Soc. Anon. de Commentry Fourchambault et Decazeville, Paris, France.
[†] Trade name of John Chatillon & Sons.
[‡] Trade name of Elgin National Watch Company.
[**] Trade name of Hamilton Watch Company.

7) Loading — static, dynamic, or shock;

8) Operating temperature;

9) Design of spring — spring index, sharp bends, hooks.

Consideration should also be given to other factors that affect spring life: corrosion, buckling, friction, and hydrogen embrittlement decrease spring life; manufacturing operations such as high-heat stress-equalizing, presetting, and shot-peening increase spring life.

Item 5, the type of service to which a spring is subjected, is a major factor in determining a safe working stress once consideration has been given to type of spring, kind and size of material, temperature, type of loading, and so on. The types of service are:

Light Service: This includes springs subjected to static loads or small deflections and seldom-used springs such as those in bomb fuses, projectiles, and safety devices. This service is for 1,000 to 10,000 deflections.

Average Service: This includes springs in general use in machine tools, mechanical products, and electrical components. Normal frequency of deflections not exceeding 18,000 per hour permit such springs to withstand 100,000 to 1,000,000 deflections.

Severe Service: This includes springs subjected to rapid deflections over long periods of time and to shock loading such as in pneumatic hammers, hydraulic controls and valves. This service is for 1,000,000 deflections, and above. Lowering the values 10 per cent permits 10,000,000 deflections.

Figs. 1 through 6 show curves that relate the three types of service conditions to allowable working stresses and wire sizes for compression and extension springs, and safe values are provided. Figs. 7 through 10 provide similar information for helical torsion springs. In each chart, the values obtained from the curves may be increased by 20 per cent (but not beyond the top curves on the charts if permanent set is to be avoided) for springs that are baked, and shot-peened, and compression springs that are pressed. Springs stressed slightly above the Light Service curves will take a permanent set.

A curvature correction factor is included in all curves, and is used in spring design calculations (see examples beginning page 300). The curves may be used for materials other than those designated in Figs. 1 through 10, by applying multiplication factors as given in Table 1.

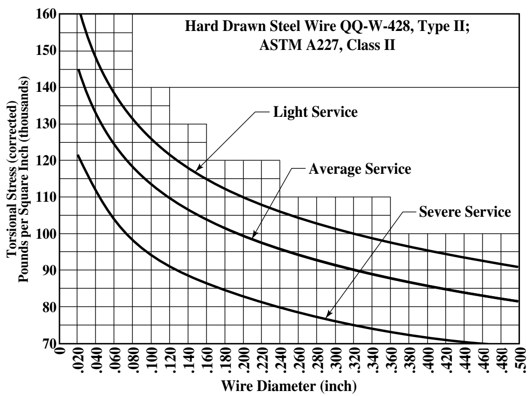


Fig. 1. Allowable Working Stresses for Compression Springs — Hard Drawn Steel Wire[a]

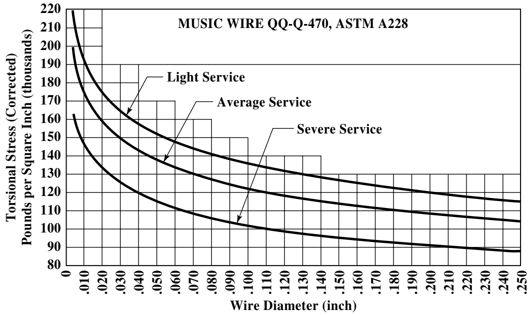


Fig. 2. Allowable Working Stresses for Compression Springs — Music Wire[a]

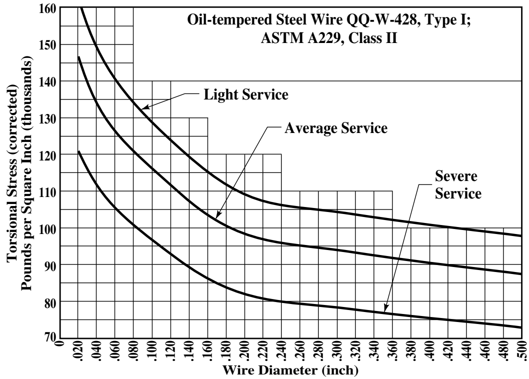


Fig. 3. Allowable Working Stresses for Compression Springs — Oil-Tempered[a]

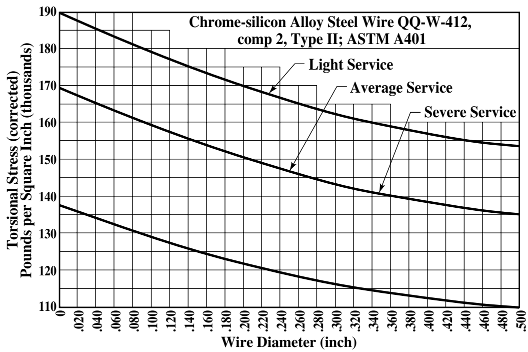


Fig. 4. Allowable Working Stresses for Compression Springs — Chrome-Silicon Alloy Steel Wire[a]

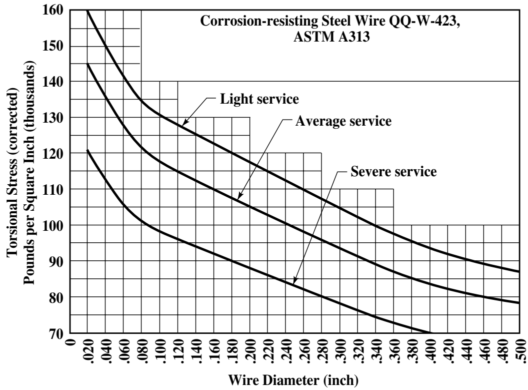


Fig. 5. Allowable Working Stresses for Compression Springs — Corrosion-Resisting Steel Wire[a]

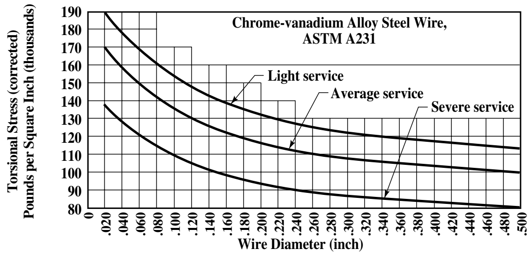


Fig. 6. Allowable Working Stresses for Compression Springs — Chrome-Vanadium Alloy Steel Wire[a]

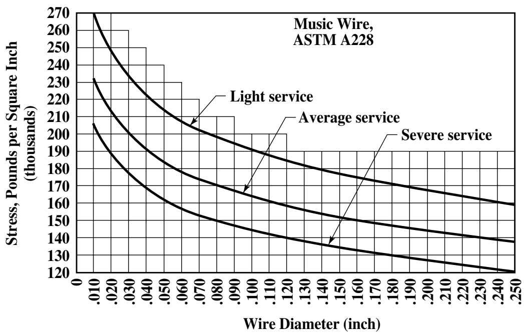


Fig. 7. Recommended Design Stresses in Bending for Helical Torsion Springs — Round Music Wire

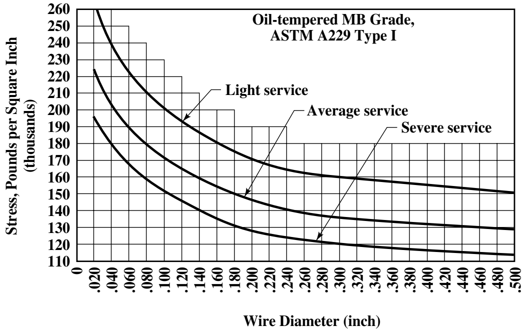


Fig. 8. Recommended Design Stresses in Bending for Helical Torsion Springs — Oil-Tempered MB Round Wire

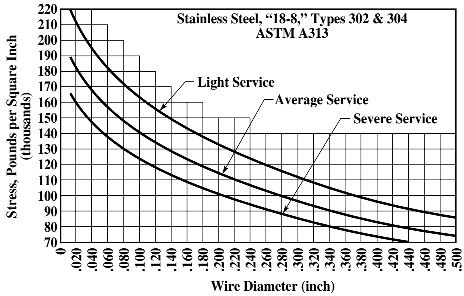


Fig. 9. Recommended Design Stresses in Bending for Helical Torsion Springs — Stainless Steel Round Wire

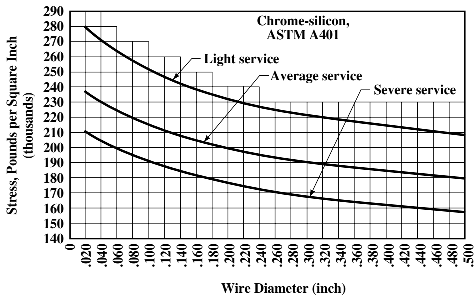


Fig. 10. Recommended Design Stresses in Bending for Helical Torsion Springs — Chrome-Silicon Round Wire

[a] Although Figs. 1 through 6 are for compression springs, they may also be used for extension springs; for extension springs, *reduce* the values obtained from the curves by 10 to 15 per cent.

Table 1. Correction Factors for Other Materials

Compression and Tension Springs			
Material	**Factor**	**Material**	**Factor**
Silicon-manganese	Multiply the values in the chromium-vanadium curves (Fig. 6) by 0.90	Stainless Steel, 316	Multiply the values in the corrosion-resisting steel curves (Fig. 5) by 0.90
Valve-spring quality wire	Use the values in the chromium-vanadium curves (Fig. 6)		
Stainless Steel, 304 and 420	Multiply the values in the corrosion-resisting steel curves (Fig. 5) by 0.95	Stainless Steel, 431 and 17-7PH	Multiply the values in the music wire curves (Fig. 2) by 0.90

Helical Torsion Springs			
Material	**Factor[a]**	**Material**	**Factor[a]**
Hard Drawn MB	0.70	Stainless Steel, 431	
Stainless Steel, 316		Up to 1/32 inch diameter	0.80
Up to 1/32 inch diameter	0.75	Over 1/32 to 1/16 inch	0.85
Over 1/32 to 3/16 inch	0.70	Over 1/16 to 1/8 inch	0.95
Over 3/16 to 1/4 inch	0.65	Over 1/8 inch	1.00
Over 1/4 inch	0.50	Chromium-Vanadium	
Stainless Steel, 17-7 PH		Up to 1/16 inch diameter	1.05
Up to 1/8 inch diameter	1.00	Over 1/16 inch	1.10
Over 1/8 to 3/16 inch	1.07	Phosphor Bronze	
Over 3/16 inch	1.12	Up to 1/8 inch diameter	0.45
Stainless Steel, 420		Over 1/8 inch	0.55
Up to 1/32 inch diameter	0.70	Beryllium Copper[b]	
Over 1/32 to 1/16 inch	0.75	Up to 1/32 inch diameter	0.55
Over 1/16 to 1/8 inch	0.80	Over 1/32 to 1/16 inch	0.60
Over 1/8 to 3/16 inch	0.90	Over 1/16 to 1/8 inch	0.70
Over 3/16 inch	1.00	Over 1/8 inch	0.80

[a] Multiply the values in the curves for oil-tempered MB grade ASTM A229 Type 1 steel (Fig. 8) by these factors to obtain required values.
[b] Hard drawn and heat treated after coiling.

For use with design stress curves shown in Figs. 2, 5, 6, and 8.

Endurance Limit for Spring Materials.—When a spring is deflected continually it will become "tired" and fail at a stress far below its elastic limit. This type of failure is called *fatigue failure* and usually occurs without warning. *Endurance limit* is the highest stress, or range of stress, in pounds per square inch that can be repeated indefinitely without failure of the spring. Usually ten million cycles of deflection is called "infinite life" and is satisfactory for determining this limit.

For severely worked springs of long life, such as those used in automobile or aircraft engines and in similar applications, it is best to determine the allowable working stresses by referring to the endurance limit curves seen in Fig. 11. These curves are based principally upon the range or difference between the stress caused by the first or initial load and the stress caused by the final load. Experience with springs designed to stresses within the limits of these curves indicates that they should have infinite or unlimited fatigue life. All values include Wahl curvature correction factor. The stress ranges shown may be increased 20 to 30 per cent for springs that have been properly heated, pressed to remove set, and then shot peened, provided that the increased values are lower than the torsional elastic limit by at least 10 per cent.

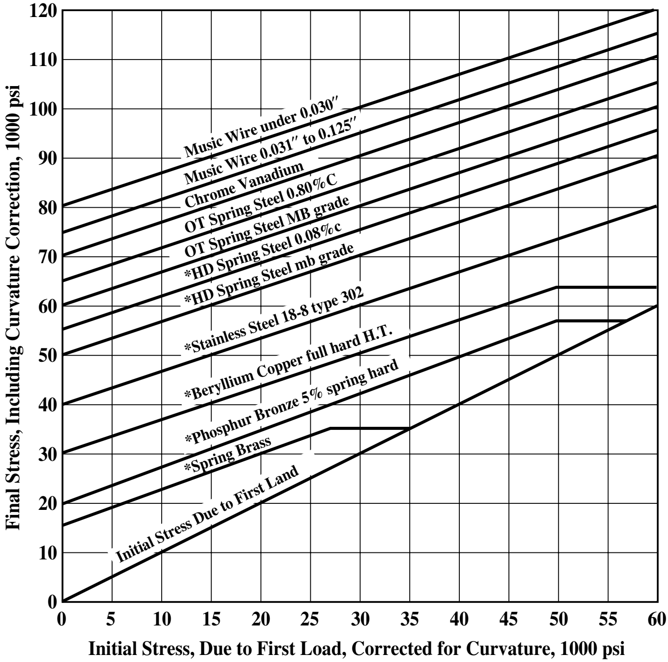


Fig. 11. Endurance Limit Curves for Compression Springs

Notes: For commercial spring materials with wire diameters up to ¼ inch except as noted. Stress ranges may be increased by approximately 30 per cent for properly heated, preset, shot-peened springs.

Materials preceeded by * are not ordinarily recommended for long continued service under severe operating conditions.

Working Stresses at Elevated Temperatures.—Since modulus of elasticity decreases with increase in temperature, springs used at high temperatures exert less load and have larger deflections under load than at room temperature. The torsional modulus of elasticity for steel may be 11,200,000 pounds per square inch at room temperature, but it will drop to 10,600,000 pounds per square inch at 400°F. and will be only 10,000,000 pounds per square inch at 600°F. Also, the elastic limit is reduced, thereby lowering the permissible working stress.

Design stresses should be as low as possible for all springs used at elevated temperatures. In addition, corrosive conditions that usually exist at high temperatures, especially with steam, may require the use of corrosion-resistant material. Table 2 shows the permissible elevated temperatures at which various spring materials may be operated, together with the maximum recommended working stresses at these temperatures. The loss in load at the temperatures shown is less than 5 per cent in 48 hours; however, if the temperatures listed are increased by 20 to 40 degrees, the loss of load may be nearer 10 per cent. Maximum stresses shown in the table are for compression and extension springs and may be increased by 75 per cent for torsion and flat springs. In using the data in Table 2 it should be noted that the values given are for materials in the heat-treated or spring temper condition.

Table 2. Recommended Maximum Working Temperatures and Corresponding Maximum Working Stresses for Springs

Spring Material	Maximum Working Temperature, Degrees, F.	Maximum Working Stress, Pounds per Square Inch
Brass Spring Wire	150	30,000
Phosphor Bronze	225	35,000
Music Wire	250	75,000
Beryllium-Copper	300	40,000
Hard Drawn Steel Wire	325	50,000
Carbon Spring Steels	375	55,000
Alloy Spring Steels	400	65,000
Monel	425	40,000
K-Monel	450	45,000
Permanickel[a]	500	50,000
Stainless Steel 18-8	550	55,000
Stainless Chromium 431	600	50,000
Inconel	700	50,000
High Speed Steel	775	70,000
Inconel X	850	55,000
Chromium-Molybdenum-Vanadium	900	55,000
Cobenium, Elgiloy	1000	75,000

[a] Formerly called Z-Nickel, Type B.

Loss of load at temperatures shown is less than 5 per cent in 48 hours.

Spring Design Data

Spring Characteristics.—This section provides tables of spring characteristics, tables of principal formulas, and other information of a practical nature for designing the more commonly used types of springs.

Standard wire gages for springs: Information on wire gages is given in the section beginning on page 2499, and gages in decimals of an inch are given in the table on page 2500. It should be noted that the range in this table extends from Number 7⁄0 through Number 80. However, in spring design, the range most commonly used extends only from Gage Number 4⁄0 through Number 40. When selecting wire use Steel Wire Gage or Washburn and Moen gage for all carbon steels and alloy steels except music wire; use Brown & Sharpe gage for brass and phosphor bronze wire; use Birmingham gage for flat spring steels, and cold rolled strip; and use piano or music wire gage for music wire.

Spring index: The spring index is the ratio of the mean coil diameter of a spring to the wire diameter (D/d). This ratio is one of the most important considerations in spring design because the deflection, stress, number of coils, and selection of either annealed or tempered material depend to a considerable extent on this ratio. The best proportioned springs have an index of 7 through 9. Indexes of 4 through 7, and 9 through 16 are often used. Springs with values larger than 16 require tolerances wider than standard for manufacturing; those with values less than 5 are difficult to coil on automatic coiling machines.

Direction of helix: Unless functional requirements call for a definite hand, the helix of compression and extension springs should be specified as optional. When springs are designed to operate, one inside the other, the helices should be opposite hand to prevent intermeshing. For the same reason, a spring that is to operate freely over a threaded member should have a helix of opposite hand to that of the thread. When a spring is to engage with a screw or bolt, it should, of course, have the same helix as that of the thread.

Helical Compression Spring Design.—After selecting a suitable material and a safe stress value for a given spring, designers should next determine the type of end coil formation best suited for the particular application. Springs with unground ends are less expensive but they do not stand perfectly upright; if this requirement has to be met, closed ground ends are used. Helical compression springs with different types of ends are shown in Fig. 12.

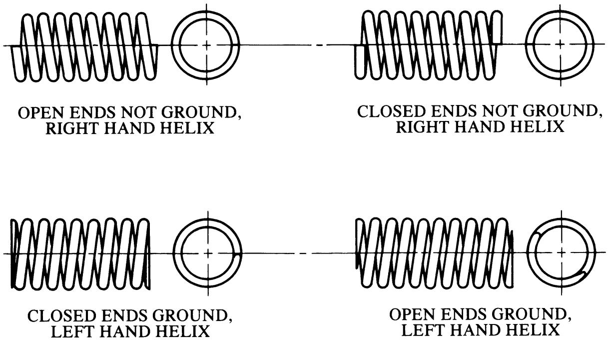


Fig. 12. Types of Helical Compression Spring Ends

Spring design formulas: Table 3 gives formulas for compression spring dimensional characteristics, and Table 4 gives design formulas for compression and extension springs.

Curvature correction: In addition to the stress obtained from the formulas for load or deflection, there is a direct shearing stress and an increased stress on the inside of the section due to curvature. Therefore, the stress obtained by the usual formulas should be multiplied by a factor *K* taken from the curve in Fig. 13. The corrected stress thus obtained is used only for comparison with the allowable working stress (fatigue strength) curves to determine if it is a safe stress and should not be used in formulas for deflection. The curvature correction factor *K* is for compression and extension springs made from round wire. For square wire reduce the *K* value by approximately 4 per cent.

Design procedure: The limiting dimensions of a spring are often determined by the available space in the product or assembly in which it is to be used. The loads and deflections on a spring may also be known or can be estimated, but the wire size and number of coils are usually unknown. Design can be carried out with the aid of the tabular data that appears later in this section (see Table , which is a simple method, or by calculation alone using the formulas in Tables 3 and 4.

Table 3. Formulas for Compression Springs

Feature	Type of End: Open or Plain (not ground)	Open or Plain (with ends ground)	Squared or Closed (not ground)	Closed and Ground
	Formula			
Pitch (p)	$\frac{FL - d}{N}$	$\frac{FL}{TC}$	$\frac{FL - 3d}{N}$	$\frac{FL - 2d}{N}$
Solid Height (SH)	$(TC + 1)d$	$TC \times d$	$(TC + I)d$	$TC \times d$
Number of Active Coils (N)	$N = TC$ $= \frac{FL - d}{p}$	$N = TC - 1$ $= \frac{FL}{p} - 1$	$N = TC - 2$ $= \frac{FL - 3d}{p}$	$N = TC - 2$ $= \frac{FL - 2d}{p}$
Total Coils (TC)	$\frac{FL - d}{p}$	$\frac{FL}{p}$	$\frac{FL - 3d}{p} + 2$	$\frac{FL - 2d}{p} + 2$
Free Length (FL)	$(p \times TC) + d$	$p \times TC$	$(p \times N) + 3d$	$(p \times N) + 2d$

The symbol notation is given on page 285.

Table 4. Formulas for Compression and Extension Springs

Feature	Formula[a]: Springs made from round wire	Springs made from square wire
Load, P Pounds	$P = \frac{0.393Sd^3}{D} = \frac{Gd^4F}{8ND^3}$	$P = \frac{0.416Sd^3}{D} = \frac{Gd^4F}{5.58ND^3}$
Stress, Torsional, S Pounds per square inch	$S = \frac{GdF}{\pi ND^2} = \frac{PD}{0.393d^3}$	$S = \frac{GdF}{2.32ND^2} = P\frac{D}{0.416d^3}$
Deflection, F Inch	$F = \frac{8PND^3}{Gd^4} = \frac{\pi SND^2}{Gd}$	$F = \frac{5.58PND^3}{Gd^4} = \frac{2.32SND^2}{Gd}$
Number of Active Coils, N	$N = \frac{Gd^4F}{8PD^3} = \frac{GdF}{\pi SD^2}$	$N = \frac{Gd^4F}{5.58PD^3} = \frac{GdF}{2.32SD^2}$
Wire Diameter, d Inch	$d = \frac{\pi SND^2}{GF} = \sqrt[3]{\frac{2.55PD}{S}}$	$d = \frac{2.32SND^2}{GF} = \sqrt[3]{\frac{PD}{0.416S}}$
Stress due to Initial Tension, S_{it}	$S_{it} = \frac{S}{P} \times IT$	$S_{it} = \frac{S}{P} \times IT$

[a] Two formulas are given for each feature, and designers can use the one found to be appropriate for a given design. The end result from either of any two formulas is the same.

The symbol notation is given on page 285.

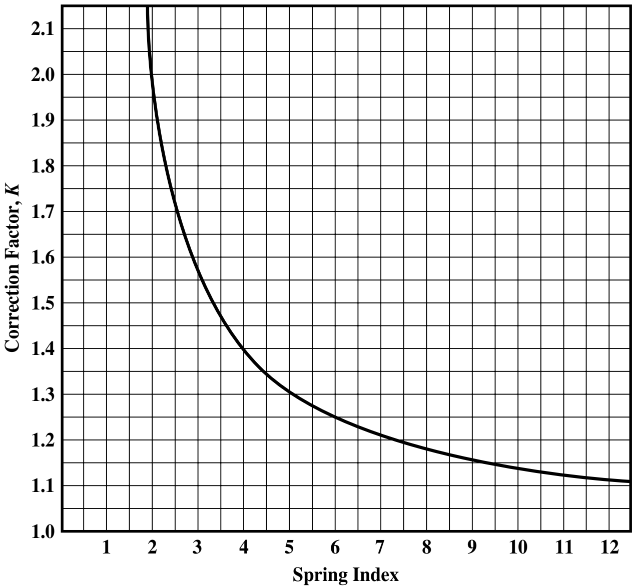


Fig. 13. Compression and Extension Spring-Stress Correction for Curvature*

Example: A compression spring with closed and ground ends is to be made from ASTM A229 high carbon steel wire, as shown in Fig. 14. Determine the wire size and number of coils.

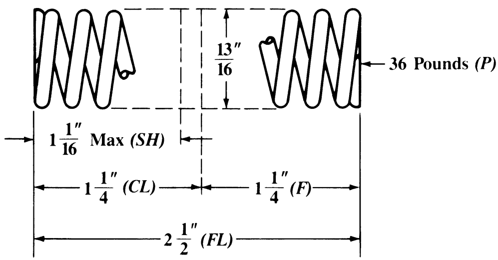


Fig. 14. Compression Spring Design Example

Method 1, using table: Referring to Table , starting on page 302, locate the spring outside diameter (13⁄16 inches, from Fig. 14) in the left-hand column. Note from the drawing that the spring load is 36 pounds. Move to the right in the table to the figure nearest this value, which is 41.7 pounds. This is somewhat above the required value but safe. Immediately above the load value, the deflection *f* is given, which in this instance is 0.1594 inch.

* For springs made from round wire. For springs made from square wire, reduce the *K* factor values by approximately 4 per cent.

This is the deflection of one coil under a load of 41.7 pounds with an uncorrected torsional stress S of 100,000 pounds per square inch (for ASTM A229 oil-tempered MB steel, see table on page 320). Moving vertically in the table from the load entry, the wire diameter is found to be 0.0915 inch.

The remaining spring design calculations are completed as follows:

Step 1: The stress with a load of 36 pounds is obtained by proportion, as follows: The 36 pound load is 86.3 per cent of the 41.7 pound load; therefore, the stress S at 36 pounds = $0.863 \times 100{,}000 = 86{,}300$ pounds per square inch.

Step 2: The 86.3 per cent figure is also used to determine the deflection per coil f at 36 pounds load: $0.863 \times 0.1594 = 0.1375$ inch.

Step 3: The number of active coils $AC = \frac{F}{f} = \frac{1.25}{0.1375} = 9.1$

Step 4: Total Coils $TC = AC + 2$ (Table 3) $= 9 + 2 = 11$
Therefore, a quick answer is: 11 coils of 0.0915 inch diameter wire. However, the design procedure should be completed by carrying out these remaining steps:

Step 5: From Table 3, Solid Height $= SH = TC \times d = 11 \times 0.0915 \cong 1$ inch

Therefore, Total Deflection $= FL - SH = 1.5$ inches

Step 6: Stress Solid $= \frac{86{,}300}{1.25} \times 1.5 = 103{,}500$ pounds per square inch

Step 7: Spring Index $= \frac{O.D.}{d} - 1 = \frac{0.8125}{0.0915} - 1 = 7.9$

Step 8: From Fig. 13, the curvature correction factor $K = 1.185$

Step 9: Total Stress at 36 pounds load $= S \times K = 86{,}300 \times 1.185 = 102{,}300$ pounds per square inch. This stress is below the 117,000 pounds per square inch permitted for 0.0915 inch wire shown on the middle curve in Fig. 3, so it is a safe working stress.

Step 10: Total Stress at Solid $= 103{,}500 \times 1.185 = 122{,}800$ pounds per square inch. This stress is also safe, as it is below the 131,000 pounds per square inch shown on the top curve Fig. 3, and therefore the spring will not set.

Method 2, using formulas: The procedure for design using formulas is as follows (the design example is the same as in Method I, and the spring is shown in Fig. 14):

Step 1: Select a safe stress S below the middle fatigue strength curve Fig. 8 for ASTM A229 steel wire, say 90,000 pounds per square inch. Assume a mean diameter D slightly below the $^{13}\!/_{16}$-inch $O.D.$, say 0.7 inch. Note that the value of G is 11,200,000 pounds per square inch (Table 20).

Step 2: A trial wire diameter d and other values are found by formulas from Table 4 as follows:

$$d = \sqrt[3]{\frac{2.55PD}{S}} = \sqrt[3]{\frac{2.55 \times 36 \times 0.7}{90{,}000}}$$

$$= \sqrt[3]{0.000714} = 0.0894 \text{ inch}$$

Note: Table 21 can be used to avoid solving the cube root.

Step 3: From the table on page 2500, select the nearest wire gauge size, which is 0.0915 inch diameter. Using this value, the mean diameter $D = {}^{13}\!/_{16}$ inch $- 0.0915 = 0.721$ inch.

Table 5. Compression and Extension Spring Deflections

Outside Diam.		Wire Size or Washburn and Moen Gauge, and Decimal Equivalent[a]																		
																	19	18	17	16
		.010	.012	.014	.016	.018	.020	.022	.024	.026	.028	.030	.032	.034	.036	.038	.041	.0475	.054	.0625
Nom.	Dec.	Deflection *f* (inch) per coil, at Load *P* (pounds)[b]																		
7/64	.1094	.0277	.0222	.01824	.01529	.01302	.01121	.00974	.00853	.00751	.00664	.00589	…	…	…	…	…	…	…	…
		.395	.697	1.130	1.722	2.51	3.52	4.79	6.36	8.28	10.59	13.35	…	…	…	…	…	…	…	…
1/8	.125	.0371	.0299	.0247	.0208	.01784	.01548	.01353	.01192	.01058	.00943	.00844	.00758	.00683	.00617	…	…	…	…	…
		.342	.600	.971	1.475	2.14	2.99	4.06	5.37	6.97	8.89	11.16	13.83	16.95	20.6	…	…	…	…	…
9/64	.1406	.0478	.0387	.0321	.0272	.0234	.0204	.01794	.01590	.01417	.01271	.01144	.01034	.00937	.00852	.00777	…	…	…	…
		.301	.528	.852	1.291	1.868	2.61	3.53	4.65	6.02	7.66	9.58	11.84	14.47	17.51	21.0	…	…	…	…
5/32	.1563	.0600	.0487	.0406	.0345	.0298	.0261	.0230	.0205	.01832	0.1649	.01491	.01354	.01234	.01128	.01033	.00909	…	…	…
		.268	.470	.758	1.146	1.656	2.31	3.11	4.10	5.30	6.72	8.39	10.35	12.62	15.23	18.22	23.5	…	…	…
11/64	.1719	.0735	.0598	.0500	.0426	.0369	.0324	.0287	.0256	.0230	.0208	.01883	.01716	.01569	.01439	.01324	.01172	.00914	…	…
		.243	.424	.683	1.031	1.488	2.07	2.79	3.67	4.73	5.99	7.47	9.19	11.19	13.48	16.09	21.8	33.8	…	…
3/16	.1875	.0884	.0720	.0603	.0516	.0448	.0394	.0349	.0313	.0281	.0255	.0232	.0212	.01944	.01788	.01650	.01468	.01157	.00926	…
		.221	.387	.621	.938	1.351	1.876	2.53	3.32	4.27	5.40	6.73	8.27	10.05	12.09	14.41	18.47	30.07	46.3	…
13/64	.2031	.1046	.0854	.0717	.0614	.0534	.0470	.0418	.0375	.0338	.0307	.0280	.0257	.0236	.0218	.0201	.01798	.01430	.01155	…
		.203	.355	.570	.859	1.237	1.716	2.31	3.03	3.90	4.92	6.12	7.52	9.13	10.96	13.05	16.69	27.1	41.5	…
7/32	.2188	…	.1000	.0841	.0721	.0628	.0555	.0494	.0444	.0401	.0365	.0333	.0306	.0282	.0260	.0241	.0216	.01733	.01411	.01096
		…	.328	.526	.793	1.140	1.580	2.13	2.79	3.58	4.52	5.61	6.88	8.35	10.02	11.92	15.22	24.6	37.5	61.3
15/64	.2344	…	.1156	.0974	.0836	.0730	.0645	.0575	.0518	.0469	.0427	.0391	.0359	.0331	.0307	.0285	.0256	.0206	.01690	.01326
		…	.305	.489	.736	1.058	1.465	1.969	2.58	3.21	4.18	5.19	6.35	7.70	9.23	10.97	13.99	22.5	34.3	55.8
1/4	.250	…	…	.1116	.0960	.0839	.0742	.0663	.0597	.0541	.0494	.0453	.0417	.0385	.0357	.0332	.0299	.0242	.01996	.01578
		…	…	.457	.687	.987	1.366	1.834	2.40	3.08	3.88	4.82	5.90	7.14	8.56	10.17	12.95	20.8	31.6	51.1
9/32	.2813	…	…	.1432	.1234	.1080	.0958	.0857	.0774	.0703	.0643	.0591	.0545	.0505	.0469	.0437	.0395	.0323	.0268	.0215
		…	…	.403	.606	.870	1.202	1.613	2.11	2.70	3.40	4.22	5.16	6.24	7.47	8.86	11.26	18.01	27.2	43.8
5/16	.3125	…	…	…	.1541	.1351	.1200	.1076	.0973	.0886	.0811	.0746	.0690	.0640	.0596	.0556	.0504	.0415	.0347	.0281
		…	…	…	.542	.778	1.074	1.440	1.881	2.41	3.03	3.75	4.58	5.54	6.63	7.85	9.97	15.89	23.9	38.3
11/32	.3438	…	…	…	…	.1633	.1470	.1321	.1196	.1090	.0999	.0921	.0852	.0792	.0733	.0690	.0627	.0518	.0436	.0355
		…	…	…	…	.703	.970	1.300	1.697	2.17	2.73	3.38	4.12	4.98	5.95	7.05	8.94	14.21	21.3	34.1
3/8	.375	…	…	…	…	…	.1768	.1589	.1440	.1314	.1206	.1113	.1031	.0960	.0895	.0839	.0764	.0634	.0535	.0438
		…	…	…	…	…	.885	1.185	1.546	1.978	2.48	3.07	3.75	4.53	5.40	6.40	8.10	12.85	19.27	30.7

[a] Round wire. For square wire, multiply *f* by 0.707, and *p*, by 1.2

[b] The upper figure is the deflection and the lower figure the load as read against each spring size.

Table 5. *(Continued)* Compression and Extension Spring Deflections

Outside Diam.		Wire Size or Washburn and Moen Gauge, and Decimal Equivalent																	
									19	18	17	16	15	14	13	3/32	12	11	1/8
		.026	.028	.030	.032	.034	.036	.038	.041	.0475	.054	.0625	.072	.080	.0915	.0938	.1055	.1205	.125
Nom.	Dec.	Deflection *f* (inch) per coil, at Load *P* (pounds)																	
13/32	.4063	.1560	.1434	.1324	.1228	.1143	.1068	.1001	.0913	.0760	.0645	.0531	.0436	.0373	.0304	.0292	.0241	…	…
		1.815	2.28	2.82	3.44	4.15	4.95	5.85	7.41	11.73	17.56	27.9	43.9	61.6	95.6	103.7	153.3	…	…
7/16	.4375	.1827	.1680	.1553	.1441	.1343	.1256	.1178	.1075	.0898	.0764	.0631	.0521	.0448	.0367	.0353	.0293	.0234	.0219
		1.678	2.11	2.60	3.17	3.82	4.56	5.39	6.82	10.79	16.13	25.6	40.1	56.3	86.9	94.3	138.9	217.	245.
15/32	.4688	.212	.1947	.1800	.1673	.1560	.1459	.1370	.1252	.1048	.0894	.0741	.0614	.0530	.0437	.0420	.0351	.0282	.0265
		1.559	1.956	2.42	2.94	3.55	4.23	5.00	6.33	9.99	14.91	23.6	37.0	51.7	79.7	86.4	126.9	197.3	223.
1/2	.500	.243	.223	.207	.1920	.1792	.1678	.1575	.1441	.1209	.1033	.0859	.0714	.0619	.0512	.0494	.0414	.0335	.0316
		1.456	1.826	2.26	2.75	3.31	3.95	4.67	5.90	9.30	13.87	21.9	34.3	47.9	73.6	80.0	116.9	181.1	205.
17/32	.5313	.276	.254	.235	.219	.204	.1911	.1796	.1645	.1382	.1183	.0987	.0822	.0714	.0593	.0572	.0482	.0393	.0371
		1.366	1.713	2.12	2.58	3.10	3.70	4.37	5.52	8.70	12.96	20.5	31.9	44.6	68.4	74.1	108.3	167.3	188.8
9/16	.5625	…	.286	.265	.247	.230	.216	.203	.1861	.1566	.1343	.1122	.0937	.0816	.0680	.0657	.0555	.0455	.0430
		…	1.613	1.991	2.42	2.92	3.48	4.11	5.19	8.18	12.16	19.17	29.9	41.7	63.9	69.1	100.9	155.5	175.3
19/32	.5938	…	…	.297	.277	.259	.242	.228	.209	.1762	.1514	.1267	.1061	.0926	.0774	.0748	.0634	.0522	.0493
		…	…	1.880	2.29	2.76	3.28	3.88	4.90	7.71	11.46	18.04	28.1	39.1	60.0	64.8	94.4	145.2	163.6
5/8	.625	…	…	.331	.308	.288	.270	.254	.233	.1969	.1693	.1420	.1191	.1041	.0873	.0844	.0718	.0593	.0561
		…	…	1.782	2.17	2.61	3.11	3.67	4.63	7.29	10.83	17.04	26.5	36.9	56.4	61.0	88.7	136.2	153.4
21/32	.6563	…	…	…	.342	.320	.300	.282	.259	.219	.1884	.1582	.1330	.1164	.0978	.0946	.0807	.0668	.0634
		…	…	…	2.06	2.48	2.95	3.49	4.40	6.92	10.27	16.14	25.1	34.9	53.3	57.6	83.7	128.3	144.3
11/16	.6875	…	…	…	…	.352	.331	.311	.286	.242	.208	.1753	.1476	.1294	.1089	.1054	.0901	.0748	.0710
		…	…	…	…	2.36	2.81	3.32	4.19	6.58	9.76	15.34	23.8	33.1	50.5	54.6	79.2	121.2	136.3
23/32	.7188	…	…	…	…	…	.363	.342	.314	.266	.230	.1933	.1630	.1431	.1206	.1168	.1000	.0833	.0791
		…	…	…	…	…	2.68	3.17	3.99	6.27	9.31	14.61	22.7	31.5	48.0	51.9	75.2	114.9	129.2
3/4	.750	…	…	…	…	…	…	.374	.344	.291	.252	.212	.1791	.1574	.1329	.1288	.1105	.0923	.0877
		…	…	…	…	…	…	3.03	3.82	5.99	8.89	13.94	21.6	30.0	45.7	49.4	71.5	109.2	122.7
25/32	.7813	…	…	…	…	…	…	…	.375	.318	.275	.232	.1960	.1724	.1459	.1413	.1214	.1017	.0967
		…	…	…	…	…	…	…	3.66	5.74	8.50	13.34	20.7	28.7	43.6	47.1	68.2	104.0	116.9
13/16	.8125	…	…	…	…	…	…	…	.407	.346	.299	.253	.214	.1881	.1594	.1545	.1329	.1115	.1061
		…	…	…	…	…	…	…	3.51	5.50	8.15	12.78	19.80	27.5	41.7	45.1	65.2	99.3	111.5

Table 5. *(Continued)* Compression and Extension Spring Deflections

Outside Diam.		Wire Size or Washburn and Moen Gauge, and Decimal Equivalent																
		15	14	13	3/32	12	11	1/8	10	9	5/32	8	7	3/16	6	5	7/32	4
		.072	.080	.0915	.0938	.1055	.1205	.125	.135	.1483	.1563	.162	.177	.1875	.192	.207	.2188	.2253
Nom.	Dec.	Deflection *f* (inch) per coil, at Load *P* (pounds)																
7/8	.875	.251	.222	.1882	.1825	.1574	.1325	.1262	.1138	.0999	.0928	.0880	.0772	.0707	.0682	.0605	.0552	.0526
		18.26	25.3	39.4	41.5	59.9	91.1	102.3	130.5	176.3	209.	234.	312.	377.	407.	521.	626.	691.
29/32	.9063	.271	.239	.204	.1974	.1705	.1438	.1370	.1236	.1087	.1010	.0959	.0843	.0772	.0746	.0663	.0606	.0577
		17.57	24.3	36.9	39.9	57.6	87.5	98.2	125.2	169.0	199.9	224.	299.	360.	389.	498.	598.	660.
15/16	.9375	.292	.258	.219	.213	.1841	.1554	.1479	.1338	.1178	.1096	.1041	.0917	.0842	.0812	.0723	.0662	.0632
		16.94	23.5	35.6	38.4	55.4	84.1	94.4	120.4	162.3	191.9	215.	286.	345.	373.	477.	572.	631.
31/32	.9688	.313	.277	.236	.229	.1982	.1675	.1598	.1445	.1273	.1183	.1127	.0994	.0913	.0882	.0786	.0721	.0688
		16.35	22.6	34.3	37.0	53.4	81.0	90.9	115.9	156.1	184.5	207.	275.	332.	358.	457.	548.	604.
1	1.000	.336	.297	.253	.246	.213	.1801	.1718	.1555	.1372	.1278	.1216	.1074	.0986	.0954	.0852	.0783	.0747
		15.80	21.9	33.1	35.8	51.5	78.1	87.6	111.7	150.4	177.6	198.8	264.	319.	344.	439.	526.	580.
1 1/32	1.031	.359	.317	.271	.263	.228	.1931	.1843	.1669	.1474	.1374	.1308	.1157	.1065	.1029	.0921	.0845	.0809
		15.28	21.1	32.0	34.6	49.8	75.5	84.6	107.8	145.1	171.3	191.6	255.	307.	331.	423.	506.	557.
1 1/16	1.063	.382	.338	.289	.281	.244	.207	.1972	.1788	.1580	.1474	.1404	.1243	.1145	.1107	.0993	.0913	.0873
		14.80	20.5	31.0	33.5	48.2	73.0	81.8	104.2	140.1	165.4	185.0	246.	296.	319.	407.	487.	537.
1 3/32	1.094	.407	.360	.308	.299	.260	.221	.211	.1910	.1691	.1578	.1503	.1332	.1229	.1188	.1066	.0982	.0939
		14.34	19.83	30.0	32.4	46.7	70.6	79.2	100.8	135.5	159.9	178.8	238.	286.	308.	393.	470.	517.
1 1/8	1.125	.432	.383	.328	.318	.277	.235	.224	.204	.1804	.1685	.1604	.1424	.1315	.1272	.1142	.1053	.1008
		13.92	19.24	29.1	31.4	45.2	68.4	76.7	97.6	131.2	154.7	173.0	230.	276.	298.	379.	454.	499.
1 3/16	1.188	.485	.431	.368	.358	.311	.265	.254	.231	.204	.1908	.1812	.1620	.1496	.1448	.1303	.1203	.1153
		13.14	18.15	27.5	29.6	42.6	64.4	72.1	91.7	123.3	145.4	162.4	215.	259.	279.	355.	424.	467.
1 1/4	1.250	.541	.480	.412	.400	.349	.297	.284	.258	.230	.215	.205	.1824	.1690	.1635	.1474	.1363	.1308
		12.44	17.19	26.0	28.0	40.3	60.8	68.2	86.6	116.2	137.0	153.1	203.	244.	263.	334.	399.	438.
1 5/16	1.313	.600	.533	.457	.444	.387	.331	.317	.288	.256	.240	.229	.205	.1894	.1836	.1657	.1535	.1472
		11.81	16.31	24.6	26.6	38.2	57.7	64.6	82.0	110.1	129.7	144.7	191.6	230.	248.	315.	376.	413.
1 3/8	1.375	.662	.588	.506	.491	.429	.367	.351	.320	.285	.267	.255	.227	.211	.204	.1848	.1713	.1650
		11.25	15.53	23.4	25.3	36.3	54.8	61.4	77.9	104.4	123.0	137.3	181.7	218.	235.	298.	356.	391
1 7/16	1.438	.727	.647	.556	.540	.472	.404	.387	.353	.314	.295	.282	.252	.234	.227	.205	.1905	.1829
		10.73	14.81	22.3	24.1	34.6	52.2	58.4	74.1	99.4	117.0	130.6	172.6	207.	223.	283.	337.	371.

Table 5. *(Continued)* **Compression and Extension Spring Deflections**

Outside Diam.		Wire Size or Washburn and Moen Gauge, and Decimal Equivalent																	
		11	1/8	10	9	5/32	8	7	3/16	6	5	7/32	4	3	1/4	2	9/32	0	5/16
		.1205	.125	.135	.1483	.1563	.162	.177	.1875	.192	.207	.2188	.2253	.2437	.250	.2625	.2813	.3065	.3125
Nom.	Dec.	Deflection *f* (inch) per coil, at Load *P* (pounds)																	
1½	1.500	.443	.424	.387	.350	.324	.310	.277	.258	.250	.227	.210	.202	.1815	.1754	.1612	.1482	.1305	.1267
		49.8	55.8	70.8	94.8	111.5	124.5	164.6	197.1	213.	269.	321.	352.	452.	499.	574.	717.	947.	1008.
1⅝	1.625	.527	.505	.461	.413	.387	.370	.332	.309	.300	.273	.254	.244	.220	.212	.1986	.1801	.1592	.1547
		45.7	51.1	64.8	86.7	102.0	113.9	150.3	180.0	193.9	246.	292.	321.	411.	446.	521.	650.	858.	912.
1¾	1.750	.619	.593	.542	.485	.456	.437	.392	.366	.355	.323	.301	.290	.261	.253	.237	.215	.1908	.1856
		42.2	47.2	59.8	80.0	94.0	104.9	138.5	165.6	178.4	226.	269.	295.	377.	409.	477.	595.	783.	833.
1⅞	1.875	.717	.687	.629	.564	.530	.508	.457	.426	.414	.377	.351	.339	.306	.296	.278	.253	.225	.219
		39.2	43.8	55.5	74.2	87.2	97.3	128.2	153.4	165.1	209.	248.	272.	348.	378.	440.	548.	721.	767.
1 15/16	1.938	.769	.738	.676	.605	.569	.546	.492	.458	.446	.405	.379	.365	.331	.320	.300	.273	.243	.237
		37.8	42.3	53.6	71.6	84.2	93.8	123.6	147.9	159.2	201.	239.	262.	335.	364.	425.	528.	693.	737.
2	2.000	.823	.789	.723	.649	.610	.585	.527	.492	.478	.436	.407	.392	.355	.344	.323	.295	.263	.256
		36.6	40.9	51.8	69.2	81.3	90.6	119.4	142.8	153.7	194.3	231.	253.	324.	351.	409.	509.	668.	710.
2 1/16	2.063	.878	.843	.768	.693	.652	.626	.564	.526	.512	.467	.436	.421	.381	.369	.346	.316	.282	.275
		35.4	39.6	50.1	66.9	78.7	87.6	115.4	138.1	148.5	187.7	223.	245.	312.	339.	395.	491.	644.	685.
2⅛	2.125	.936	.898	.823	.739	.696	.667	.602	.562	.546	.499	.466	.449	.407	.395	.371	.339	.303	.295
		34.3	38.3	48.5	64.8	76.1	84.9	111.8	133.6	143.8	181.6	216.	236.	302.	327.	381.	474.	622.	661.
2 3/16	2.188	.995	.955	.876	.786	.740	.711	.641	.598	.582	.532	.497	.479	.435	.421	.396	.362	.324	.316
		33.3	37.2	47.1	62.8	73.8	82.2	108.3	129.5	139.2	175.8	209.	229.	292.	317.	369.	459.	601.	639.
2¼	2.250	1.056	1.013	.930	.835	.787	.755	.681	.637	.619	.566	.529	.511	.463	.449	.423	.387	.346	.337
		32.3	36.1	45.7	60.9	71.6	79.8	105.7	125.5	135.0	170.5	202.	222.	283.	307.	357.	444.	582.	618.
2 5/16	2.313	1.119	1.074	.986	.886	.834	.801	.723	.676	.657	.601	.562	.542	.493	.478	.449	.411	.368	.359
		31.4	35.1	44.4	59.2	69.5	77.5	101.9	121.8	131.0	165.4	196.3	215.	275.	298.	347.	430.	564.	599.
2⅜	2.375	1.184	1.136	1.043	.938	.884	.848	.763	.716	.696	.637	.596	.576	.523	.507	.477	.437	.392	.382
		30.5	34.1	43.1	57.5	67.6	75.3	99.1	118.3	127.3	160.7	190.7	209.	267.	289.	336.	417.	547.	581.
2 7/16	2.438	…	1.201	1.102	.991	.934	.897	.810	.757	.737	.674	.631	.609	.554	.537	.506	.464	.416	.405
		…	33.2	42.0	56.0	65.7	73.2	96.3	115.1	123.7	156.1	185.3	203.	259.	281.	327.	405.	531.	564.
2½	2.500	…	1.266	1.162	1.046	.986	.946	.855	.800	.778	.713	.667	.644	.586	.568	.536	.491	.441	.430
		…	32.3	40.9	54.5	64.0	71.3	93.7	111.6	120.4	151.9	180.2	197.5	252.	273.	317.	394.	516.	548.

Note: Intermediate values can be obtained within reasonable accuracy by interpolation.

The table is for ASTM A229 oil tempered spring steel with a torsional modulus *G* of 11,200,000 psi, and an uncorrected torsional stress of 100,000 psi. For other materials use the following factors: stainless steel, multiply *f* by 1.067; spring brass, multiply *f* by 2.24; phosphor bronze, multiply *f* by 1.867; Monel metal, multiply *f* by 1.244; beryllium copper, multiply *f* by 1.725; Inconel (non-magnetic), multiply *f* by 1.045.

Step 4: The stress $S = \dfrac{PD}{0.393d^3} = \dfrac{36 \times 0.721}{0.393 \times 0.0915^3} = 86{,}300 \text{ lb/in}^2$

Step 5: The number of active coils is $N = \dfrac{GdF}{\pi SD^2}$

$$= \frac{11{,}200{,}000 \times 0.0915 \times 1.25}{3.1416 \times 86{,}300 \times 0.721^2} = 9.1 \text{ (say 9)}$$

The answer is the same as before, which is to use 11 total coils of 0.0915-inch diameter wire. The total coils, solid height, etc., are determined in the same manner as in Method 1.

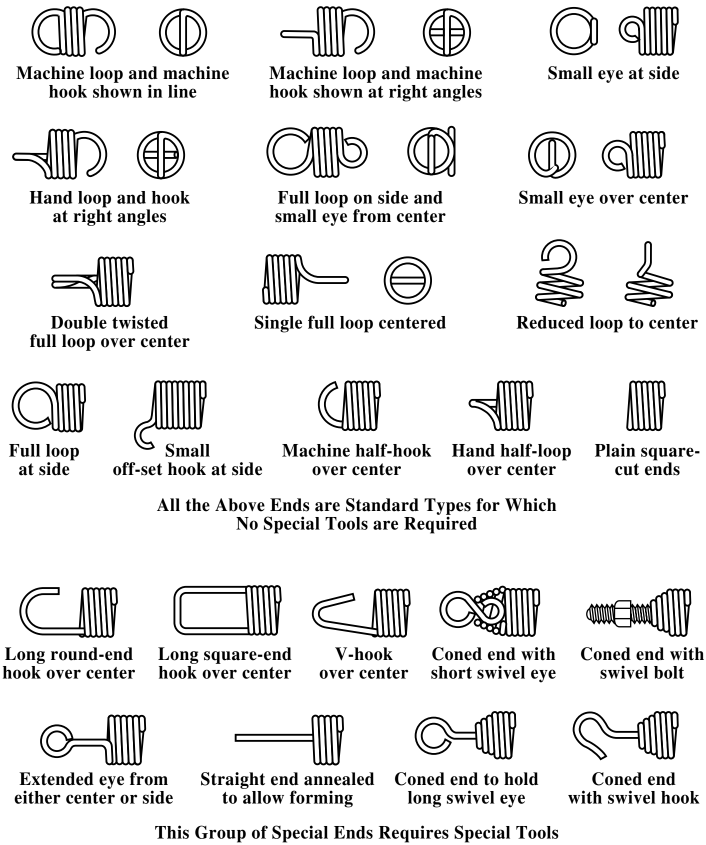


Fig. 15. Types of Helical Extension Spring Ends

Table of Spring Characteristics.—Table 5 gives characteristics for compression and extension springs made from ASTM A229 oil-tempered MB spring steel having a torsional modulus of elasticity *G* of 11,200,000 pounds per square inch, and an uncorrected torsional stress *S* of 100,000 pounds per square inch. The deflection *f* for one coil under a load *P* is shown in the body of the table. The method of using these data is explained in the problems for compression and extension spring design. The table may be used for other materials by applying factors to *f*. The factors are given in a footnote to the table.

Extension Springs.—About 10 per cent of all springs made by many companies are of this type, and they frequently cause trouble because insufficient consideration is given to stress due to initial tension, stress and deflection of hooks, special manufacturing methods, secondary operations and overstretching at assembly. Fig. 15 shows types of ends used on these springs.

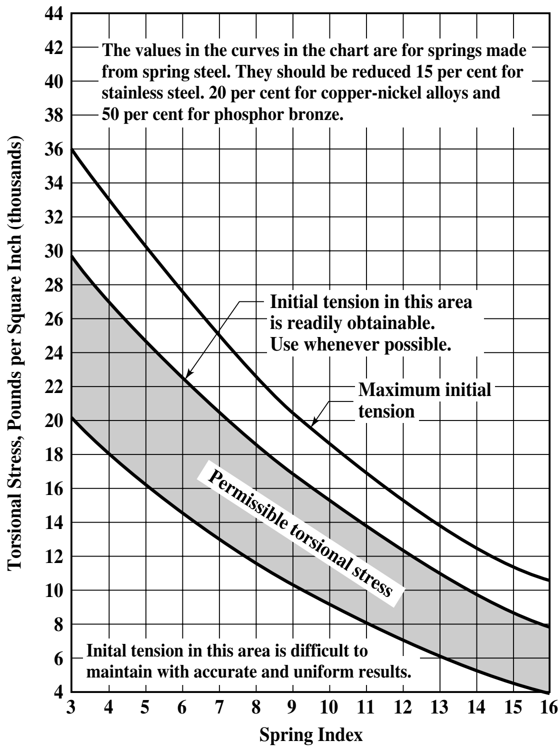


Fig. 16. Permissible Torsional Stress Caused by Initial Tension in Coiled Extension Springs for Different Spring Indexes

Initial tension: In the spring industry, the term "Initial tension" is used to define a force or load, measurable in pounds or ounces, which presses the coils of a close wound extension spring against one another. This force must be overcome before the coils of a spring begin to open up.

Initial tension is wound into extension springs by bending each coil as it is wound away from its normal plane, thereby producing a slight twist in the wire which causes the coil to spring back tightly against the adjacent coil. Initial tension can be wound into cold-coiled extension springs only. Hot-wound springs and springs made from annealed steel are hardened and tempered after coiling, and therefore initial tension cannot be produced. It is possible to make a spring having initial tension only when a high tensile strength, obtained by cold drawing or by heat-treatment, is possessed by the material as it is being wound into springs. Materials that possess the required characteristics for the manufacture of such springs include hard-drawn wire, music wire, pre-tempered wire, 18-8 stainless steel, phosphor-bronze, and many of the hard-drawn copper-nickel, and nonferrous alloys. Permissible torsional stresses resulting from initial tension for different spring indexes are shown in Fig. 16.

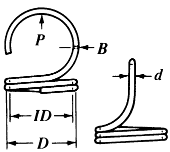
Hook failure: The great majority of breakages in extension springs occurs in the hooks. Hooks are subjected to both bending and torsional stresses and have higher stresses than the coils in the spring.

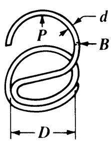
Stresses in regular hooks: The calculations for the stresses in hooks are quite complicated and lengthy. Also, the radii of the bends are difficult to determine and frequently vary between specifications and actual production samples. However, regular hooks are more highly stressed than the coils in the body and are subjected to a bending stress at section B (see Table 6.) The bending stress S_b at section B should be compared with allowable stresses for torsion springs and with the elastic limit of the material in tension (See Figs. 7 through 10.)

Stresses in cross over hooks: Results of tests on springs having a normal average index show that the cross over hooks last longer than regular hooks. These results may not occur on springs of small index or if the cross over bend is made too sharply.

Inasmuch as both types of hooks have the same bending stress, it would appear that the fatigue life would be the same. However, the large bend radius of the regular hooks causes some torsional stresses to coincide with the bending stresses, thus explaining the earlier breakages. If sharper bends were made on the regular hooks, the life should then be the same as for cross over hooks.

Table 6. Formula for Bending Stress at Section B

<table>
<tr><th>Type of Hook</th><th>Stress in Bending</th></tr>
<tr><td>

Regular Hook</td><td rowspan="2">$S_b = \frac{5PD^2}{\mathrm{I.D.}d^3}$</td></tr>
<tr><td>

Cross-over Hook</td></tr>
</table>

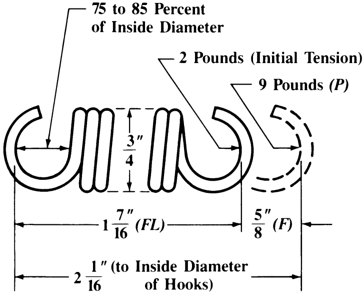


Fig. 17. Extension Spring Design Example

Stresses in half hooks: The formulas for regular hooks can also be used for half hooks, because the smaller bend radius allows for the increase in stress. It will therefore be observed that half hooks have the same stress in bending as regular hooks.

Frequently overlooked facts by many designers are that one full hook deflects an amount equal to one half a coil and each half hook deflects an amount equal to one tenth of a coil. Allowances for these deflections should be made when designing springs. Thus, an extension spring, with regular full hooks and having 10 coils, will have a deflection equal to 11 coils, or 10 per cent more than the calculated deflection.

Extension Spring Design.—The available space in a product or assembly usually determines the limiting dimensions of a spring, but the wire size, number of coils, and initial tension are often unknown.

Example: An extension spring is to be made from spring steel ASTM A229, with regular hooks as shown in Fig. 17. Calculate the wire size, number of coils and initial tension.

Note: Allow about 20 to 25 per cent of the 9 pound load for initial tension, say 2 pounds, and then design for a 7 pound load (not 9 pounds) at ⅝ inch deflection. Also use lower stresses than for a compression spring to allow for overstretching during assembly and to obtain a safe stress on the hooks. Proceed as for compression springs, but locate a load in the tables somewhat higher than the 9 pound load.

Method 1, using table: From Table locate ¾ inch outside diameter in the left column and move to the right to locate a load P of 13.94 pounds. A deflection f of 0.212 inch appears above this figure. Moving vertically from this position to the top of the column a suitable wire diameter of 0.0625 inch is found.

The remaining design calculations are completed as follows:

Step 1: The stress with a load of 7 pounds is obtained as follows:
The 7 pound load is 50.2 per cent of the 13.94 pound load. Therefore, the stress S at 7 pounds = 0.502 per cent × 100,000 = 50,200 pounds per square inch.

Step 2: The 50.2 per cent figure is also used to determine the deflection per coil f: 0.502 per cent × 0.212 = 0.1062 inch.

Step 3: The number of active coils. (say 6)

$$AC = \frac{F}{f} = \frac{0.625}{0.1062} = 5.86$$

This result should be reduced by 1 to allow for deflection of 2 hooks (see notes 1 and 2 that follow these calculations.) Therefore, a quick answer is: 5 coils of 0.0625 inch diameter wire. However, the design procedure should be completed by carrying out the following steps:

Step 4: The body length = $(TC + 1) \times d = (5 + 1) \times 0.0625 = \frac{3}{8}$ inch.

Step 5: The length from the body to inside hook

$$= \frac{FL - \text{Body}}{2} = \frac{1.4375 - 0.375}{2} = 0.531 \text{ inch}$$

$$\text{Percentage of I.D.} = \frac{0.531}{\text{I.D.}} = \frac{0.531}{0.625} = 85 \text{ per cent}$$

This length is satisfactory, see Note 3 following this proceedure.

Step 6:

$$\text{The spring index} = \frac{\text{O.D.}}{d} - 1 = \frac{0.75}{0.0625} - 1 = 11$$

Step 7: The initial tension stress is

$$S_{it} = \frac{S \times IT}{P} = \frac{50{,}200 \times 2}{7}$$

$$= 14{,}340 \text{ pounds per square inch}$$

This stress is satisfactory, as checked against curve in Fig. 16.

Step 8: The curvature correction factor $K = 1.12$ (Fig. 13).

Step 9: The total stress = (50,200 + 14,340) × 1.12 = 72.285 pounds per square inch

This result is less than 106,250 pounds per square inch permitted by the middle curve for 0.0625 inch wire in Fig. 3 and therefore is a safe working stress that permits some additional deflection that is usually necessary for assembly purposes.

Step 10: The large majority of hook breakage is due to high stress in bending and should be checked as follows:

From Table 6, stress on hook in bending is:

$$S_b = \frac{5PD^2}{\text{I.D.}d^3}$$

$$= \frac{5 \times 9 \times 0.6875^2}{0.625 \times 0.0625^3} = 139{,}200 \text{ pounds per square inch}$$

This result is less than the top curve value, Fig. 8, for 0.0625 inch diameter wire, and is therefore safe. Also see Note 5 that follows.

Notes: The following points should be noted when designing extension springs:

1) All coils are active and thus $AC = TC$.

2) Each full hook deflection is approximately equal to ½ coil. Therefore for 2 hooks, reduce the total coils by 1. (Each half hook deflection is nearly equal to $\frac{1}{10}$ of a coil.)

3) The distance from the body to the inside of a regular full hook equals 75 to 85 per cent (90 per cent maximum) of the I.D. For a cross over center hook, this distance equals the I.D.

4) Some initial tension should usually be used to hold the spring together. Try not to exceed the maximum curve shown on Fig. 16. Without initial tension, a long spring with many coils will have a different length in the horizontal position than it will when hung vertically.

5) The hooks are stressed in bending, therefore their stress should be less than the maximum bending stress as used for torsion springs — use top fatigue strength curves Figs. 7 through 10.

Method 2, using formulas: The sequence of steps for designing extension springs by formulas is similar to that for compression springs. The formulas for this method are given in Table 3.

Tolerances for Compression and Extension Springs.—Tolerances for coil diameter, free length, squareness, load, and the angle between loop planes for compression and extension springs are given in Tables 7 through 12. To meet the requirements of load, rate, free length, and solid height, it is necessary to vary the number of coils for compression springs by ± 5 per cent. For extension springs, the tolerances on the numbers of coils are: for 3 to 5 coils, ± 20 per cent; for 6 to 8 coils, ± 30 per cent; for 9 to 12 coils, ± 40 per cent. For each additional coil, a further 1½ per cent tolerance is added to the extension spring values. Closer tolerances on the number of coils for either type of spring lead to the need for trimming after coiling, and manufacturing time and cost are increased. Fig. 18 shows deviations allowed on the ends of extension springs, and variations in end alignments.

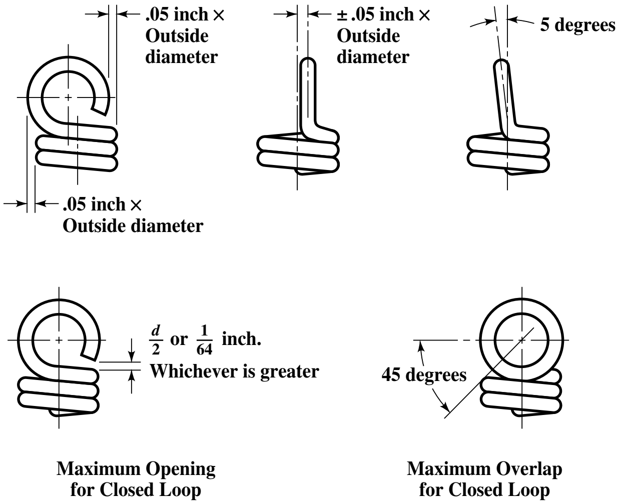


Fig. 18. Maximum Deviations Allowed on Ends and Variation in Alignment of Ends (Loops) for Extension Springs

Table 7. Compression and Extension Spring Coil Diameter Tolerances

Wire Diameter, Inch	Spring Index						
	4	6	8	10	12	14	16
	Tolerance, ± inch						
0.015	0.002	0.002	0.003	0.004	0.005	0.006	0.007
0.023	0.002	0.003	0.004	0.006	0.007	0.008	0.010
0.035	0.002	0.004	0.006	0.007	0.009	0.011	0.013
0.051	0.003	0.005	0.007	0.010	0.012	0.015	0.017
0.076	0.004	0.007	0.010	0.013	0.016	0.019	0.022
0.114	0.006	0.009	0.013	0.018	0.021	0.025	0.029
0.171	0.008	0.012	0.017	0.023	0.028	0.033	0.038
0.250	0.011	0.015	0.021	0.028	0.035	0.042	0.049
0.375	0.016	0.020	0.026	0.037	0.046	0.054	0.064
0.500	0.021	0.030	0.040	0.062	0.080	0.100	0.125

Courtesy of the Spring Manufacturers Institute

Table 8. Compression Spring Normal Free-Length Tolerances, Squared and Ground Ends

Number of Active Coils per Inch	Spring Index						
	4	6	8	10	12	14	16
	Tolerance, ± Inch per Inch of Free Length[a]						
0.5	0.010	0.011	0.012	0.013	0.015	0.016	0.016
1	0.011	0.013	0.015	0.016	0.017	0.018	0.019
2	0.013	0.015	0.017	0.019	0.020	0.022	0.023
4	0.016	0.018	0.021	0.023	0.024	0.026	0.027
8	0.019	0.022	0.024	0.026	0.028	0.030	0.032
12	0.021	0.024	0.027	0.030	0.032	0.034	0.036
16	0.022	0.026	0.029	0.032	0.034	0.036	0.038
20	0.023	0.027	0.031	0.034	0.036	0.038	0.040

[a] For springs less than 0.5 inch long, use the tolerances for 0.5 inch long springs. For springs with unground closed ends, multiply the tolerances by 1.7.

Courtesy of the Spring Manufacturers Institute

Table 9. Extension Spring Normal Free-Length and End Tolerances

Free-Length Tolerances		End Tolerances	
Spring Free-Length (inch)	Tolerance (inch)	Total Number of Coils	Angle Between Loop Planes (degrees)
Up to 0.5	±0.020		
Over 0.5 to 1.0	±0.030	3 to 6	±25
Over 1.0 to 2.0	±0.040	7 to 9	±35
Over 2.0 to 4.0	±0.060	10 to 12	±45
Over 4.0 to 8.0	±0.093	13 to 16	±60
Over 8.0 to 16.0	±0.156	Over 16	Random
Over 16.0 to 24.0	±0.218		

Courtesy of the Spring Manufacturers Institute

Table 10. Compression Spring Squareness Tolerances

Slenderness Ratio *FL/D*[a]	Spring Index						
	4	6	8	10	12	14	16
	Squareness Tolerances (± degrees)						
0.5	3.0	3.0	3.5	3.5	3.5	3.5	4.0
1.0	2.5	3.0	3.0	3.0	3.0	3.5	3.5
1.5	2.5	2.5	2.5	3.0	3.0	3.0	3.0
2.0	2.5	2.5	2.5	2.5	3.0	3.0	3.0
3.0	2.0	2.5	2.5	2.5	2.5	2.5	3.0
4.0	2.0	2.0	2.5	2.5	2.5	2.5	2.5
6.0	2.0	2.0	2.0	2.5	2.5	2.5	2.5
8.0	2.0	2.0	2.0	2.0	2.5	2.5	2.5
10.0	2.0	2.0	2.0	2.0	2.0	2.5	2.5
12.0	2.0	2.0	2.0	2.0	2.0	2.0	2.5

[a] Slenderness Ratio = *FL*÷*D*

Springs with closed and ground ends, in the free position. Squareness tolerances closer than those shown require special process techniques which increase cost. Springs made from fine wire sizes, and with high spring indices, irregular shapes or long free lengths, require special attention in determining appropriate tolerance and feasibility of grinding ends.

Table 11. Compression Spring Normal Load Tolerances

Length Tolerance, ± inch	Deflection (inch)[a]							
	0.05	0.10	0.15	0.20	0.25	0.30	0.40	0.50
	Tolerance, ± Per Cent of Load							
0.005	12	7	6	5	…	…	…	…
0.010	…	12	8.5	7	6.5	5.5	5	…
0.020	…	22	15.5	12	10	8.5	7	6
0.030	…	…	22	17	14	12	9.5	8
0.040	…	…	…	22	18	15.5	12	10
0.050	…	…	…	…	22	19	14.5	12
0.060	…	…	…	…	25	22	17	14
0.070	…	…	…	…	…	25	19.5	16
0.080	…	…	…	…	…	…	22	18
0.090	…	…	…	…	…	…	25	20
0.100	…	…	…	…	…	…	…	22
0.200	…	…	…	…	…	…	…	…
0.300	…	…	…	…	…	…	…	…
0.400	…	…	…	…	…	…	…	…
0.500	…	…	…	…	…	…	…	…

Length Tolerance, ± inch	Deflection (inch)[a]						
	0.75	1.00	1.50	2.00	3.00	4.00	6.00
	Tolerance, ± Per Cent of Load						
0.005	…	…	…	…	…	…	…
0.010	…	…	…	…	…	…	…
0.020	5	…	…	…	…	…	…
0.030	6	5	…	…	…	…	…
0.040	7.5	6	5	…	…	…	…
0.050	9	7	5.5	…	…	…	…
0.060	10	8	6	5	…	…	…
0.070	11	9	6.5	5.5	…	…	…
0.080	12.5	10	7.5	6	5	…	…
0.090	14	11	8	6	5	…	…
0.100	15.5	12	8.5	7	5.5	…	…
0.200	…	22	15.5	12	8.5	7	5.5
0.300	…	…	22	17	12	9.5	7
0.400	…	…	…	21	15	12	8.5
0.500	…	…	…	25	18.5	14.5	10.5

[a] From free length to loaded position.

Table 12. Extension Spring Normal Load Tolerances

Spring Index	$\frac{FL}{F}$	Wire Diameter (inch)										
		0.015	0.022	0.032	0.044	0.062	0.092	0.125	0.187	0.250	0.375	0.437
		Tolerance, ± Per Cent of Load										
4	12	20.0	18.5	17.6	16.9	16.2	15.5	15.0	14.3	13.8	13.0	12.6
	8	18.5	17.5	16.7	15.8	15.0	14.5	14.0	13.2	12.5	11.5	11.0
	6	16.8	16.1	15.5	14.7	13.8	13.2	12.7	11.8	11.2	9.9	9.4
	4.5	15.0	14.7	14.1	13.5	12.6	12.0	11.5	10.3	9.7	8.4	7.9
	2.5	13.1	12.4	12.1	11.8	10.6	10.0	9.1	8.5	8.0	6.8	6.2
	1.5	10.2	9.9	9.3	8.9	8.0	7.5	7.0	6.5	6.1	5.3	4.8
	0.5	6.2	5.4	4.8	4.6	4.3	4.1	4.0	3.8	3.6	3.3	3.2
6	12	17.0	15.5	14.6	14.1	13.5	13.1	12.7	12.0	11.5	11.2	10.7
	8	16.2	14.7	13.9	13.4	12.6	12.2	11.7	11.0	10.5	10.0	9.5
	6	15.2	14.0	12.9	12.3	11.6	10.9	10.7	10.0	9.4	8.8	8.3
	4.5	13.7	12.4	11.5	11.0	10.5	10.0	9.6	9.0	8.3	7.6	7.1
	2.5	11.9	10.8	10.2	9.8	9.4	9.0	8.5	7.9	7.2	6.2	6.0
	1.5	9.9	9.0	8.3	7.7	7.3	7.0	6.7	6.4	6.0	4.9	4.7
	0.5	6.3	5.5	4.9	4.7	4.5	4.3	4.1	4.0	3.7	3.5	3.4
8	12	15.8	14.3	13.1	13.0	12.1	12.0	11.5	10.8	10.2	10.0	9.5
	8	15.0	13.7	12.5	12.1	11.4	11.0	10.6	10.1	9.4	9.0	8.6
	6	14.2	13.0	11.7	11.2	10.6	10.0	9.7	9.3	8.6	8.1	7.6
	4.5	12.8	11.7	10.7	10.1	9.7	9.0	8.7	8.3	7.8	7.2	6.6
	2.5	11.2	10.2	9.5	8.8	8.3	7.9	7.7	7.4	6.9	6.1	5.6
	1.5	9.5	8.6	7.8	7.1	6.9	6.7	6.5	6.2	5.8	4.9	4.5
	0.5	6.3	5.6	5.0	4.8	4.5	4.4	4.2	4.1	3.9	3.6	3.5
10	12	14.8	13.3	12.0	11.9	11.1	10.9	10.5	9.9	9.3	9.2	8.8
	8	14.2	12.8	11.6	11.2	10.5	10.2	9.7	9.2	8.6	8.3	8.0
	6	13.4	12.1	10.8	10.5	9.8	9.3	8.9	8.6	8.0	7.6	7.2
	4.5	12.3	10.8	10.0	9.5	9.0	8.5	8.1	7.8	7.3	6.8	6.4
	2.5	10.8	9.6	9.0	8.4	8.0	7.7	7.3	7.0	6.5	5.9	5.5
	1.5	9.2	8.3	7.5	6.9	6.7	6.5	6.3	6.0	5.6	5.0	4.6
	0.5	6.4	5.7	5.1	4.9	4.7	4.5	4.3	4.2	4.0	3.8	3.7
12	12	14.0	12.3	11.1	10.8	10.1	9.8	9.5	9.0	8.5	8.2	7.9
	8	13.2	11.8	10.7	10.2	9.6	9.3	8.9	8.4	7.9	7.5	7.2
	6	12.6	11.2	10.2	9.7	9.0	8.5	8.2	7.9	7.4	6.9	6.4
	4.5	11.7	10.2	9.4	9.0	8.4	8.0	7.6	7.2	6.8	6.3	5.8
	2.5	10.5	9.2	8.5	8.0	7.8	7.4	7.0	6.6	6.1	5.6	5.2
	1.5	8.9	8.0	7.2	6.8	6.5	6.3	6.1	5.7	5.4	4.8	4.5
	0.5	6.5	5.8	5.3	5.1	4.9	4.7	4.5	4.3	4.2	4.0	3.3
14	12	13.1	11.3	10.2	9.7	9.1	8.8	8.4	8.1	7.6	7.2	7.0
	8	12.4	10.9	9.8	9.2	8.7	8.3	8.0	7.6	7.2	6.8	6.4
	6	11.8	10.4	9.3	8.8	8.3	7.7	7.5	7.2	6.8	6.3	5.9
	4.5	11.1	9.7	8.7	8.2	7.8	7.2	7.0	6.7	6.3	5.8	5.4
	2.5	10.1	8.8	8.1	7.6	7.1	6.7	6.5	6.2	5.7	5.2	5.0
	1.5	8.6	7.7	7.0	6.7	6.3	6.0	5.8	5.5	5.2	4.7	4.5
	0.5	6.6	5.9	5.4	5.2	5.0	4.8	4.6	4.4	4.3	4.2	4.0
16	12	12.3	10.3	9.2	8.6	8.1	7.7	7.4	7.2	6.8	6.3	6.1
	8	11.7	10.0	8.9	8.3	7.8	7.4	7.2	6.8	6.5	6.0	5.7
	6	11.0	9.6	8.5	8.0	7.5	7.1	6.9	6.5	6.2	5.7	5.4
	4.5	10.5	9.1	8.1	7.5	7.2	6.8	6.5	6.2	5.8	5.3	5.1
	2.5	9.7	8.4	7.6	7.0	6.7	6.3	6.1	5.7	5.4	4.9	4.7
	1.5	8.3	7.4	6.6	6.2	6.0	5.8	5.6	5.3	5.1	4.6	4.4
	0.5	6.7	5.9	5.5	5.3	5.1	5.0	4.8	4.6	4.5	4.3	4.1

FL/F = the ratio of the spring free length FL to the deflection F.

Torsion Spring Design.—Fig. 19 shows the types of ends most commonly used on torsion springs. To produce them requires only limited tooling. The straight torsion end is the least expensive and should be used whenever possible. After determining the spring load or torque required and selecting the end formations, the designer usually estimates suitable space or size limitations. However, the space should be considered approximate until the wire size and number of coils have been determined. The wire size is dependent principally upon the torque. Design data can be devoloped with the aid of the tabular data, which is a simple method, or by calculation alone, as shown in the following sections. Many other factors affecting the design and operation of torsion springs are also covered in the section, *Torsion Spring Design Recommendations* on page page 325. Design formulas are shown in Table .

Curvature correction: In addition to the stress obtained from the formulas for load or deflection, there is a direct shearing stress on the inside of the section due to curvature. Therefore, the stress obtained by the usual formulas should be multiplied by the factor K obtained from the curve in Fig. 20. The corrected stress thus obtained is used only for comparison with the allowable working stress (fatigue strength) curves to determine if it is a safe value, and should not be used in the formulas for deflection.

Torque: Torque is a force applied to a moment arm and tends to produce rotation. Torsion springs exert torque in a circular arc and the arms are rotated about the central axis. It should be noted that the stress produced is in bending, not in torsion. In the spring industry it is customary to specify torque in conjunction with the deflection or with the arms of a spring at a definite position. Formulas for torque are expressed in pound-inches. If ounce-inches are specified, it is necessary to divide this value by 16 in order to use the formulas.

When a load is specified at a distance from a centerline, the torque is, of course, equal to the load multiplied by the distance. The load can be in pounds or ounces with the distances in inches or the load can be in grams or kilograms with the distance in centimeters or millimeters, but to use the design formulas, all values must be converted to pounds and inches. Design formulas for torque are based on the tangent to the arc of rotation and presume that a rod is used to support the spring. The stress in bending caused by the moment $P \times R$ is identical in magnitude to the torque T, provided a rod is used.

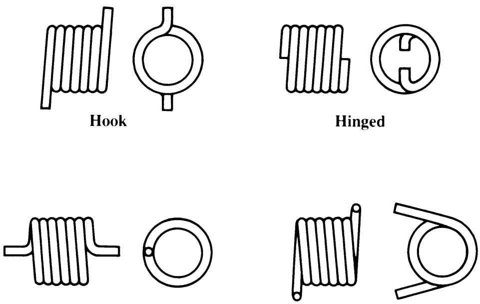


Fig. 19. The Most Commonly Used Types of Ends for Torsion Springs

Theoretically, it makes no difference how or where the load is applied to the arms of torsion springs. Thus, in Fig. 21, the loads shown multiplied by their respective distances produce the same torque; i.e., $20 \times 0.5 = 10$ pound-inches; $10 \times 1 = 10$ pound-inches; and $5 \times 2 = 10$ pound-inches. To further simplify the understanding of torsion spring torque, observe in both Fig. 22 and Fig. 23 that although the turning force is in a circular arc the torque is not

Table 13. Formulas for Torsion Springs

Feature	Springs made from round wire	Springs made from square wire	Feature	Springs made from round wire	Springs made from square wire
	Formula[a]			Formula[a]	
d = Wire diameter, Inches	$\sqrt[3]{\frac{10.18T}{S_b}}$	$\sqrt[3]{\frac{6T}{S_b}}$	F° = Deflection	$\frac{392S_bND}{Ed}$	$\frac{392S_bND}{Ed}$
	$\sqrt[4]{\frac{4000TND}{EF^\circ}}$	$\sqrt[4]{\frac{2375TND}{EF^\circ}}$		$\frac{4000TND}{Ed^4}$	$\frac{2375TND}{Ed^4}$
S_b = Stress, bending pounds per square inch	$\frac{10.18T}{d^3}$	$\frac{6T}{d^3}$	T = Torque Inch lbs. (Also = $P \times R$)	$0.0982S_bd^3$	$0.1666S_bd^3$
	$\frac{EdF^\circ}{392ND}$	$\frac{EdF^\circ}{392ND}$		$\frac{Ed^4F^\circ}{4000ND}$	$\frac{Ed^4F^\circ}{2375ND}$
N = Active Coils	$\frac{EdF^\circ}{392S_bD}$	$\frac{EdF^\circ}{392S_bD}$	ID_1 = Inside Diameter After Deflection, Inches	$\frac{N(ID\ \text{free})}{N + \frac{F^\circ}{360}}$	$\frac{N(ID\ \text{free})}{N + \frac{F^\circ}{360}}$
	$\frac{Ed^4F^\circ}{4000TD}$	$\frac{Ed^4F^\circ}{2375TD}$	The symbol notation is given on page 285.		

[a] Where two formulas are given for one feature, the designer should use the one found to be appropriate for the given design. The end result from either of any two formulas is the same.

equal to P times the radius. The torque in both designs equals $P \times R$ because the spring rests against the support rod at point a.

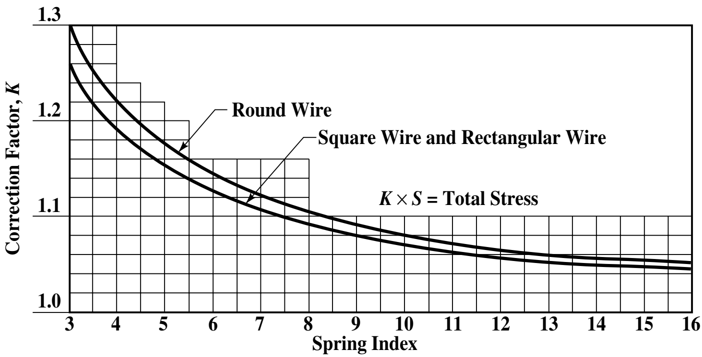


Fig. 20. Torsion Spring Stress Correction for Curvature

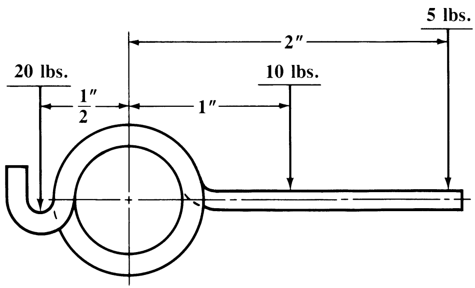


Fig. 21. Right-Hand Torsion Spring

Design Procedure: Torsion spring designs require more effort than other kinds because consideration has to be given to more details such as the proper size of a supporting rod, reduction of the inside diameter, increase in length, deflection of arms, allowance for friction, and method of testing.

Example: What music wire diameter and how many coils are required for the torsion spring shown in Fig. 24, which is to withstand at least 1000 cycles? Determine the corrected stress and the reduced inside diameter after deflection.

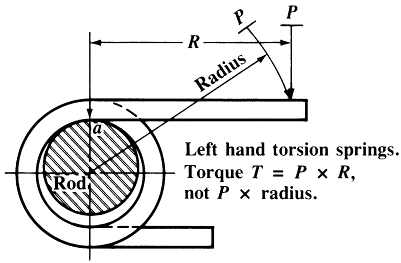
Method 1, using table: From Table 15, page 321, locate the $\frac{1}{2}$ inch inside diameter for the spring in the left-hand column. Move to the right and then vertically to locate a torque value nearest to the required 10 pound-inches, which is 10.07 pound-inches. At the top of the same column, the music wire diameter is found, which is Number 31 gauge (0.085 inch). At the bottom of the same column the deflection for one coil is found, which is 15.81 degrees. As a 90-degree deflection is required, the number of coils needed is 90/15.81 = 5.69 (say $5\frac{3}{4}$ coils).

The spring index $\frac{D}{d} = \frac{0.500 + 0.085}{0.085} = 6.88$ and thus the curvature correction factor K from Fig. 20 = 1.13. Therefore the corrected stress equals 167,000 × 1.13 = 188,700 pounds per square inch which is below the Light Service curve (Fig. 7) and therefore should provide a fatigue life of over 1,000 cycles. The reduced inside diameter due to deflection is found from the formula in Table :

$$ID_1 = \frac{N(ID\ \text{free})}{N + \frac{F}{360}} = \frac{5.75 \times 0.500}{5.75 + \frac{90}{360}} = 0.479 \text{ in.}$$

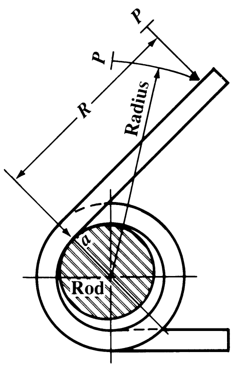
This reduced diameter easily clears a suggested $\frac{7}{16}$ inch diameter supporting rod: 0.479 − 0.4375 = 0.041 inch clearance, and it also allows for the standard tolerance. The overall length of the spring equals the total number of coils plus one, times the wire diameter. Thus, $6\frac{3}{4} \times 0.085 = 0.574$ inch. If a small space of about $\frac{1}{64}$ in. is allowed between the coils to eliminate coil friction, an overall length of $\frac{21}{32}$ inch results.

Although this completes the design calculations, other tolerances should be applied in accordance with the Torsion Spring Tolerance Tables 16 through 18 shown at the end of this section.



The Torque is $T = P \times R$, Not $P \times$ Radius, because the Spring is Resting Against the Support Rod at Point a

Fig. 22. Left-Hand Torsion Spring



As with the Spring in Fig. 22, the Torque is $T = P \times R$, Not $P \times$ Radius, Because the Support Point Is at a

Fig. 23. Left-Hand Torsion Spring

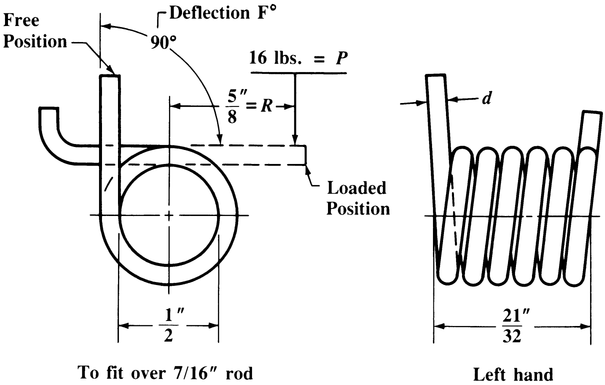


Fig. 24. Torsion Spring Design Example. The Spring Is to be Assembled on a 7⁄16-Inch Support Rod

Longer fatigue life: If a longer fatigue life is desired, use a slightly larger wire diameter. Usually the next larger gage size is satisfactory. The larger wire will reduce the stress and still exert the same torque, but will require more coils and a longer overall length.

Percentage method for calculating longer life: The spring design can be easily adjusted for longer life as follows:

1) Select the next larger gage size, which is Number 32 (0.090 inch) from Table 15. The torque is 11.88 pound-inches, the design stress is 166,000 pounds per square inch, and the deflection is 14.9 degrees per coil. As a percentage the torque is 10/11.88 × 100 = 84 per cent.

2) The new stress is 0.84 × 166,000 = 139,440 pounds per square inch. This value is under the bottom or Severe Service curve, Fig. 7, and thus assures longer life.

3) The new deflection per coil is 0.84 × 14.97 = 12.57 degrees. Therefore, the total number of coils required = 90/12.57 = 7.16 (say 7 1⁄8). The new overall length = 8 1⁄8 × 0.090 = 0.73 inch (say 3⁄4 inch). A slight increase in the overall length and new arm location are thus necessary.

Method 2, using formulas: When using this method, it is often necessary to solve the formulas several times because assumptions must be made initially either for the stress or for a wire size. The procedure for design using formulas is as follows (the design example is the same as in Method 1, and the spring is shown in Fig. 24):

Step 1: Note from Table , page 315 that the wire diameter formula is:

$$d = \sqrt[3]{\frac{10.18T}{S_b}}$$

Step 2: Referring to Fig. 7, select a trial stress, say 150,000 pounds per square inch.

Step 3: Apply the trial stress, and the 10 pound-inches torque value in the wire diameter formula:

$$d = \sqrt[3]{\frac{10.18T}{S_b}} = \sqrt[3]{\frac{10.18 \times 10}{150{,}000}} = \sqrt[3]{0.000679} = 0.0879 \text{ inch}$$

The nearest gauge sizes are 0.085 and 0.090 inch diameter. *Note*: Table 21, page 330, can be used to avoid solving the cube root.

Step 4: Select 0.085 inch wire diameter and solve the equation for the actual stress:

$$S_b = \frac{10.18T}{d^3} = \frac{10.18 \times 10}{0.085^3} = 165{,}764 \text{ pounds per square inch}$$

Step 5: Calculate the number of coils from the equation, Table :

$$N = \frac{EdF^\circ}{392 S_b D}$$

$$= \frac{28{,}500{,}000 \times 0.085 \times 90}{392 \times 165{,}764 \times 0.585} = 5.73 \text{ (say } 5\tfrac{3}{4}\text{)}$$

Step 6: Calculate the total stress. The spring index is 6.88, and the correction factor K is 1.13, therefore total stress = 165,764 × 1.13 = 187,313 pounds per square inch. *Note*: The corrected stress should not be used in any of the formulas as it does not determine the torque or the deflection.

Table of Torsion Spring Characteristics.—Table 15 shows design characteristics for the most commonly used torsion springs made from wire of standard gauge sizes. The deflection for one coil at a specified torque and stress is shown in the body of the table. The figures are based on music wire (ASTM A228) and oil-tempered MB grade (ASTM A229), and can be used for several other materials which have similar values for the modulus of elasticity E. However, the design stress may be too high or too low, and the design stress, torque, and deflection per coil should each be multiplied by the appropriate correction factor in Table 14 when using any of the materials given in that table.

Table 14. Correction Factors for Other Materials

Material	Factor	Material	Factor
Hard Drawn MB	0.75	Stainless 316	
Chrome-Vanadium	1.10	Up to ⅛ inch diameter	0.75
Chrome-Silicon	1.20	Over ⅛ to ¼ inch diameter	0.65
Stainless 302 and 304		Over ¼ inch diameter	0.65
Up to ⅛ inch diameter	0.85	Stainless 17–7 PH	
Over ⅛ to ¼ inch diameter	0.75	Up to ⅛ inch diameter	1.00
Over ¼ inch diameter	0.65	Over ⅛ to 3⁄16 inch diameter	1.07
Stainless 431	0.80	Over 3⁄16 inch diameter	1.12
Stainless 420	0.85	…	…

For use with values in Table 15. *Note*: The figures in Table 15 are for music wire (ASTM A228) and oil-tempered MB grade (ASTM A229) and can be used for several other materials that have a similar modulus of elasticity E. However, the design stress may be too high or too low, and therefore the design stress, torque, and deflection per coil should each be multiplied by the appropriate correction factor when using any of the materials given in this table (Table 14).

Table 15. Torsion Spring Deflections

Inside Diam.		AMW Wire Gauge and Decimal Equivalent[a]															
		1 .010	2 .011	3 .012	4 .013	5 .014	6 .016	7 .018	8 .020	9 .022	10 .024	11 .026	12 .029	13 .031	14 .033	15 .035	16 .037
		Design Stress, pounds per sq. in. (thousands)															
		232	229	226	224	221	217	214	210	207	205	202	199	197	196	194	192
		Torque, pound-inch															
		.0228	.0299	.0383	.0483	.0596	.0873	.1226	.1650	.2164	.2783	.3486	.4766	.5763	.6917	.8168	.9550
Fractional	Decimal	Deflection, degrees per coil															
1/16	0.0625	22.35	20.33	18.64	17.29	16.05	14.15	18.72	11.51	10.56	9.818	9.137	8.343	7.896	…	…	…
5/64	0.078125	27.17	24.66	22.55	20.86	19.32	16.96	15.19	13.69	12.52	11.59	10.75	9.768	9.215	…	…	…
3/32	0.09375	31.98	28.98	26.47	24.44	22.60	19.78	17.65	15.87	14.47	13.36	12.36	11.19	10.53	10.18	9.646	9.171
7/64	0.109375	36.80	33.30	30.38	28.02	25.88	22.60	20.12	18.05	16.43	15.14	13.98	12.62	11.85	11.43	10.82	10.27
1/8	0.125	41.62	37.62	34.29	31.60	29.16	25.41	22.59	20.23	18.38	16.91	15.59	14.04	13.17	12.68	11.99	11.36
9/64	0.140625	46.44	41.94	38.20	35.17	32.43	28.23	25.06	22.41	20.33	18.69	17.20	15.47	14.49	13.94	13.16	12.46
5/32	0.15625	51.25	46.27	42.11	38.75	35.71	31.04	27.53	24.59	22.29	20.46	18.82	16.89	15.81	15.19	14.33	13.56
3/16	0.1875	60.89	54.91	49.93	45.91	42.27	36.67	32.47	28.95	26.19	24.01	22.04	19.74	18.45	17.70	16.67	15.75
7/32	0.21875	70.52	63.56	57.75	53.06	48.82	42.31	37.40	33.31	30.10	27.55	25.27	22.59	21.09	20.21	19.01	17.94
1/4	0.250	80.15	72.20	65.57	60.22	55.38	47.94	42.34	37.67	34.01	31.10	28.49	25.44	23.73	22.72	21.35	20.13

Inside Diam.		AMW Wire Gauge and Decimal Equivalent[a]														
		17 .039	18 .041	19 .043	20 .045	21 .047	22 .049	23 .051	24 .055	25 .059	26 .063	27 .067	28 .071	29 .075	30 .080	31 .085
		Design Stress, pounds per sq. in. (thousands)														
		190	188	187	185	184	183	182	180	178	176	174	173	171	169	167
		Torque, pound-inch														
		1.107	1.272	1.460	1.655	1.876	2.114	2.371	2.941	3.590	4.322	5.139	6.080	7.084	8.497	10.07
Fractional	Decimal	Deflection, degrees per coil														
7/64	0.109375	9.771	9.320	8.957	…	…	…	…	…	…	…	…	…	…	…	…
1/8	0.125	10.80	10.29	9.876	9.447	9.102	8.784	…	…	…	…	…	…	…	…	…
9/64	0.140625	11.83	11.26	10.79	10.32	9.929	9.572	9.244	8.654	8.141	…	…	…	…	…	…
5/32	0.15625	12.86	12.23	11.71	11.18	10.76	10.36	9.997	9.345	8.778	8.279	7.975	…	…	…	…
3/16	0.1875	14.92	14.16	13.55	12.92	12.41	11.94	11.50	10.73	10.05	9.459	9.091	8.663	8.232	7.772	7.364
7/32	0.21875	16.97	16.10	15.39	14.66	14.06	13.52	13.01	12.11	11.33	10.64	10.21	9.711	9.212	8.680	8.208
1/4	0.250	19.03	18.04	17.22	16.39	15.72	15.09	14.52	13.49	12.60	11.82	11.32	10.76	10.19	9.588	9.053

[a] For sizes up to 13 gauge, the table values are for music wire with a modulus *E* of 29,000,000 psi; and for sizes from 27 to 31 guage, the values are for oil-tempered MB with a modulus of 28,500,000 psi.

Table 15. *(Continued)* Torsion Spring Deflections

Inside Diam.		AMW Wire Gauge and Decimal Equivalent[a]															
		8 .020	9 .022	10 .024	11 .026	12 .029	13 .031	14 .033	15 .035	16 .037	17 .039	18 .041	19 .043	20 .045	21 .047	22 .049	23 .051
		Design Stress, pounds per sq. in. (thousands)															
		210	207	205	202	199	197	196	194	192	190	188	187	185	184	183	182
		Torque, pound-inch															
		.1650	.2164	.2783	.3486	.4766	.5763	.6917	.8168	.9550	1.107	1.272	1.460	1.655	1.876	2.114	2.371
Fractional	Decimal	Deflection, degrees per coil															
9⁄32	0.28125	42.03	37.92	34.65	31.72	28.29	26.37	25.23	23.69	22.32	21.09	19.97	19.06	18.13	17.37	16.67	16.03
5⁄16	0.3125	46.39	41.82	38.19	34.95	31.14	29.01	27.74	26.04	24.51	23.15	21.91	20.90	19.87	19.02	18.25	17.53
11⁄32	0.34375	50.75	45.73	41.74	38.17	33.99	31.65	30.25	28.38	26.71	25.21	23.85	22.73	21.60	20.68	19.83	19.04
3⁄8	0.375	55.11	49.64	45.29	41.40	36.84	34.28	32.76	30.72	28.90	27.26	25.78	24.57	23.34	22.33	21.40	20.55
13⁄32	0.40625	59.47	53.54	48.85	44.63	39.69	36.92	35.26	33.06	31.09	29.32	27.72	26.41	25.08	23.99	22.98	22.06
7⁄16	0.4375	63.83	57.45	52.38	47.85	42.54	39.56	37.77	35.40	33.28	31.38	29.66	28.25	26.81	25.64	24.56	23.56
15⁄32	0.46875	68.19	61.36	55.93	51.00	45.39	42.20	40.28	37.74	35.47	33.44	31.59	30.08	28.55	27.29	26.14	25.07
1⁄2	0.500	72.55	65.27	59.48	54.30	48.24	44.84	42.79	40.08	37.67	35.49	33.53	31.92	30.29	28.95	27.71	26.58

Inside Diam.		AMW Wire Gauge and Decimal Equivalent[a]														
		24 .055	25 .059	26 .063	27 .067	28 .071	29 .075	30 .080	31 .085	32 .090	33 .095	34 .100	35 .106	36 .112	37 .118	1⁄8 125
		Design Stress, pounds per sq. in. (thousands)														
		180	178	176	174	173	171	169	167	166	164	163	161	160	158	156
		Torque, pound-inch														
		2.941	3.590	4.322	5.139	6.080	7.084	8.497	10.07	11.88	13.81	16.00	18.83	22.07	25.49	29.92
Fractional	Decimal	Deflection, degrees per coil														
9⁄32	0.28125	14.88	13.88	13.00	12.44	11.81	11.17	10.50	9.897	9.418	8.934	8.547	8.090	7.727	7.353	6.973
5⁄16	0.3125	16.26	15.15	14.18	13.56	12.85	12.15	11.40	10.74	10.21	9.676	9.248	8.743	8.341	7.929	7.510
11⁄32	0.34375	17.64	16.42	15.36	14.67	13.90	13.13	12.31	11.59	11.00	10.42	9.948	9.396	8.955	8.504	8.046
3⁄8	0.375	19.02	17.70	16.54	15.79	14.95	14.11	13.22	12.43	11.80	11.16	10.65	10.05	9.569	9.080	8.583
13⁄32	0.40625	20.40	18.97	17.72	16.90	15.99	15.09	14.13	13.28	12.59	11.90	11.35	10.70	10.18	9.655	9.119
7⁄16	0.4375	21.79	20.25	18.90	18.02	17.04	16.07	15.04	14.12	13.38	12.64	12.05	11.35	10.80	10.23	9.655
15⁄32	0.46875	23.17	21.52	20.08	19.14	18.09	17.05	15.94	14.96	14.17	13.39	12.75	12.01	11.41	10.81	10.19
1⁄2	0.500	24.55	22.80	21.26	20.25	19.14	18.03	16.85	15.81	14.97	14.13	13.45	12.66	12.03	11.38	10.73

[a] For sizes up to 13 gauge, the table values are for music wire with a modulus *E* of 29,000,000 psi; and for sizes from 27 to 31 guage, the values are for oil-tempered MB with a modulus of 28,500,000 psi.

Table 15. *(Continued)* Torsion Spring Deflections

Inside Diam.		AMW Wire Gauge and Decimal Equivalent[a]														
		16 .037	17 .039	18 .041	19 .043	20 .045	21 .047	22 .049	23 .051	24 .055	25 .059	26 .063	27 .067	28 .071	29 .075	30 .080
		Design Stress, pounds per sq. in. (thousands)														
		192	190	188	187	185	184	183	182	180	178	176	174	173	171	169
		Torque, pound-inch														
		.9550	1.107	1.272	1.460	1.655	1.876	2.114	2.371	2.941	3.590	4.322	5.139	6.080	7.084	8.497
Fractional	Decimal	Deflection, degrees per coil														
17/32	0.53125	39.86	37.55	35.47	33.76	32.02	30.60	29.29	28.09	25.93	24.07	22.44	21.37	20.18	19.01	17.76
9/16	0.5625	42.05	39.61	37.40	35.59	33.76	32.25	30.87	29.59	27.32	25.35	23.62	22.49	21.23	19.99	18.67
19/32	0.59375	44.24	41.67	39.34	37.43	35.50	33.91	32.45	31.10	28.70	26.62	24.80	23.60	22.28	20.97	19.58
5/8	0.625	46.43	43.73	41.28	39.27	37.23	35.56	34.02	32.61	30.08	27.89	25.98	24.72	23.33	21.95	20.48
21/32	0.65625	48.63	45.78	43.22	41.10	38.97	37.22	35.60	34.12	31.46	29.17	27.16	25.83	24.37	22.93	21.39
11/16	0.6875	50.82	47.84	45.15	42.94	40.71	38.87	37.18	35.62	32.85	30.44	28.34	26.95	25.42	23.91	22.30
23/32	0.71875	53.01	49.90	47.09	44.78	42.44	40.52	38.76	37.13	34.23	31.72	29.52	28.07	26.47	24.89	23.21
3/4	0.750	55.20	51.96	49.03	46.62	44.18	42.18	40.33	38.64	35.61	32.99	30.70	29.18	27.52	25.87	24.12

Inside Diam.		Wire Gauge and Decimal Equivalent[ab]															
		31 .085	32 .090	33 .095	34 .100	35 .106	36 .112	37 .118	1/8 .125	10 .135	9 .1483	5/32 .1563	8 .162	7 .177	3/16 .1875	6 .192	5 .207
		Design Stress, pounds per sq. in. (thousands)															
		167	166	164	163	161	160	158	156	161	158	156	154	150	149	146	143
		Torque, pound-inch															
		10.07	11.88	13.81	16.00	18.83	22.07	25.49	29.92	38.90	50.60	58.44	64.30	81.68	96.45	101.5	124.6
Fractional	Decimal	Deflection, degrees per coil															
17/32	0.53125	16.65	15.76	14.87	14.15	13.31	12.64	11.96	11.26	10.93	9.958	9.441	9.064	8.256	7.856	7.565	7.015
9/16	0.5625	17.50	16.55	15.61	14.85	13.97	13.25	12.53	11.80	11.44	10.42	9.870	9.473	8.620	8.198	7.891	7.312
19/32	0.59375	18.34	17.35	16.35	15.55	14.62	13.87	13.11	12.34	11.95	10.87	10.30	9.882	8.984	8.539	8.218	7.609
5/8	0.625	19.19	18.14	17.10	16.25	15.27	14.48	13.68	12.87	12.47	11.33	10.73	10.29	9.348	8.881	8.545	7.906
21/32	0.65625	20.03	18.93	17.84	16.95	15.92	15.10	14.26	13.41	12.98	11.79	11.16	10.70	9.713	9.222	8.872	8.202
11/16	0.6875	20.88	19.72	18.58	17.65	16.58	15.71	14.83	13.95	13.49	12.25	11.59	11.11	10.08	9.564	9.199	8.499
23/32	0.71875	21.72	20.52	19.32	18.36	17.23	16.32	15.41	14.48	14.00	12.71	12.02	11.52	10.44	9.905	9.526	8.796
3/4	0.750	22.56	21.31	20.06	19.06	17.88	16.94	15.99	15.02	14.52	13.16	12.44	11.92	10.81	10.25	9.852	9.093

[a] For sizes up to 26 gauge, the table values are for music wire with a modulus *E* of 29,500,000 psi; for sizes from 27 to 1/8 inch diameter the table values are for music wire with a modulus of 28,500,000 psi; for sizes from 10 gauge to 1/8 inch diameter, the values are for oil-tempered MB with a modulus of 28,500,000 psi.

[b] Gauges 31 through 37 are AMW gauges. Gauges 10 through 5 are Washburn and Moen.

Table 15. *(Continued)* Torsion Spring Deflections

Inside Diam.		AMW Wire Gauge and Decimal Equivalent[a]														
		24 .055	25 .059	26 .063	27 .067	28 .071	29 .075	30 .080	31 .085	32 .090	33 .095	34 .100	35 .106	36 .112	37 .118	⅛ .125
		Design Stress, pounds per sq. in. (thousands)														
		180	178	176	174	173	171	169	167	166	164	163	161	160	158	156
		Torque, pound-inch														
		2.941	3.590	4.322	5.139	6.080	7.084	8.497	10.07	11.88	13.81	16.00	18.83	22.07	25.49	29.92
Fractional	Decimal	Deflection, degrees per coil														
13/16	0.8125	38.38	35.54	33.06	31.42	29.61	27.83	25.93	24.25	22.90	21.55	20.46	19.19	18.17	17.14	16.09
7/8	0.875	41.14	38.09	35.42	33.65	31.70	29.79	27.75	25.94	24.58	23.03	21.86	20.49	19.39	18.29	17.17
15/16	0.9375	43.91	40.64	37.78	35.88	33.80	31.75	29.56	27.63	26.07	24.52	23.26	21.80	20.62	19.44	18.24
1	1.000	46.67	43.19	40.14	38.11	35.89	33.71	31.38	29.32	27.65	26.00	24.66	23.11	21.85	20.59	19.31
1 1/16	1.0625	49.44	45.74	42.50	40.35	37.99	35.67	33.20	31.01	29.24	27.48	26.06	24.41	23.08	21.74	20.38
1 1/8	1.125	52.20	48.28	44.86	42.58	40.08	37.63	35.01	32.70	30.82	28.97	27.46	25.72	24.31	22.89	21.46
1 3/16	1.1875	54.97	50.83	47.22	44.81	42.18	39.59	36.83	34.39	32.41	30.45	28.86	27.02	25.53	24.04	22.53
1 1/4	1.250	57.73	53.38	49.58	47.04	44.27	41.55	38.64	36.08	33.99	31.94	30.27	28.33	26.76	25.19	23.60

Inside Diam.		Washburn and Moen Gauge or Size and Decimal Equivalent[a]															
		10 .135	9 .1483	5/32 .1563	8 .162	7 .177	3/16 .1875	6 .192	5 .207	7/32 .2188	4 .2253	3 .2437	1/4 .250	9/32 .2813	5/16 .3125	11/32 .3438	3/8 .375
		Design Stress, pounds per sq. in. (thousands)															
		161	158	156	154	150	149	146	143	142	141	140	139	138	137	136	135
		Torque, pound-inch															
		38.90	50.60	58.44	64.30	81.68	96.45	101.5	124.6	146.0	158.3	199.0	213.3	301.5	410.6	542.5	700.0
Fractional	Decimal	Deflection, degrees per coil															
13/16	0.8125	15.54	14.08	13.30	12.74	11.53	10.93	10.51	9.687	9.208	8.933	8.346	8.125	7.382	6.784	6.292	5.880
7/8	0.875	16.57	15.00	14.16	13.56	12.26	11.61	11.16	10.28	9.766	9.471	8.840	8.603	7.803	7.161	6.632	6.189
15/16	0.9375	17.59	15.91	15.02	14.38	12.99	12.30	11.81	10.87	10.32	10.01	9.333	9.081	8.225	7.537	6.972	6.499
1	1.000	18.62	16.83	15.88	15.19	13.72	12.98	12.47	11.47	10.88	10.55	9.827	9.559	8.647	7.914	7.312	6.808
1 1/16	1.0625	19.64	17.74	16.74	16.01	14.45	13.66	13.12	12.06	11.44	11.09	10.32	10.04	9.069	8.291	7.652	7.118
1 1/8	1.125	20.67	18.66	17.59	16.83	15.18	14.35	13.77	12.66	12.00	11.62	10.81	10.52	9.491	8.668	7.993	7.427
1 3/16	1.1875	21.69	19.57	18.45	17.64	15.90	15.03	14.43	13.25	12.56	12.16	11.31	10.99	9.912	9.045	8.333	7.737
1 1/4	1.250	22.72	20.49	19.31	18.46	16.63	15.71	15.08	13.84	13.11	12.70	11.80	11.47	10.33	9.422	8.673	8.046

[a] For sizes up to 26 gauge, the table values are for music wire with a modulus E of 29,500,000 psi; for sizes from 27 to ⅛ inch diameter the table values are for music wire with a modulus of 28,500,000 psi; for sizes from 10 gauge to ⅛ inch diameter, the values are for oil-tempered MB with a modulus of 28,500,000 psi.

For an example in the use of the table, see the example starting on page 317. Note: Intermediate values may be interpolated within reasonable accuracy.

Torsion Spring Design Recommendations.—The following recommendations should be taken into account when designing torsion springs:

Hand: The hand or direction of coiling should be specified and the spring designed so deflection causes the spring to wind up and to have more coils. This increase in coils and overall length should be allowed for during design. Deflecting the spring in an unwinding direction produces higher stresses and may cause early failure. When a spring is sighted down the longitudinal axis, it is "right hand" when the direction of the wire into the spring takes a clockwise direction or if the angle of the coils follows an angle similar to the threads of a standard bolt or screw, otherwise it is "left hand." A spring must be coiled right-handed to engage the threads of a standard machine screw.

Rods: Torsion springs should be supported by a rod running through the center whenever possible. If unsupported, or if held by clamps or lugs, the spring will buckle and the torque will be reduced or unusual stresses may occur.

Diameter Reduction: The inside diameter reduces during deflection. This reduction should be computed and proper clearance provided over the supporting rod. Also, allowances should be considered for normal spring diameter tolerances.

Winding: The coils of a spring may be closely or loosely wound, but they seldom should be wound with the coils pressed tightly together. Tightly wound springs with initial tension on the coils do not deflect uniformly and are difficult to test accurately. A small space between the coils of about 20 to 25 per cent of the wire thickness is desirable. Square and rectangular wire sections should be avoided whenever possible as they are difficult to wind, expensive, and are not always readily available.

Arm Length: All the wire in a torsion spring is active between the points where the loads are applied. Deflection of long extended arms can be calculated by allowing one third of the arm length, from the point of load contact to the body of the spring, to be converted into coils. However, if the length of arm is equal to or less than one-half the length of one coil, it can be safely neglected in most applications.

Total Coils: Torsion springs having less than three coils frequently buckle and are difficult to test accurately. When thirty or more coils are used, light loads will not deflect all the coils simultaneously due to friction with the supporting rod. To facilitate manufacturing it is usually preferable to specify the total number of coils to the nearest fraction in eighths or quarters such as $5\frac{1}{8}$, $5\frac{1}{4}$, $5\frac{1}{2}$, etc.

Double Torsion: This design consists of one left-hand-wound series of coils and one series of right-hand-wound coils connected at the center. These springs are difficult to manufacture and are expensive, so it often is better to use two separate springs. For torque and stress calculations, each series is calculated separately as individual springs; then the torque values are added together, but the deflections are not added.

Bends: Arms should be kept as straight as possible. Bends are difficult to produce and often are made by secondary operations, so they are therefore expensive. Sharp bends raise stresses that cause early failure. Bend radii should be as large as practicable. Hooks tend to open during deflection; their stresses can be calculated by the same procedure as that for tension springs.

Spring Index: The spring index must be used with caution. In design formulas it is D/d. For shop measurement it is O.D./d. For arbor design it is I.D./d. Conversions are easily performed by either adding or subtracting 1 from D/d.

Proportions: A spring index between 4 and 14 provides the best proportions. Larger ratios may require more than average tolerances. Ratios of 3 or less, often cannot be coiled on automatic spring coiling machines because of arbor breakage. Also, springs with smaller or larger spring indexes often do not give the same results as are obtained using the design formulas.

Torsion Spring Tolerances.—Torsion springs are coiled in a different manner from other types of coiled springs and therefore different tolerances apply. The commercial tolerance on loads is ± 10 per cent and is specified with reference to the angular deflection. For example: 100 pound-inches ± 10 per cent at 45 degrees deflection. One load specified usually suffices. If two loads and two deflections are specified, the manufacturing and testing times are increased. Tolerances smaller than ± 10 per cent require each spring to be individually tested and adjusted, which adds considerably to manufacturing time and cost. Tables 16, 17, and 18 give, respectively, free angle tolerances, coil diameter tolerances, and tolerances on the number of coils.

Table 16. Torsion Spring Tolerances for Angular Relationship of Ends

Number of Coils (*N*)	Spring Index								
	4	6	8	10	12	14	16	18	20
	Free Angle Tolerance, ± degrees								
1	2	3	3.5	4	4.5	5	5.5	5.5	6
2	4	5	6	7	8	8.5	9	9.5	10
3	5.5	7	8	9.5	10.5	11	12	13	14
4	7	9	10	12	14	15	16	16.5	17
5	8	10	12	14	16	18	20	20.5	21
6	9.5	12	14.5	16	19	20.5	21	22.5	24
8	12	15	18	20.5	23	25	27	28	29
10	14	19	21	24	27	29	31.5	32.5	34
15	20	25	28	31	34	36	38	40	42
20	25	30	34	37	41	44	47	49	51
25	29	35	40	44	48	52	56	60	63
30	32	38	44	50	55	60	65	68	70
50	45	55	63	70	77	84	90	95	100

Table 17. Torsion Spring Coil Diameter Tolerances

Wire Diameter, Inch	Spring Index						
	4	6	8	10	12	14	16
	Coil Diameter Tolerance, ± inch						
0.015	0.002	0.002	0.002	0.002	0.003	0.003	0.004
0.023	0.002	0.002	0.002	0.003	0.004	0.005	0.006
0.035	0.002	0.002	0.003	0.004	0.006	0.007	0.009
0.051	0.002	0.003	0.005	0.007	0.008	0.010	0.012
0.076	0.003	0.005	0.007	0.009	0.012	0.015	0.018
0.114	0.004	0.007	0.010	0.013	0.018	0.022	0.028
0.172	0.006	0.010	0.013	0.020	0.027	0.034	0.042
0.250	0.008	0.014	0.022	0.030	0.040	0.050	0.060

Table 18. Torsion Spring Tolerance on Number of Coils

Number of Coils	Tolerance	Number of Coils	Tolerance
up to 5	±5°	over 10 to 20	±15°
over 5 to 10	±10°	over 20 to 40	±30°

Miscellaneous Springs.—This section provides information on various springs, some in common use, some less commonly used.

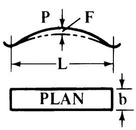
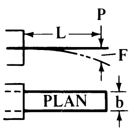
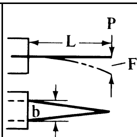
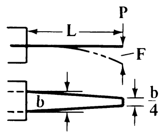
Conical compression: These springs taper from top to bottom and are useful where an increasing (instead of a constant) load rate is needed, where solid height must be small, and where vibration must be damped. Conical springs with a uniform pitch are easiest to coil. Load and deflection formulas for compression springs can be used – using the average mean coil diameter, and providing the deflection does not cause the largest active coil to lie against the bottom coil. When this happens, each coil must be calculated separately, using the standard formulas for compression springs.

Constant force springs: Those springs are made from flat spring steel and are finding more applications each year. Complicated design procedures can be eliminated by selecting a standard design from thousands now available from several spring manufacturers.

Spiral, clock, and motor springs: Although often used in wind-up type motors for toys and other products, these springs are difficult to design and results cannot be calculated with precise accuracy. However, many useful designs have been developed and are available from spring manufacturing companies.

Flat springs: These springs are often used to overcome operating space limitations in various products such as electric switches and relays. Table 19 lists formulas for designing flat springs. The formulas are based on standard beam formulas where the deflection is small.

Table 19. Formulas for Flat Springs

Feature				
Deflect., f Inches	$f = \frac{PL^3}{4Ebt^3} = \frac{S_bL^2}{6Et}$	$f = \frac{4PL^3}{Ebt^3} = \frac{2S_bL^2}{3Et}$	$f = \frac{6PL^3}{Ebt^3} = \frac{S_bL^2}{Et}$	$f = \frac{5.22PL^3}{Ebt^3} = \frac{0.87S_bL^2}{Et}$
Load, P Pounds	$P = \frac{2S_bbt^2}{3L} = \frac{4Ebt^3F}{L^3}$	$P = \frac{S_bbt^2}{6L} = \frac{Ebt^3F}{4L^3}$	$P = \frac{S_bbt^2}{6L} = \frac{Ebt^3F}{6L^3}$	$P = \frac{S_bbt^2}{6L} = \frac{Ebt^3F}{5.22L^3}$
Stress, S_b Bending Pounds per sq. inch	$S_b = \frac{3PL}{2bt^2} = \frac{6EtF}{L^2}$	$S_b = \frac{6PL}{bt^2} = \frac{3EtF}{2L^2}$	$S_b = \frac{6PL}{bt^2} = \frac{EtF}{L^2}$	$S_b = \frac{6PL}{bt^2} = \frac{EtF}{0.87L^2}$
Thickness, t Inches	$t = \frac{S_bL^2}{6EF} = \sqrt[3]{\frac{PL^3}{4EbF}}$	$t = \frac{2S_bL^2}{3EF} = \sqrt[3]{\frac{4PL^3}{EbF}}$	$t = \frac{S_bL^2}{EF} = \sqrt[3]{\frac{6PL^3}{EbF}}$	$t = \frac{0.87S_bL^2}{EF} = \sqrt[3]{\frac{5.22PL^3}{EbF}}$

Based on standard beam formulas where the deflection is small

See page 285 for notation.

Note: Where two formulas are given for one feature, the designer should use the one found to be appropriate for the given design. The result from either of any two formulas is the same.

Belleville washers: These washer type springs can sustain relatively large loads with small deflections, and the loads and deflections can be increased by stacking the springs as shown in Fig. 25.

Design data is not given here because the wide variations in ratios of O.D. to I.D., height to thickness, and other factors require too many formulas for convenient use and involve constants obtained from more than 24 curves. It is now practicable to select required sizes from the large stocks carried by several of the larger spring manufacturing companies. Most of these companies also stock curved and wave washers.

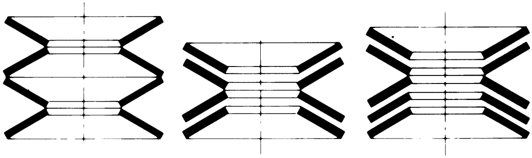


Fig. 25. Examples of Belleville Spring Combinations

Volute springs: These springs are often used on army tanks and heavy field artillery, and seldom find additional uses because of their high cost, long production time, difficulties in manufacture, and unavailability of a wide range of materials and sizes. Small volute springs are often replaced with standard compression springs.

Torsion bars: Although the more simple types are often used on motor cars, the more complicated types with specially forged ends are finding fewer applications as time goes on.

Moduli of Elasticity of Spring Materials.—The modulus of elasticity in tension, denoted by the letter E, and the modulus of elasticity in torsion, denoted by the letter G, are used in formulas relating to spring design. Values of these moduli for various ferrous and nonferrous spring materials are given in Table .

General Heat Treating Information for Springs.—The following is general information on the heat treatment of springs, and is applicable to pre-tempered or hard-drawn spring materials only.

Compression springs are baked after coiling (before setting) to relieve residual stresses and thus permit larger deflections before taking a permanent set.

Extension springs also are baked, but heat removes some of the initial tension. Allowance should be made for this loss. Baking at 500 degrees F for 30 minutes removes approximately 50 per cent of the initial tension. The shrinkage in diameter however, will slightly increase the load and rate.

Outside diameters shrink when springs of music wire, pretempered MB, and other carbon or alloy steels are baked. Baking also slightly increases the free length and these changes produce a little stronger load and increase the rate.

Outside diameters expand when springs of stainless steel (18-8) are baked. The free length is also reduced slightly and these changes result in a little lighter load and a decrease the spring rate.

Inconel, Monel, and nickel alloys do not change much when baked.

Beryllium-copper shrinks and deforms when heated. Such springs usually are baked in fixtures or supported on arbors or rods during heating.

Brass and phosphor bronze springs should be given a light heat only. Baking above 450 degrees F will soften the material. Do not heat in salt pots.

Torsion springs do not require baking because coiling causes residual stresses in a direction that is helpful, but such springs frequently are baked so that jarring or handling will not cause them to lose the position of their ends.

Table 20. Moduli of Elasticity in Torsion and Tension of Spring Materials

Ferrous Materials			Nonferrous Materials		
	Modulus of Elasticity, pounds per square inch			Modulus of Elasticity, pounds per square inch	
Material (Commercial Name)	In Torsion, *G*	In Tension, *E*	Material (Commercial Name)	In Torsion, *G*	In Tension, *E*
Hard Drawn MB			Spring Brass		
Up to 0.032 inch	11,700,000	28,800,000	Type 70–30	5,000,000	15,000,000
0.033 to 0.063 inch	11,600,000	28,700,000	Phosphor Bronze		
0.064 to 0.125 inch	11,500,000	28,600,000	5 per cent tin	6,000,000	15,000,000
0.126 to 0.625 inch	11,400,000	28,500,000	Beryllium-Copper		
Music Wire			Cold Drawn 4 Nos.	7,000,000	17,000,000
Up to 0.032 inch	12,000,000	29,500,000	Pretempered,		
0.033 to 0.063 inch	11,850,000	29,000,000	fully hard	7,250,000	19,000,000
0.064 to 0.125 inch	11,750,000	28,500,000	Inconel[a] 600	10,500,000	31,000,000[b]
0.126 to 0.250 inch	11,600,000	28,000,000	Inconel[a] X 750	10,500,000	31,000,000[b]
Oil-Tempered MB	11,200,000	28,500,000			
Chrome-Vanadium	11,200,000	28,500,000	Monel[a] 400	9,500,000	26,000,000
Chrome-Silicon	11,200,000	29,500,000	Monel[a] K 500	9,500,000	26,000,000
Silicon-Manganese	10,750,000	29,000,000	Duranickel[a] 300	11,000,000	30,000,000
Stainless Steel			Permanickel[a]	11,000,000	30,000,000
Types 302, 304, 316	10,000,000	28,000,000[b]			
Type 17–7 PH	10,500,000	29,500,000	Ni Span C[a] 902	10,000,000	27,500,000
Type 420	11,000,000	29,000,000	Elgiloy[c]	12,000,000	29,500,000
Type 431	11,400,000	29,500,000	Iso-Elastic[d]	9,200,000	26,000,000

[a] Trade name of International Nickel Company.
[b] May be 2,000,000 pounds per square inch less if material is not fully hard.
[c] Trade name of Hamilton Watch Company.
[d] Trade name of John Chatillon & Sons.

Note: Modulus *G* (shear modulus) is used for compression and extension springs; modulus *E* (Young's modulus) is used for torsion, flat, and spiral springs.

Spring brass and phosphor bronze springs that are not very highly stressed and are not subject to severe operating use may be stress relieved after coiling by immersing them in boiling water for a period of 1 hour.

Positions of loops will change with heat. Parallel hooks may change as much as 45 degrees during baking. Torsion spring arms will alter position considerably. These changes should be allowed for during looping or forming.

Quick heating after coiling either in a high-temperature salt pot or by passing a spring through a gas flame is not good practice. Samples heated in this way will not conform with production runs that are properly baked. A small, controlled-temperature oven should be used for samples and for small lot orders.

Plated springs should always be baked before plating to relieve coiling stresses and again after plating to relieve hydrogen embrittlement.

Hardness values fall with high heat—but music wire, hard drawn, and stainless steel will increase 2 to 4 points Rockwell C.

Table 21. Squares, Cubes, and Fourth Powers of Wire Diameters

Steel Wire Gage (U.S.)	Music or Piano Wire Gage	Diameter Inch	Section Area	Square	Cube	Fourth Power
7-0	…	0.4900	0.1886	0.24010	0.11765	0.05765
6-0	…	0.4615	0.1673	0.21298	0.09829	0.04536
5-0	…	0.4305	0.1456	0.18533	0.07978	0.03435
4-0	…	0.3938	0.1218	0.15508	0.06107	0.02405
3-0	…	0.3625	0.1032	0.13141	0.04763	0.01727
2-0	…	0.331	0.0860	0.10956	0.03626	0.01200
1-0	…	0.3065	0.0738	0.09394	0.02879	0.008825
1	…	0.283	0.0629	0.08009	0.02267	0.006414
2	…	0.2625	0.0541	0.06891	0.01809	0.004748
3	…	0.2437	0.0466	0.05939	0.01447	0.003527
4	…	0.2253	0.0399	0.05076	0.01144	0.002577
5	…	0.207	0.0337	0.04285	0.00887	0.001836
6	…	0.192	0.0290	0.03686	0.00708	0.001359
…	45	0.180	0.0254	0.03240	0.00583	0.001050
7	…	0.177	0.0246	0.03133	0.00555	0.000982
…	44	0.170	0.0227	0.02890	0.00491	0.000835
8	43	0.162	0.0206	0.02624	0.00425	0.000689
…	42	0.154	0.0186	0.02372	0.00365	0.000563
9	…	0.1483	0.0173	0.02199	0.00326	0.000484
…	41	0.146	0.0167	0.02132	0.00311	0.000455
…	40	0.138	0.0150	0.01904	0.00263	0.000363
10	…	0.135	0.0143	0.01822	0.00246	0.000332
…	39	0.130	0.0133	0.01690	0.00220	0.000286
…	38	0.124	0.0121	0.01538	0.00191	0.000237
11	…	0.1205	0.0114	0.01452	0.00175	0.000211
…	37	0.118	0.0109	0.01392	0.00164	0.000194
…	36	0.112	0.0099	0.01254	0.00140	0.000157
…	35	0.106	0.0088	0.01124	0.00119	0.000126
12	…	0.1055	0.0087	0.01113	0.001174	0.0001239
…	34	0.100	0.0078	0.0100	0.001000	0.0001000
…	33	0.095	0.0071	0.00902	0.000857	0.0000815
13	…	0.0915	0.0066	0.00837	0.000766	0.0000701
…	32	0.090	0.0064	0.00810	0.000729	0.0000656
…	31	0.085	0.0057	0.00722	0.000614	0.0000522
14	30	0.080	0.0050	0.0064	0.000512	0.0000410
…	29	0.075	0.0044	0.00562	0.000422	0.0000316
15	…	0.072	0.0041	0.00518	0.000373	0.0000269
…	28	0.071	0.0040	0.00504	0.000358	0.0000254
…	27	0.067	0.0035	0.00449	0.000301	0.0000202
…	26	0.063	0.0031	0.00397	0.000250	0.0000158
16	…	0.0625	0.0031	0.00391	0.000244	0.0000153
…	25	0.059	0.0027	0.00348	0.000205	0.0000121
…	24	0.055	0.0024	0.00302	0.000166	0.00000915
17	…	0.054	0.0023	0.00292	0.000157	0.00000850
…	23	0.051	0.0020	0.00260	0.000133	0.00000677
…	22	0.049	0.00189	0.00240	0.000118	0.00000576
18	…	0.0475	0.00177	0.00226	0.000107	0.00000509
…	21	0.047	0.00173	0.00221	0.000104	0.00000488
…	20	0.045	0.00159	0.00202	0.000091	0.00000410
…	19	0.043	0.00145	0.00185	0.0000795	0.00000342
19	18	0.041	0.00132	0.00168	0.0000689	0.00000283
…	17	0.039	0.00119	0.00152	0.0000593	0.00000231
…	16	0.037	0.00108	0.00137	0.0000507	0.00000187
…	15	0.035	0.00096	0.00122	0.0000429	0.00000150
20	…	0.0348	0.00095	0.00121	0.0000421	0.00000147
…	14	0.033	0.00086	0.00109	0.0000359	0.00000119
21	…	0.0317	0.00079	0.00100	0.0000319	0.00000101
…	13	0.031	0.00075	0.00096	0.0000298	0.000000924
…	12	0.029	0.00066	0.00084	0.0000244	0.000000707
22	…	0.0286	0.00064	0.00082	0.0000234	0.000000669
…	11	0.026	0.00053	0.00068	0.0000176	0.000000457
23	…	0.0258	0.00052	0.00067	0.0000172	0.000000443
…	10	0.024	0.00045	0.00058	0.0000138	0.000000332
24	…	0.023	0.00042	0.00053	0.0000122	0.000000280
…	9	0.022	0.00038	0.00048	0.0000106	0.000000234

Table 22. Causes of Spring Failure

	Cause	Comments and Recommendations
Group 1	High stress	The majority of spring failures are due to high stresses caused by large deflections and high loads. High stresses should be used only for statically loaded springs. Low stresses lengthen fatigue life.
	Hydrogen embrittlement	Improper electroplating methods and acid cleaning of springs, without proper baking treatment, cause spring steels to become brittle, and are a frequent cause of failure. Nonferrous springs are immune.
	Sharp bends and holes	Sharp bends on extension, torsion, and flat springs, and holes or notches in flat springs, cause high concentrations of stress, resulting in failure. Bend radii should be as large as possible, and tool marks avoided.
	Fatigue	Repeated deflections of springs, especially above 1,000,000 cycles, even with medium stresses, may cause failure. Low stresses should be used if a spring is to be subjected to a very high number of operating cycles.
Group 2	Shock loading	Impact, shock, and rapid loading cause far higher stresses than those computed by the regular spring formulas. High-carbon spring steels do not withstand shock loading as well as do alloy steels.
	Corrosion	Slight rusting or pitting caused by acids, alkalis, galvanic corrosion, stress corrosion cracking, or corrosive atmosphere weakens the material and causes higher stresses in the corroded area.
	Faulty heat treatment	Keeping spring materials at the hardening temperature for longer periods than necessary causes an undesirable growth in grain structure, resulting in brittleness, even though the hardness may be correct.
	Faulty material	Poor material containing inclusions, seams, slivers, and flat material with rough, slit, or torn edges is a cause of early failure. Overdrawn wire, improper hardness, and poor grain structure also cause early failure.
Group 3	High temperature	High operating temperatures reduce spring temper (or hardness) and lower the modulus of elasticity, thereby causing lower loads, reducing the elastic limit, and increasing corrosion. Corrosion-resisting or nickel alloys should be used.
	Low temperature	Temperatures below –40 degrees F reduce the ability of carbon steels to withstand shock loads. Carbon steels become brittle at -70 degrees F. Corrosion-resisting, nickel, or nonferrous alloys should be used.
	Friction	Close fits on rods or in holes result in a wearing away of material and occasional failure. The outside diameters of compression springs expand during deflection but they become smaller on torsion springs.
	Other causes	Enlarged hooks on extension springs increase the stress at the bends. Carrying too much electrical current will cause failure. Welding and soldering frequently destroy the spring temper. Tool marks, nicks, and cuts often raise stresses. Deflecting torsion springs outwardly causes high stresses and winding them tightly causes binding on supporting rods. High speed of deflection, vibration, and surging due to operation near natural periods of vibration or their harmonics cause increased stresses.

Spring failure may be breakage, high permanent set, or loss of load. The causes are listed in groups in this table. Group 1 covers causes that occur most frequently; Group 2 covers causes that are less frequent; and Group 3 lists causes that occur occasionally.

Table 23. Arbor Diameters for Springs Made from Music Wire

Wire Diam. (inch)	Spring Outside Diameter (inch)												
	1/16	3/32	1/8	5/32	3/16	7/32	1/4	9/32	5/16	11/32	3/8	7/16	1/2
	Arbor Diameter (inch)												
0.008	0.039	0.060	0.078	0.093	0.107	0.119	0.129	…	…	…	…	…	…
0.010	0.037	0.060	0.080	0.099	0.115	0.129	0.142	0.154	0.164	…	…	…	…
0.012	0.034	0.059	0.081	0.101	0.119	0.135	0.150	0.163	0.177	0.189	0.200	…	…
0.014	0.031	0.057	0.081	0.102	0.121	0.140	0.156	0.172	0.187	0.200	0.213	0.234	…
0.016	0.028	0.055	0.079	0.102	0.123	0.142	0.161	0.178	0.194	0.209	0.224	0.250	0.271
0.018	…	0.053	0.077	0.101	0.124	0.144	0.161	0.182	0.200	0.215	0.231	0.259	0.284
0.020	…	0.049	0.075	0.096	0.123	0.144	0.165	0.184	0.203	0.220	0.237	0.268	0.296
0.022	…	0.046	0.072	0.097	0.122	0.145	0.165	0.186	0.206	0.224	0.242	0.275	0.305
0.024	…	0.043	0.070	0.095	0.120	0.144	0.166	0.187	0.207	0.226	0.245	0.280	0.312
0.026	…	…	0.067	0.093	0.118	0.143	0.166	0.187	0.208	0.228	0.248	0.285	0.318
0.028	…	…	0.064	0.091	0.115	0.141	0.165	0.187	0.208	0.229	0.250	0.288	0.323
0.030	…	…	0.061	0.088	0.113	0.138	0.163	0.187	0.209	0.229	0.251	0.291	0.328
0.032	…	…	0.057	0.085	0.111	0.136	0.161	0.185	0.209	0.229	0.251	0.292	0.331
0.034	…	…	…	0.082	0.109	0.134	0.159	0.184	0.208	0.229	0.251	0.292	0.333
0.036	…	…	…	0.078	0.106	0.131	0.156	0.182	0.206	0.229	0.250	0.294	0.333
0.038	…	…	…	0.075	0.103	0.129	0.154	0.179	0.205	0.227	0.251	0.293	0.335
0.041	…	…	…	…	0.098	0.125	0.151	0.176	0.201	0.226	0.250	0.294	0.336
0.0475	…	…	…	…	0.087	0.115	0.142	0.168	0.194	0.220	0.244	0.293	0.337
0.054	…	…	…	…	…	0.103	0.132	0.160	0.187	0.212	0.245	0.287	0.336
0.0625	…	…	…	…	…	…	0.108	0.146	0.169	0.201	0.228	0.280	0.330
0.072	…	…	…	…	…	…	…	0.129	0.158	0.186	0.214	0.268	0.319
0.080	…	…	…	…	…	…	…	…	0.144	0.173	0.201	0.256	0.308
0.0915	…	…	…	…	…	…	…	…	…	…	0.181	0.238	0.293
0.1055	…	…	…	…	…	…	…	…	…	…	…	0.215	0.271
0.1205	…	…	…	…	…	…	…	…	…	…	…	…	0.215
0.125	…	…	…	…	…	…	…	…	…	…	…	…	0.239

Wire Diam. (inch)	Spring Outside Diameter (inches)													
	9/16	5/8	11/16	3/4	13/16	7/8	15/16	1	1 1/8	1 1/4	1 3/8	1 1/2	1 3/4	2
	Arbor Diameter (inches)													
0.022	0.332	0.357	0.380	…	…	…	…	…	…	…	…	…	…	…
0.024	0.341	0.367	0.393	0.415	…	…	…	…	…	…	…	…	…	…
0.026	0.350	0.380	0.406	0.430	…	…	…	…	…	…	…	…	…	…
0.028	0.356	0.387	0.416	0.442	0.467	…	…	…	…	…	…	…	…	…
0.030	0.362	0.395	0.426	0.453	0.481	0.506	…	…	…	…	…	…	…	…
0.032	0.367	0.400	0.432	0.462	0.490	0.516	0.540	…	…	…	…	…	…	…
0.034	0.370	0.404	0.437	0.469	0.498	0.526	0.552	0.557	…	…	…	…	…	…
0.036	0.372	0.407	0.442	0.474	0.506	0.536	0.562	0.589	…	…	…	…	…	…
0.038	0.375	0.412	0.448	0.481	0.512	0.543	0.572	0.600	0.650	…	…	…	…	…
0.041	0.378	0.416	0.456	0.489	0.522	0.554	0.586	0.615	0.670	0.718	…	…	…	…
0.0475	0.380	0.422	0.464	0.504	0.541	0.576	0.610	0.643	0.706	0.763	0.812	…	…	…
0.054	0.381	0.425	0.467	0.509	0.550	0.589	0.625	0.661	0.727	0.792	0.850	0.906	…	…
0.0625	0.379	0.426	0.468	0.512	0.556	0.597	0.639	0.678	0.753	0.822	0.889	0.951	1.06	1.17
0.072	0.370	0.418	0.466	0.512	0.555	0.599	0.641	0.682	0.765	0.840	0.911	0.980	1.11	1.22
0.080	0.360	0.411	0.461	0.509	0.554	0.599	0.641	0.685	0.772	0.851	0.930	1.00	1.13	1.26
0.0915	0.347	0.398	0.448	0.500	0.547	0.597	0.640	0.685	0.776	0.860	0.942	1.02	1.16	1.30
0.1055	0.327	0.381	0.433	0.485	0.535	0.586	0.630	0.683	0.775	0.865	0.952	1.04	1.20	1.35
0.1205	0.303	0.358	0.414	0.468	0.520	0.571	0.622	0.673	0.772	0.864	0.955	1.04	1.22	1.38
0.125	0.295	0.351	0.406	0.461	0.515	0.567	0.617	0.671	0.770	0.864	0.955	1.05	1.23	1.39

STRENGTH AND PROPERTIES OF WIRE ROPE

Strength and Properties of Wire Rope

Wire Rope Construction.—Essentially, a wire rope is made up of a number of strands laid helically about a metallic or non-metallic core. Each strand consists of a number of wires also laid helically about a metallic or non-metallic center. Various types of wire rope have been developed to meet a wide range of uses and operating conditions. These types are distinguished by the kind of core; the number of strands; the number, sizes, and arrangement of the wires in each strand; and the way in which the wires and strands are wound or laid about each other. The following descriptive material is based largely on information supplied by the Bethlehem Steel Co.

Rope Wire Materials: Materials used in the manufacture of rope wire are, in order of increasing strength: iron, phosphor bronze, traction steel, plow steel, improved plow steel, and bridge rope steel. Iron wire rope is largely used for low-strength applications such as elevator ropes not used for hoisting, and for stationary guy ropes.

Phosphor bronze wire rope is used occasionally for elevator governor-cable rope and for certain marine applications as life lines, clearing lines, wheel ropes and rigging.

Traction steel wire rope is used primarily as hoist rope for passenger and freight elevators of the traction drive type, an application for which it was specifically designed.

Ropes made of galvanized wire or wire coated with zinc by the electrodeposition process are used in certain applications where additional protection against rusting is required. As will be noted from the tables of wire-rope sizes and strengths, the breaking strength of galvanized wire rope is 10 per cent less than that of ungalvanized (bright) wire rope. Bethanized (zinc-coated) wire rope can be furnished to bright wire rope strength when so specified.

Galvanized carbon steel, tinned carbon steel, and stainless steel are used for small cords and strands ranging in diameter from $\frac{1}{64}$ to $\frac{3}{8}$ inch and larger.

Marline clad wire rope has each strand wrapped with a layer of tarred marline. The cladding provides hand protection for workers and wear protection for the rope.

Rope Cores: Wire-rope cores are made of fiber, cotton, asbestos, polyvinyl plastic, a small wire rope (independent wire-rope core), a multiple-wire strand (wire-strand core) or a cold-drawn wire-wound spring.

Fiber: (manila or sisal) is the type of core most widely used when loads are not too great. It supports the strands in their relative positions and acts as a cushion to prevent nicking of the wires lying next to the core.

Cotton: is used for small ropes such as sash cord and aircraft cord.

Asbestos cores: can be furnished for certain special operations where the rope is used in oven operations.

Polyvinyl plastics cores: are offered for use where exposure to moisture, acids, or caustics is excessive.

A wire-strand core: often referred to as WSC, consists of a multiple-wire strand that may be the same as one of the strands of the rope. It is smoother and more solid than the independent wire rope core and provides a better support for the rope strands.

The *independent wire rope core*, often referred to as IWRC, is a small 6 × 7 wire rope with a wire-strand core and is used to provide greater resistance to crushing and distortion of the wire rope. For certain applications it has the advantage over a wire-strand core in that it stretches at a rate closer to that of the rope itself.

Wire ropes with wire-strand cores are, in general, less flexible than wire ropes with independent wire-rope or non-metallic cores.

Ropes with metallic cores are rated 7½ per cent stronger than those with non-metallic cores.

Wire-Rope Lay: The lay of a wire rope is the direction of the helical path in which the strands are laid and, similarly, the lay of a strand is the direction of the helical path in which the wires are laid. If the wires in the strand or the strands in the rope form a helix similar to the threads of a right-hand screw, i.e., they wind around to the right, the lay is called right hand and, conversely, if they wind around to the left, the lay is called left hand. In the *regular lay*, the wires in the strands are laid in the opposite direction to the lay of the strands in the rope. In right-regular lay, the strands are laid to the right and the wires to the left. In left-regular lay, the strands are laid to the left, the wires to the right. In *Lang lay*, the wires and strands are laid in the same direction, i.e., in right Lang lay, both the wires and strands are laid to the right and in left Lang they are laid to the left.

Alternate lay ropes having alternate right and left laid strands are used to resist distortion and prevent clamp slippage, but because other advantages are missing, have limited use.

The regular lay wire rope is most widely used and right regular lay rope is customarily furnished. Regular lay rope has less tendency to spin or untwist when placed under load and is generally selected where long ropes are employed and the loads handled are frequently removed. Lang lay ropes have greater flexibility than regular lay ropes and are more resistant to abrasion and fatigue.

In preformed wire ropes the wires and strands are preshaped into a helical form so that when laid to form the rope they tend to remain in place. In a non-preformed rope, broken wires tend to "wicker out" or protrude from the rope and strands that are not seized tend to spring apart. Preforming also tends to remove locked-in stresses, lengthen service life, and make the rope easier to handle and to spool.

Strand Construction: Various arrangements of wire are used in the construction of wire rope strands. In the simplest arrangement six wires are grouped around a central wire thus making seven wires, all of the same size. Other types of construction known as "filler-wire," Warrington, Seale, etc. make use of wires of different sizes. Their respective patterns of arrangement are shown diagrammatically in the table of wire weights and strengths.

Specifying Wire Rope.—In specifying wire rope the following information will be required: length, diameter, number of strands, number of wires in each strand, type of rope construction, grade of steel used in rope, whether preformed or not preformed, type of center, and type of lay. The manufacturer should be consulted in selecting the best type of wire rope for a new application.

Properties of Wire Rope.—Important properties of wire rope are strength, wear resistance, flexibility, and resistance to crushing and distortion.

Strength: The strength of wire rope depends upon its size, kind of material of which the wires are made and their number, the type of core, and whether the wire is galvanized or not. Strengths of various types and sizes of wire ropes are given in the accompanying tables together with appropriate factors to apply for ropes with steel cores and for galvanized wire ropes.

Wear Resistance: When wire rope must pass back and forth over surfaces that subject it to unusual wear or abrasion, it must be specially constructed to give satisfactory service.

Such construction may make use of 1) relatively large outer wires; 2) Lang lay in which wires in each strand are laid in the same direction as the strand; and 3) flattened strands.

The object in each type is to provide a greater outside surface area to take the wear or abrasion. From the standpoint of material, improved plow steel has not only the highest tensile strength but also the greatest resistance to abrasion in regularly stocked wire rope.

Flexibility: Wire rope that undergoes repeated and severe bending, such as in passing around small sheaves and drums, must have a high degree of flexibility to prevent premature breakage and failure due to fatigue. Greater flexibility in wire rope is obtained by

1) using small wires in larger numbers; 2) using Lang lay; and 3) preforming, that is, the wires and strands of the rope are shaped during manufacture to fit the position they will assume in the finished rope.

Resistance to Crushing and Distortion: Where wire rope is to be subjected to transverse loads that may crush or distort it, care should be taken to select a type of construction that will stand up under such treatment.

Wire rope designed for such conditions may have 1) large outer wires to spread the load per wire over a greater area; and 2) an independent wire core or a high-carbon cold-drawn wound spring core.

Standard Classes of Wire Rope.—Wire rope is commonly designated by two figures, the first indicating the number of strands and the second, the number of wires per strand, as: 6 × 7, a six-strand rope having seven wires per strand, or 8 × 19, an eight-strand rope having 19 wires per strand. When such numbers are used as designations of standard wire rope classes, the second figure in the designation may be purely nominal in that the number of wires per strand for various ropes in the class may be slightly less or slightly more than the nominal as will be seen from the following brief descriptions. (For ropes with a wire strand core, a second group of two numbers may be used to indicate the construction of the wire core, as 1 × 21, 1 × 43, and so on.)

6 × 7 Class (Standard Coarse Laid Rope): Wire ropes in this class are for use where resistance to wear, as in dragging over the ground or across rollers, is an important requirement. Heavy hauling, rope transmissions, and well drilling are common applications. These wire ropes are furnished in right regular lay and occasionally in Lang lay. The cores may be of fiber, independent wire rope, or wire strand. Since this class is a relatively stiff type of construction, these ropes should be used with large sheaves and drums. Because of the small number of wires, a larger factor of safety may be called for.

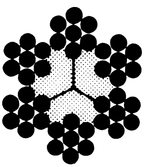


Fig. 1a.
6 × 7 with fiber core

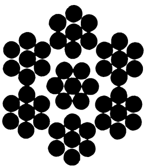


Fig. 1b.
6 × 7 with 1 × 7 WSC

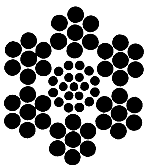


Fig. 1c.
6 × 7 with 1 × 19 WSC

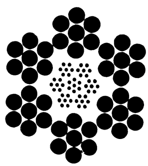


Fig. 1d.
6 × 7 with IWRC

As shown in Figs. 1a through Figs. 1d, this class includes a 6 × 7 construction with fiber core: a 6 × 7 construction with 1 × 7 wire strand core (sometimes called 7 × 7); a 6 × 7 construction with 1 × 19 wire strand core; and a 6 × 7 construction with independent wire rope core. Table 1 provides strength and weight data for this class.

Two special types of wire rope in this class are: aircraft cord, a 6 × 6 or 7 × 7 Bethanized wire rope of high tensile strength and sash cord, a 6 × 7 iron rope used for a variety of purposes where strength is not an important factor.

Table 1. Weights and Strengths of 6 × 7 (Standard Coarse Laid) Wire Ropes, Preformed and Not Preformed

Diam., Inches	Approx. Weight per Ft., Pounds	Breaking Strength, Tons of 2000 Lbs.			Diam., Inches	Approx. Weight per Ft., Pounds	Breaking Strength, Tons of 2000 Lbs.		
		Impr. Plow Steel	Plow Steel	Mild Plow Steel			Impr. Plow Steel	Plow Steel	Mild Plow Steel
1/4	0.094	2.64	2.30	2.00	3/4	0.84	22.7	19.8	17.2
5/16	0.15	4.10	3.56	3.10	7/8	1.15	30.7	26.7	23.2
3/8	0.21	5.86	5.10	4.43	1	1.50	39.7	34.5	30.0
7/16	0.29	7.93	6.90	6.00	1 1/8	1.90	49.8	43.3	37.7
1/2	0.38	10.3	8.96	7.79	1 1/4	2.34	61.0	53.0	46.1
9/16	0.48	13.0	11.3	9.82	1 3/8	2.84	73.1	63.6	55.3
5/8	0.59	15.9	13.9	12.0	1 1/2	3.38	86.2	75.0	65.2

For ropes with steel cores, add 7½ per cent to above strengths.

For galvanized ropes, deduct 10 per cent from above strengths.

Source: Rope diagrams, Bethlehem Steel Co. All data, U.S. Simplified Practice Recommendation 198–50.

6 × 19 Class (Standard Hoisting Rope): This rope is the most popular and widely used class. Ropes in this class are furnished in regular or Lang lay and may be obtained preformed or not preformed. Cores may be of fiber, independent wire rope, or wire strand. As can be seen from Table 2 and Figs. 2a through 2h, there are four common types: 6 × 25 filler wire construction with fiber core (not illustrated), independent wire core, or wire strand core (1 × 25 or 1 × 43); 6 × 19 Warrington construction with fiber core; 6 × 21 filler wire construction with fiber core; and 6 × 19, 6 × 21, and 6 × 17 Seale construction with fiber core.

Table 2. Weights and Strengths of 6 × 19 (Standard Hoisting) Wire Ropes, Preformed and Not Preformed

Dia., Inches	Approx. Weight per Ft., Pounds	Breaking Strength, Tons of 2000 Lbs.			Dia., Inches	Approx. Weight per Ft., Pounds	Breaking Strength, Tons of 2000 Lbs.		
		Impr. Plow Steel	Plow Steel	Mild Plow Steel			Impr. Plow Steel	Plow Steel	Mild Plow Steel
1/4	0.10	2.74	2.39	2.07	1 1/4	2.50	64.6	56.2	48.8
5/16	0.16	4.26	3.71	3.22	1 3/8	3.03	77.7	67.5	58.8
3/8	0.23	6.10	5.31	4.62	1 1/2	3.60	92.0	80.0	69.6
7/16	0.31	8.27	7.19	6.25	1 5/8	4.23	107	93.4	81.2
1/2	0.40	10.7	9.35	8.13	1 3/4	4.90	124	108	93.6
9/16	0.51	13.5	11.8	10.2	1 7/8	5.63	141	123	107
5/8	0.63	16.7	14.5	12.6	2	6.40	160	139	121
3/4	0.90	23.8	20.7	18.0	2 1/8	7.23	179	156	…
7/8	1.23	32.2	28.0	24.3	2 1/4	8.10	200	174	…
1	1.60	41.8	36.4	31.6	2 1/2	10.00	244	212	…
1 1/8	2.03	52.6	45.7	39.8	2 3/4	12.10	292	254	…

The 6 × 25 filler wire with fiber core not illustrated.

For ropes with steel cores, add 7½ per cent to above strengths.

For galvanized ropes, deduct 10 per cent from above strengths.

Source: Rope diagrams, Bethlehem Steel Co. All data, U.S. Simplified Practice Recommendation 198–50.

6 × 37 Class (Extra Flexible Hoisting Rope): For a given size of rope, the component wires are of smaller diameter than those in the two classes previously described and hence have less resistance to abrasion. Ropes in this class are furnished in regular and Lang lay with fiber core or independent wire rope core, preformed or not preformed.

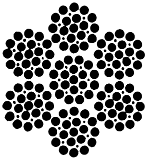


Fig. 2a.
6 × 25 filler wire
with WSC (1 × 25)

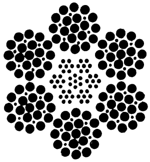


Fig. 2b.
6 × 25 filler wire
with IWRC

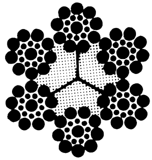


Fig. 2c.
6 × 19 Seale
with fiber core

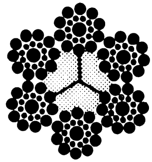


Fig. 2d.
6 × 21 Seale
with fiber core

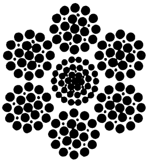


Fig. 2e.
6 × 25 filler wire
with WSC (1 × 43)

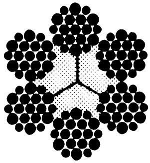


Fig. 2f.
6 × 19 Warrington
with fiber core

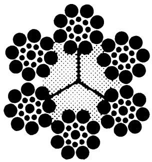


Fig. 2g.
6 × 17 Seale
with fiber core

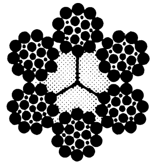


Fig. 2h.
6 × 21 filler wire
with fiber core

Table 3. Weights and Strengths of 6 × 37 (Extra Flexible Hoisting) Wire Ropes, Preformed and Not Preformed

Dia., Inches	Approx. Weight per Ft., Pounds	Breaking Strength, Tons of 2000 Lbs.		Dia., Inches	Approx. Weight per Ft., Pounds	Breaking Strength, Tons of 2000 Lbs.	
		Impr. Plow Steel	Plow Steel			Impr. Plow Steel	Plow Steel
¼	0.10	2.59	2.25	1½	3.49	87.9	76.4
5⁄16	0.16	4.03	3.50	1⅝	4.09	103	89.3
⅜	0.22	5.77	5.02	1¾	4.75	119	103
7⁄16	0.30	7.82	6.80	1⅞	5.45	136	118
½	0.39	10.2	8.85	2	6.20	154	134
9⁄16	0.49	12.9	11.2	2⅛	7.00	173	150
⅝	0.61	15.8	13.7	2¼	7.85	193	168
¾	0.87	22.6	19.6	2½	9.69	236	205
⅞	1.19	30.6	26.6	2¾	11.72	284	247
1	1.55	39.8	34.6	3	14.0	335	291
1⅛	1.96	50.1	43.5	3¼	16.4	390	339
1¼	2.42	61.5	53.5	3½	19.0	449	390
1⅜	2.93	74.1	64.5	…	…	…	…

For ropes with steel cores, add 7½ per cent to above strengths.

For galvanized ropes, deduct 10 per cent from above strengths.

Source: Rope diagrams, Bethlehem Steel Co. All data, U. S. Simplified Practice Recommendation 198-50.

As shown in Table 3 and Figs. 3a through 3h, there are four common types: 6 × 29 filler wire construction with fiber core and 6 × 36 filler wire construction with independent wire rope core, a special rope for construction equipment; 6 × 35 (two operations) construction with fiber core and 6 × 41 Warrington Seale construction with fiber core, a standard crane rope in this class of rope construction; 6 × 41 filler wire construction with fiber core or independent wire core, a special large shovel rope usually furnished in Lang lay; and 6 × 46 filler wire construction with fiber core or independent wire rope core, a special large shovel and dredge rope.

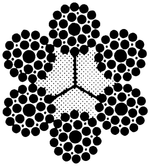


Fig. 3a.
6 × 29 filler wire
with fiber core

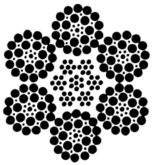


Fig. 3b.
6 × 36 filler wire
with IWRC

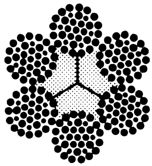


Fig. 3c.
6 × 35 with
fiber core

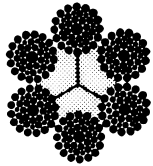


Fig. 3d.
6 × 41 Warrington-Seale
with fiber core

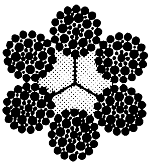


Fig. 3e.
6 × 41 filler wire
with fiber core

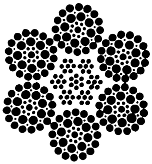


Fig. 3f.
6 × 41 filler wire
with IWRC

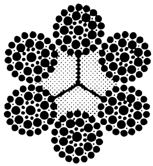


Fig. 3g.
6 × 46 filler wire
with fiber core

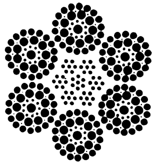


Fig. 3h.
6 × 46 filler wire
with IWRC

8 × 19 Class (Special Flexible Hoisting Rope): This rope is stable and smooth-running, and is especially suitable, because of its flexibility, for high speed operation with reverse bends. Ropes in this class are available in regular lay with fiber core.

As shown in Table 4 and Figs. 4a through 4d, there are four common types: 8 × 25 filler wire construction, the most flexible but the least wear resistant rope of the four types; Warrington type in 8 × 19 construction, less flexible than the 8 × 25; 8 × 21 filler wire construction, less flexible than the Warrington; and Seale type in 8 × 19 construction, which has the greatest wear resistance of the four types but is also the least flexible.

Table 4. Weights and Strengths of 8 × 19 (Special Flexible Hoisting) Wire Ropes, Preformed and Not Preformed

Dia., Inches	Approx. Weight per Ft., Pounds	Breaking Strength, Tons of 2000 Lbs.		Dia., Inches	Approx. Weight per Ft., Pounds	Breaking Strength, Tons of 2000 Lbs.	
		Impr. Plow Steel	Plow Steel			Impr. Plow Steel	Plow Steel
1/4	0.09	2.35	2.04	3/4	0.82	20.5	17.8
5/16	0.14	3.65	3.18	7/8	1.11	27.7	24.1
3/8	0.20	5.24	4.55	1	1.45	36.0	31.3
7/16	0.28	7.09	6.17	1 1/8	1.84	45.3	39.4
1/2	0.36	9.23	8.02	1 1/4	2.27	55.7	48.4
9/16	0.46	11.6	10.1	1 3/8	2.74	67.1	58.3
5/8	0.57	14.3	12.4	1 1/2	3.26	79.4	69.1

For ropes with steel cores, add 7 1/2 per cent to above strengths.

For galvanized ropes, deduct 10 per cent from above strengths.

Source: Rope diagrams, Bethlehem Steel Co. All data, U. S. Simplified Practice Recommendation 198-50.

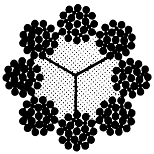


Fig. 4a.
8 × 25 filler wire
with fiber core

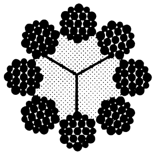


Fig. 4b.
8 × 19 Warrington
with fiber core

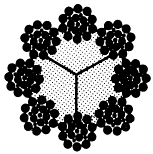


Fig. 4c.
8 × 21 filler wire
with fiber core

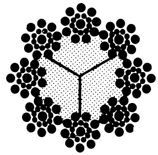


Fig. 4d.
8 × 19 Seale
with fiber core

Also in this class, but not shown in Table 4 are elevator ropes made of traction steel and iron.

18 × 7 Non-rotating Wire Rope: This rope is specially designed for use where a minimum of rotating or spinning is called for, especially in the lifting or lowering of free loads with a single-part line. It has an inner layer composed of 6 strands of 7 wires each laid in left Lang lay over a fiber core and an outer layer of 12 strands of 7 wires each laid in right regular lay. The combination of opposing lays tends to prevent rotation when the rope is stretched. However, to avoid any tendency to rotate or spin, loads should be kept to at least one-eighth and preferably one-tenth of the breaking strength of the rope. Weights and strengths are shown in Table 5.

Table 5. Weights and Strengths of Standard 18 × 7 Nonrotating Wire Rope, Preformed and Not Preformed

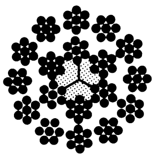


Fig. 5.
18 × 7 Non-Rotating Rope

Recommended Sheave and Drum Diameters: Single layer on drum … 36 rope diameters. Multiple layers on drum … 48 rope diameters. Mine service … 60 rope diameters.

Dia., Inches	Approx. Weight per Ft., Pounds	Breaking Strength, Tons of 2000 Lbs.		Dia., Inches	Approx. Weight per Ft., Pounds	Breaking Strength, Tons of 2000 Lbs.	
		Impr. Plow Steel	Plow Steel			Impr. Plow Steel	Plow Steel
3/16	0.061	1.42	1.24	7/8	1.32	29.5	25.7
1/4	0.108	2.51	2.18	1	1.73	38.3	33.3
5/16	0.169	3.90	3.39	1 1/8	2.19	48.2	41.9
3/8	0.24	5.59	4.86	1 1/4	2.70	59.2	51.5
7/16	0.33	7.58	6.59	1 3/8	3.27	71.3	62.0
1/2	0.43	9.85	8.57	1 1/2	3.89	84.4	73.4
9/16	0.55	12.4	10.8	1 5/8	4.57	98.4	85.6
5/8	0.68	15.3	13.3	1 3/4	5.30	114	98.8
3/4	0.97	21.8	19.0	…	…	…	…

For galvanized ropes, deduct 10 per cent from above strengths.

Source: Rope diagrams, sheave and drum diameters, and data for 3/16, 1/4 and 5/16-inch sizes, Bethlehem Steel Co. All other data, U. S. Simplified Practice Recommendation 198-50.

Flattened Strand Wire Rope: The wires forming the strands of this type of rope are wound around triangular centers so that a flattened outer surface is provided with a greater area than in the regular round rope to withstand severe conditions of abrasion. The triangular shape of the strands also provides superior resistance to crushing. Flattened strand wire rope is usually furnished in Lang lay and may be obtained with fiber core or independent wire rope core. The three types shown in Table 6 and Figs. 6a through 6c are flexible and are designed for hoisting work.

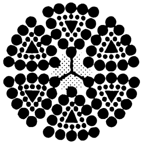


Fig. 6a.
6 × 25 with fiber core

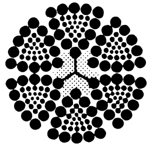


Fig. 6b.
6 × 30 with fiber core

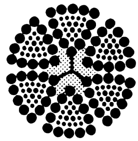


Fig. 6c.
6 × 27 with fiber core

Table 6. Weights and Strengths of Flattened Strand Wire Rope, Preformed and Not Preformed

Dia., Inches	Approx. Weight per Ft., Pounds	Breaking Strength, Tons of 2000 Lbs.		Dia., Inches	Approx. Weight per Ft., Pounds	Breaking Strength, Tons of 2000 Lbs.	
		Impr. Plow Steel	Mild Plow Steel			Impr. Plow Steel	Mild Plow Steel
3/8[a]	0.25	6.71	…	1 3/8	3.40	85.5	…
1/2[a]	0.45	11.8	8.94	1 1/2	4.05	101	…
9/16[a]	0.57	14.9	11.2	1 5/8	4.75	118	…
5/8	0.70	18.3	13.9	1 3/4	5.51	136	…
3/4	1.01	26.2	19.8	2	7.20	176	…
7/8	1.39	35.4	26.8	2 1/4	9.10	220	…
1	1.80	46.0	34.8	2 1/2	11.2	269	…
1 1/8	2.28	57.9	43.8	2 3/4	13.6	321	…
1 1/4	2.81	71.0	53.7	…	…	…	…

[a] These sizes in Type B only.

Type H is not in U.S. Simplified Practice Recommendation.

Source: Rope diagrams, Bethlehem Steel Co. All other data, U.S. Simplified Practice Recommendation 198-50.

Flat Wire Rope: This type of wire rope is made up of a number of four-strand rope units placed side by side and stitched together with soft steel sewing wire. These four-strand units are alternately right and left lay to resist warping, curling, or rotating in service. Weights and strengths are shown in Table 7.

Simplified Practice Recommendations.—Because the total number of wire rope types is large, manufacturers and users have agreed upon and adopted a U.S. Simplified Practice Recommendation to provide a simplified listing of those kinds and sizes of wire rope which are most commonly used and stocked. These, then, are the types and sizes which are most generally available. Other types and sizes for special or limited uses also may be found in individual manufacturer's catalogs.

Sizes and Strengths of Wire Rope.—The data shown in Tables 1 through 7 have been taken from U.S. Simplified Practice Recommendation 198-50 but do not include those wire ropes shown in that Simplified Practice Recommendation which are intended primarily for marine use.

Wire Rope Diameter: The diameter of a wire rope is the diameter of the circle that will just enclose it, hence when measuring the diameter with calipers, care must be taken to obtain the largest outside dimension, taken across the opposite strands, rather than the smallest dimension across opposite "valleys" or "flats." It is standard practice for the nominal diameter to be the minimum with all tolerances taken on the plus side. Limits for diameter as well as for minimum breaking strength and maximum pitch are given in Federal Specification for Wire Rope, RR-R—571a.

Wire Rope Strengths: The strength figures shown in the accompanying tables have been obtained by a mathematical derivation based on actual breakage tests of wire rope and represent from 80 to 95 per cent of the total strengths of the individual wires, depending upon the type of rope construction.

Table 7. Weights and Strengths of Standard Flat Wire Rope, Not Preformed

Flat Wire Rope

This rope consists of a number of 4-strand rope units placed side by side and stitched together with soft steel sewing wire.

Width and Thickness, Inches	No. of Ropes	Approx. Weight per Ft., Pounds	Breaking Strength, Tons of 2000 Lbs. Plow Steel	Breaking Strength, Tons of 2000 Lbs. Mild Plow-Steel	Width and Thickness, Inches	No. of Ropes	Approx. Weight per Ft., Pounds	Breaking Strength, Tons of 2000 Lbs. Plow Steel	Breaking Strength, Tons of 2000 Lbs. Mild Plow Steel
1/4 × 1 1/2	7	0.69	16.8	14.6	1/2 × 4	9	3.16	81.8	71.2
1/4 × 2	9	0.88	21.7	18.8	1/2 × 4 1/2	10	3.82	90.9	79.1
1/4 × 2 1/2	11	1.15	26.5	23.0	1/2 × 5	12	4.16	109	94.9
1/4 × 3	13	1.34	31.3	27.2	1/2 × 5 1/2	13	4.50	118	103
					1/2 × 6	14	4.85	127	111
5/16 × 1 1/2	5	0.77	18.5	16.0	1/2 × 7	16	5.85	145	126
5/16 × 2	7	1.05	25.8	22.4					
5/16 × 2 1/2	9	1.33	33.2	28.8	5/8 × 3 1/2	6	3.40	85.8	74.6
5/16 × 3	11	1.61	40.5	35.3	5/8 × 4	7	3.95	100	87.1
5/16 × 3 1/2	13	1.89	47.9	41.7	5/8 × 4 1/2	8	4.50	114	99.5
5/16 × 4	15	2.17	55.3	48.1	5/8 × 5	9	5.04	129	112
					5/8 × 5 1/2	10	5.59	143	124
3/8 × 2	6	1.25	31.4	27.3	5/8 × 6	11	6.14	157	137
3/8 × 2 1/2	8	1.64	41.8	36.4	5/8 × 7	13	7.23	186	162
3/8 × 3	9	1.84	47.1	40.9	5/8 × 8	15	8.32	214	186
3/8 × 3 1/2	11	2.23	57.5	50.0					
3/8 × 4	12	2.44	62.7	54.6	3/4 × 5	8	6.50	165	143
3/8 × 4 1/2	14	2.83	73.2	63.7	3/4 × 6	9	7.31	185	161
3/8 × 5	15	3.03	78.4	68.2	3/4 × 7	10	8.13	206	179
3/8 × 5 1/2	17	3.42	88.9	77.3	3/4 × 8	11	9.70	227	197
3/8 × 6	18	3.63	94.1	81.9					
					7/8 × 5	7	7.50	190	165
1/2 × 2 1/2	6	2.13	54.5	47.4	7/8 × 6	8	8.56	217	188
1/2 × 3	7	2.47	63.6	55.4	7/8 × 7	9	9.63	244	212
1/2 × 3 1/2	8	2.82	72.7	63.3	7/8 × 8	10	10.7	271	236

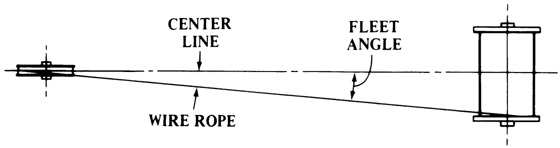
Source: Rope diagram, Bethlehem Steel Co.; all data, U.S. Simplified Practice Recommendation 198–50.

Safe Working Loads and Factors of Safety.—The maximum load for which a wire rope is to be used should take into account such associated factors as friction, load caused by bending around each sheave, acceleration and deceleration, and, if a long length of rope is to be used for hoisting, the weight of the rope at its maximum extension. The condition of the rope — whether new or old, worn or corroded — and type of attachments should also be considered.

Factors of safety for standing rope usually range from 3 to 4; for operating rope, from 5 to 12. Where there is the element of hazard to life or property, higher values are used.

Installing Wire Rope.—The main precaution to be taken in removing and installing wire rope is to avoid kinking which greatly lessens the strength and useful life. Thus, it is preferable when removing wire rope from the reel to have the reel with its axis in a horizontal position and, if possible, mounted so that it will revolve and the wire rope can be taken off straight. If the rope is in a coil, it should be unwound with the coil in a vertical position as by rolling the coil along the ground. Where a drum is to be used, the rope should be run directly onto it from the reel, taking care to see that it is not bent around the drum in a direction opposite to that on the reel, thus causing it to be subject to reverse bending. On flat or smooth-faced drums it is important that the rope be started from the proper end of the drum. A right lay rope that is being overwound on the drum, that is, it passes over the top of the drum as it is wound on, should be started from the right flange of the drum (looking at the drum from the side that the rope is to come) and a left lay rope from the left flange.

When the rope is underwound on the drum, a right lay rope should be started from the left flange and a left lay rope from the right flange, so that the rope will spool evenly and the turns will lie snugly together.



Sheaves and drums should be properly aligned to prevent undue wear. The proper position of the main or lead sheave for the rope as it comes off the drum is governed by what is called the fleet angle or angle between the rope as it stretches from drum to sheave and an imaginary center-line passing through the center of the sheave groove and a point halfway between the ends of the drum. When the rope is at one end of the drum, this angle should not exceed one and a half to two degrees. With the lead sheave mounted with its groove on this center-line, a safe fleet angle is obtained by allowing 30 feet of lead for each two feet of drum width.

Sheave and Drum Dimensions: Sheaves and drums should be as large as possible to obtain maximum rope life. However, factors such as the need for lightweight equipment for easy transport and use at high speeds, may call for relatively small sheaves with consequent sacrifice in rope life in the interest of overall economy. No hard and fast rules can be laid down for any particular rope if the utmost in economical performance is to be obtained. Where maximum rope life is of prime importance, the following recommendations of Federal Specification RR-R-571a for minimum sheave or drum diameters D in terms of rope diameter d will be of interest. For 6 × 7 rope (six strands of 7 wires each) $D = 72d$; for 6 × 19 rope, $D = 45d$; for 6 × 25 rope, $D = 45d$; for 6 × 29 rope, $D = 30d$; for 6 × 37 rope, $D = 27d$; and for 8 × 19 rope, $D = 31d$.

Too small a groove for the rope it is to carry will prevent proper seating of the rope in the bottom of the groove and result in uneven distribution of load on the rope. Too large a groove will not give the rope sufficient side support. Federal specification RR-R-571a recommends that sheave groove diameters be larger than the nominal rope diameters by the following minimum amounts: For ropes of $\frac{1}{4}$- to $\frac{5}{16}$-inch diameters, $\frac{1}{64}$ inch larger; for $\frac{3}{8}$- to $\frac{3}{4}$-inch diameter ropes, $\frac{1}{32}$ inch larger; for $\frac{13}{16}$- to $1\frac{1}{8}$-inch diameter ropes, $\frac{3}{64}$ inch larger; for $1\frac{3}{16}$- to $1\frac{1}{2}$-inch ropes, $\frac{1}{16}$ inch larger; for $1\frac{9}{16}$- to $2\frac{1}{4}$-inch ropes, $\frac{3}{32}$ inch larger; and for $2\frac{5}{16}$ and larger diameter ropes, $\frac{1}{8}$ inch larger. For new or regrooved sheaves these values should be doubled; in other words for $\frac{1}{4}$- to $\frac{5}{16}$-inch diameter ropes, the groove diameter should be $\frac{1}{32}$ inch larger, and so on.

Drum or Reel Capacity: The length of wire rope, in feet, that can be spooled onto a drum or reel, is computed by the following formula, where

A = depth of rope space on drum, inches: $A = (H - D - 2Y) \div 2$

B = width between drum flanges, inches

D = diameter of drum barrel, inches

H = diameter of drum flanges, inches

K = factor from Table 8 for size of line selected

Y = depth not filled on drum or reel where winding is to be less than full capacity

L = length of wire rope on drum or reel, feet.

$$L = (A + D) \times A \times B \times K$$

Table 8. Table 8 Factors K Used in Calculating Wire Rope Drum and Reel Capacities

Rope Dia., In.	Factor K	Rope Dia., In.	Factor K	Rope Dia., In.	Factor K
3⁄32	23.4	1⁄2	0.925	1 3⁄8	0.127
1⁄8	13.6	9⁄16	0.741	1 1⁄2	0.107
9⁄64	10.8	5⁄8	0.607	1 5⁄8	0.0886
5⁄32	8.72	11⁄16	0.506	1 3⁄4	0.0770
3⁄16	6.14	3⁄4	0.428	1 7⁄8	0.0675
7⁄32	4.59	13⁄16	0.354	2	0.0597
1⁄4	3.29	7⁄8	0.308	2 1⁄8	0.0532
5⁄16	2.21	1	0.239	2 1⁄4	0.0476
3⁄8	1.58	1 1⁄8	0.191	2 3⁄8	0.0419
7⁄16	1.19	1 1⁄4	0.152	2 1⁄2	0.0380

Note: The values of "K" allow for normal oversize of ropes, and the fact that it is practically impossible to "thread-wind" ropes of small diameter. However, the formula is based on uniform rope winding and will not give correct figures if rope is wound non-uniformly on the reel. The amount of tension applied when spooling the rope will also affect the length. The formula is based on the same number of wraps of rope in each layer, which is not strictly correct, but which does not result in appreciable error unless the width (B) of the reel is quite small compared with the flange diameter (H).

Example: Find the length in feet of 9⁄16-inch diameter rope required to fill a drum having the following dimensions: B = 24 inches, D = 18 inches, H = 30 inches,

$$A = (30 - 18 - 0) \div 2 = 6 \text{ inches}$$
$$L = (6 + 18) \times 6 \times 24 \times 0.741 = 2560.0 \text{ or } 2560 \text{ feet}$$

The above formula and factors K allow for normal oversize of ropes but will not give correct figures if rope is wound non-uniformly on the reel.

Load Capacity of Sheave or Drum: To avoid excessive wear and groove corrugation, the radial pressure exerted by the wire rope on the sheave or drum must be kept within certain maximum limits. The radial pressure of the rope is a function of the rope tension, rope diameter, and tread diameter of the sheave and can be determined by the following equation:

$$P = \frac{2T}{D \times d}$$

where P = Radial pressure in pounds per square inch (see Table 9)

T = Rope tension in pounds

D = Tread diameter of sheave or drum in inches

d = Rope diameter in inches

Table 9. Maximum Radial Pressures for Drums and Sheaves

Type of Wire Rope	Drum or Sheave Material		
	Cast Iron	Cast Steel	Manganese Steel[a]
	Recommended Maximum Radial Pressures, Pounds per Square Inch		
6 × 7	300[b]	550[b]	1500[b]
6 × 19	500[b]	900[b]	2500[b]
6 × 37	600	1075	3000
6 × 8 Flattened Strand	450	850	2200
6 × 25 Flattened Strand	800	1450	4000
6 × 30 Flattened Strand	800	1450	4000

[a] 11 to 13 per cent manganese.

[b] These values are for regular lay rope. For Lang lay rope these values may be increased by 15 per cent.

According to the Bethlehem Steel Co. the radial pressures shown in Table 9 are recommended as maximums according to the material of which the sheave or drum is made.

Rope Loads due to Bending: When a wire rope is bent around a sheave, the resulting bending stress s_b in the outer wire, and equivalent bending load P_b (amount that direct tension load on rope is increased by bending) may be computed by the following formulas: $s_b = Ed_w \div D$; $P_b = s_b A$, where $A = d^2 Q$. E is the modulus of elasticity of the wire rope (varies with the type and condition of rope from 10,000,000 to 14,000,000. An average value of 12,000,000 is frequently used), d is the diameter of the wire rope, d_w is the diameter of the component wire (for 6 × 7 rope, $d_w = 0.106d$; for 6 × 19 rope, $0.063d$; for 6 × 37 rope, $0.045d$; and for 8 × 19 rope, $d_w = 0.050d$). D is the pitch diameter of the sheave in inches, A is the metal cross-sectional area of the rope, and Q is a constant, values for which are: 6 × 7 (Fiber Core) rope, 0.380; 6 × 7 (IWRC or WSC), 0.437; 6 × 19 (Fiber Core), 0.405; 6 × 19 (IWRC or WSC), 0.475; 6 × 37 (Fiber Core), 0.400; 6 × 37 (IWRC), 0.470; 8 × 19 (Fiber Core), 0.370; and Flattened Strand Rope, 0.440.

Example: Find the bending stress and equivalent bending load due to the bending of a 6 × 19 (Fiber Core) wire rope of ½-inch diameter around a 24-inch pitch diameter sheave.

$$d_w = 0.063 \times 0.5 = 0.0315 \text{ in.} \qquad A = 0.5^2 \times 0.405 = 0.101 \text{ sq. in.}$$

$$s_b = 12{,}000{,}000 \times 0.0315 \div 24 = 15{,}750 \text{ lbs. per sq. in.}$$

$$P_b = 15{,}750 \times 0.101 = 1590 \text{ lbs.}$$

Cutting and Seizing of Wire Rope.—Wire rope can be cut with mechanical wire rope shears, an abrasive wheel, an electric resistance cutter (used for ropes of smaller diameter only), or an acetylene torch. This last method fuses the ends of the wires in the strands. It is important that the rope be seized on either side of where the cut is to be made. Any annealed low carbon steel wire may be used for seizing, the recommended sizes being as follows: For a wire rope of ¼- to 15⁄16-inch diameter, use a seizing wire of 0.054-inch (No. 17 Steel Wire Gage); for a rope of 1- to 1⅝-inch diameter, use a 0.105-inch wire (No. 12); and for rope of 1¾- to 3½-inch diameter, use a 0.135-inch wire (No. 10). Except for preformed wire ropes, a minimum of two seizings on either side of a cut is recommended. Four seizings should be used on either side of a cut for Lang lay rope, a rope with a steel core, or a non-spinning type of rope.

The following method of seizing is given in Federal specification for wire rope, RR-R-571a. Lay one end of the seizing wire in the groove between two strands of wire rope and wrap the other end tightly in a close helix over the portion in the groove. A seizing iron (round bar ½ to ⅝ inch diameter by 18 inches long) should be used to wrap the seizing tightly. This bar is placed at right angles to the rope next to the first turn or two of the seizing wire. The seizing wire is brought around the back of the seizing iron so that it can be wrapped loosely around the wire rope in the opposite direction to that of the seizing coil. As the seizing iron is now rotated around the rope it will carry the seizing wire snugly and tightly into place. When completed, both ends of the seizing should be twisted together tightly.

Maintenance of Wire Rope.—Heavy abrasion, overloading, and bending around sheaves or drums that are too small in diameter are the principal reasons for the rapid deterioration of wire rope. Wire rope in use should be inspected periodically for evidence of wear and damage by corrosion. Such inspections should take place at progressively shorter intervals over the useful life of the rope as wear tends to accelerate with use. Where wear is rapid, the outside of a wire rope will show flattened surfaces in a short time.

If there is any hazard involved in the use of the rope, it may be prudent to estimate the remaining strength and service life. This assessment should be done for the weakest point where the most wear or largest number of broken wires are in evidence. One way to arrive at a conclusion is to set an arbitrary number of broken wires in a given strand as an indication that the rope should be removed from service and an ultimate strength test run on the worn sample. The arbitrary figure can then be revised and rechecked until a practical working formula is arrived at. A piece of waste rubbed along the wire rope will help to reveal broken wires. The effects of corrosion are not easy to detect because the exterior wires may appear to be only slightly rusty, and the damaging effects of corrosion may be confined to the hidden inner wires where it cannot be seen. To prevent damage by corrosion, the rope should be kept well lubricated. Use of zinc coated wire rope may be indicated for some applications.

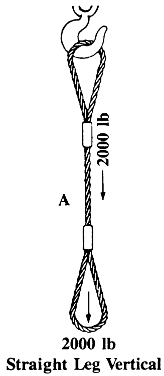
Periodic cleaning of wire rope by using a stiff brush and kerosene or with compressed air or live steam and relubricating will help to lengthen rope life and reduce abrasion and wear on sheaves and drums. Before storing after use, wire rope should be cleaned and lubricated.

Lubrication of Wire Rope.—Although wire rope is thoroughly lubricated during manufacture to protect it against corrosion and to reduce friction and wear, this lubrication should be supplemented from time to time. Special lubricants are supplied by wire rope manufacturers. These lubricants vary somewhat with the type of rope application and operating condition. Where the preferred lubricant can not be obtained from the wire rope manufacturer, an adhesive type of lubricant similar to that used for open gearing will often be found suitable. At normal temperatures, some wire rope lubricants may be practically solid and will require thinning before application. Thinning may be done by heating to 160 to 200 degrees F. or by diluting with gasoline or some other fluid that will allow the lubricant to penetrate the rope. The lubricant may be painted on the rope or the rope may be passed through a box or tank filled with the lubricant.

Replacement of Wire Rope.—When an old wire rope is to be replaced, all drums and sheaves should be examined for wear. All evidence of scoring or imprinting of grooves from previous use should be removed and sheaves with flat spots, defective bearings, and broken flanges, should be repaired or replaced. It will frequently be found that the area of maximum wear is located relatively near one end of the rope. By cutting off that portion, the remainder of the rope may be salvaged for continued use. Sometimes the life of a rope can be increased by simply changing it end for end at about one-half the estimated normal life. The worn sections will then no longer come at the points that cause the greatest wear.

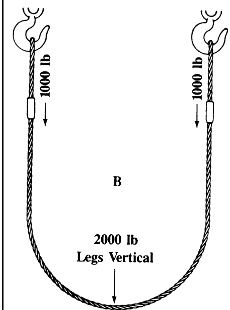
Wire Rope Slings and Fittings.—A few of the simpler sling arrangements or hitches as they are called, are shown in the accompanying illustration. Normally 6 × 19 Class wire rope is recommended where a diameter in the ¼-inch to 1⅛-inch range is to be used and 6 × 37 Class wire rope where a diameter in the 1¼-inch and larger range is to be used. However, the 6 × 19 Class may be used even in the larger sizes if resistance to abrasion is of primary importance and the 6 × 37 Class in the smaller sizes if greater flexibility is desired.

Wire Rope Slings and Fittings



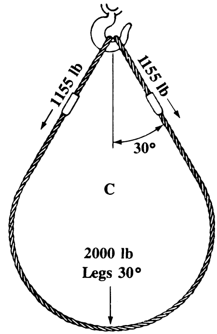
Straight Lift

One leg Vertical. Load capacity is 100 pct of a single rope.



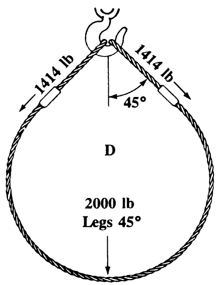
Basket Hitch

Two legs vertical. Load capacity is 200 pct of the single rope in the Straight Lift Hitch (A).



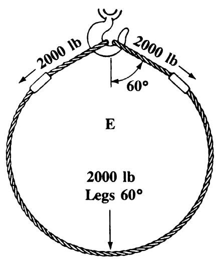
Basket Hitch

Two Legs at 30 deg with the vertical. Load capacity is 174 pct of the single rope in the Straight Lift Hitch (A).



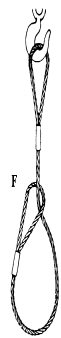
Basket Hitch

Two legs at 45 deg with the vertical. Load capacity is 141 pct of the single rope in the Straight Lift Hitch (A).



Basket Hitch

Two legs at 60 deg with the vertical. Load capacity is 100 pct of the single rope in the Stright Lift Hitch (A).



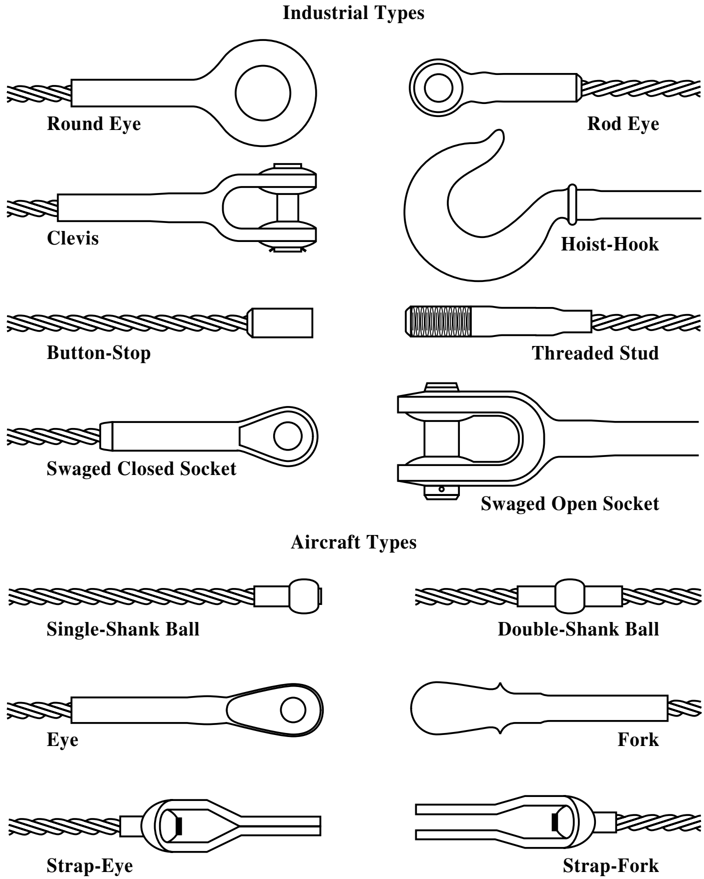
Choker Hitch

One leg vertical, with slip-through loop. Rated capacity is 75 pct of the single rope in the Straight Lift Hitch (A).

The *straight lift hitch*, shown at A, is a straight connector between crane hook and load.

The *basket hitch* may be used with two hooks so that the sides are vertical as shown at B or with a single hook with sides at various angles with the vertical as shown at C, D, and E. As the angle with the vertical increases, a greater tension is placed on the rope so that for any given load, a sling of greater lifting capacity must be used.

The *choker hitch*, shown at F, is widely used for lifting bundles of items such as bars, poles, pipe, and similar objects. The choker hitch holds these items firmly but the load must be balanced so that it rides safely. Since additional stress is imposed on the rope due to the choking action, the capacity of this type of hitch is 25 per cent less than that of the comparable straight lift. If two choker hitches are used at an angle, these angles must also be taken into consideration as with the basket hitches.



Wire Rope Fittings

Wire Rope Fittings.—Many varieties of swaged fittings are available for use with wire rope and several industrial and aircraft types are shown in the accompanying illustration. Swaged fittings on wire rope have an efficiency (ability to hold the wire rope) of approximately 100 per cent of the catalogue rope strength. These fittings are attached to the end or body of the wire rope by the application of high pressure through special dies that cause the material of the fitting to "flow" around the wires and strands of the rope to form a union that is as strong as the rope itself. The more commonly used types, of swaged fittings range from 1⁄8- to 5⁄8-inch diameter sizes in industrial types and from the 1⁄16- to 5⁄8-inch sizes in aircraft types. These fittings are furnished attached to the wire strand, rope, or cable.

Applying Clips and Attaching Sockets.—In attaching U-bolt clips for fastening the end of a wire rope to form a loop, it is essential that the saddle or base of the clip bears against the longer or "live" end of the rope loop and the U-bolt against the shorter or "dead" end. The "U" of the clips should never bear against the live end of the rope because the rope may be cut or kinked. A wire-rope thimble should be used in the loop eye of the rope to prevent kinking when rope clips are used. The strength of a clip fastening is usually less than 80 percent of the strength of the rope. Table 10 gives the proper size, number, and spacing for each size of wire rope.

Table 10. Clips Required for Fastening Wire Rope End

Rope Dia., In.	U-Bolt Dia., In.	Min. No. of Clips	Clip Spacing, In.	Rope Dia., In.	U-Bolt Dia., In.	Min. No. of Clips	Clip Spacing, In.
3⁄16	11⁄32	2	3	1 1⁄8	1 1⁄4	5	9 3⁄4
1⁄4	7⁄16	2	3 1⁄4	1 1⁄4	1 7⁄16	5	10 3⁄4
5⁄16	1⁄2	2	3 1⁄4	1 3⁄8	1 1⁄2	6	11 1⁄2
3⁄8	9⁄16	2	4	1 1⁄2	1 23⁄32	6	12 1⁄2
7⁄16	5⁄8	2	4 1⁄2	1 5⁄8	1 3⁄4	6	13 1⁄4
1⁄2	11⁄16	3	5	1 3⁄4	1 15⁄16	7	14 1⁄2
5⁄8	3⁄4	3	5 3⁄4	2	2 1⁄8	8	16 1⁄2
3⁄4	7⁄8	4	6 3⁄4	2 1⁄4	2 5⁄8	8	16 1⁄2
7⁄8	1	4	8	2 1⁄2	2 7⁄8	8	17 3⁄4
1	1 1⁄8	4	8 3⁄4	…	…	…	…

In attaching commercial sockets of forged steel to wire rope ends, the following procedure is recommended. The wire rope is seized at the end and another seizing is applied at a distance from the end equal to the length of the basket of the socket. As explained in a previous section, soft iron wire is used and particularly for the larger sizes of wire rope, it is important to use a seizing iron to secure a tight winding. For large ropes, the seizing should be several inches long.

The end seizing is now removed and the strands are separated so that the fiber core can be cut back to the next seizing. The individual wires are then untwisted and "broomed out" and for the distance they are to be inserted in the socket are carefully cleaned with benzine, naphtha, or unleaded gasoline. The wires are then dipped into commercial muriatic (hydrochloric) acid and left (usually one to three minutes) until the wires are bright and clean or, if zinc coated, until the zinc is removed. After cleaning, the wires are dipped into a hot soda solution (1 pound of soda to 4 gallons of water at 175 degrees F. minimum) to neutralize the acid. The rope is now placed in a vise. A temporary seizing is used to hold the wire ends together until the socket is placed over the rope end. The temporary seizing is then removed and the socket located so that the ends of the wires are about even with the upper end of the basket. The opening around the rope at the bottom of the socket is now sealed with putty.

A special high grade pure zinc is used to fill the socket. Babbit metal should not be used as it will not hold properly. For proper fluidity and penetration, the zinc is heated to a temperature in the 830- to 900-degree F. range. If a pyrometer is not available to measure the temperature of the molten zinc, a dry soft pine stick dipped into the zinc and quickly withdrawn will show only a slight discoloration and no zinc will adhere to it. If the wood chars, the zinc is too hot. The socket is now permitted to cool and the resulting joint is ready for use. When properly prepared, the strength of the joint should be approximately equal to that of the rope itself.

Rated Capacities for Improved Plow Steel Wire Rope and Wire Rope Slings (in tons of 2,000 lbs)—Independent Wire Rope Core

Rope Diameter (in.)	Vertical			Choker			60° Bridle			45°Bridle			30°Bridle		
	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C
Single Leg, 6 × 19 Wire Rope															
1/4	0.59	0.56	0.53	0.44	0.42	0.40	…	…	…	…	…	…	…	…	…
3/8	1.3	1.2	1.1	0.98	0.93	0.86	…	…	…	…	…	…	…	…	…
1/2	2.3	2.2	2.0	1.7	1.6	1.5	…	…	…	…	…	…	…	…	…
5/8	3.6	3.4	3.0	2.7	2.5	2.2	…	…	…	…	…	…	…	…	…
3/4	5.1	4.9	4.2	3.8	3.6	3.1	…	…	…	…	…	…	…	…	…
7/8	6.9	6.6	5.5	5.2	4.9	4.1	…	…	…	…	…	…	…	…	…
1	9.0	8.5	7.2	6.7	6.4	5.4	…	…	…	…	…	…	…	…	…
1 1/8	11	10	9.0	8.5	7.8	6.8	…	…	…	…	…	…	…	…	…
Single Leg, 6 × 37 Wire Rope															
1 1/4	13	12	10	9.9	9.2	7.9	…	…	…	…	…	…	…	…	…
1 3/8	16	15	13	12	11	9.6	…	…	…	…	…	…	…	…	…
1 1/2	19	17	15	14	13	11	…	…	…	…	…	…	…	…	…
1 3/4	26	24	20	19	18	15	…	…	…	…	…	…	…	…	…
2	33	30	26	25	23	20	…	…	…	…	…	…	…	…	…
2 1/4	41	38	33	31	29	25	…	…	…	…	…	…	…	…	…
Two-Leg Bridle or Basket Hitch, 6 × 19 Wire Rope Sling															
1/4	1.2	1.1	1.0	…	…	…	1.0	0.97	0.92	0.83	0.79	0.75	0.59	0.56	0.53
3/8	2.0	2.5	2.3	…	…	…	2.3	2.1	2.0	1.8	1.8	1.8	1.3	1.2	1.1
1/2	4.0	4.4	3.9	…	…	…	4.0	3.6	3.4	3.2	3.1	2.8	2.3	2.2	2.0
5/8	7.2	6.6	6.0	…	…	…	6.2	5.9	5.2	5.1	4.8	4.2	3.6	3.4	3.0
3/4	10	9.7	8.4	…	…	…	8.9	8.4	7.3	7.2	6.9	5.9	5.1	4.9	4.2
7/8	14	13	11	…	…	…	12	11	9.6	9.8	9.3	7.8	6.9	6.6	5.5
1	18	17	14	…	…	…	15	15	12	13	12	10	9.0	8.5	7.2
1 1/8	23	21	18	…	…	…	19	18	16	16	15	13	11	10	9.0
Two-Leg Bridle or Basket Hitch, 6 × 37 Wire Rope Sling															
1 1/4	26	24	21	…	…	…	23	21	18	19	17	15	13	12	10
1 3/8	32	29	25	…	…	…	28	25	22	22	21	18	16	15	13
1 1/2	38	35	30	…	…	…	33	30	26	27	25	21	19	17	15
1 3/4	51	47	41	…	…	…	44	41	35	36	33	29	26	24	20
2	66	61	53	…	…	…	57	53	46	47	43	37	33	30	26
2 1/4	83	76	66	…	…	…	72	66	67	58	54	47	41	38	33

Rated Capacities for Improved Plow Steel Wire Rope and Wire Rope Slings (in tons of 2,000 lbs)—Fiber Core

Rope Diameter (in.)	Vertical			Choker			60° Bridle			45° Bridle			30° Bridle		
	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C
Single Leg, 6 × 19 Wire Rope															
¼	0.55	0.51	0.49	0.41	0.38	0.37	…	…	…	…	…	…	…	…	…
⅜	1.2	1.1	1.1	0.91	0.85	0.80	…	…	…	…	…	…	…	…	…
½	2.1	2.0	1.8	1.6	1.5	1.4	…	…	…	…	…	…	…	…	…
⅝	3.3	3.1	2.8	2.5	2.3	2.1	…	…	…	…	…	…	…	…	…
¾	4.8	4.4	3.9	3.6	3.3	2.9	…	…	…	…	…	…	…	…	…
⅞	6.4	5.9	5.1	4.8	4.5	3.9	…	…	…	…	…	…	…	…	…
1	8.4	7.7	6.7	6.3	5.8	5.0	…	…	…	…	…	…	…	…	…
1⅛	10	9.5	8.4	7.9	7.1	6.3	…	…	…	…	…	…	…	…	…
Single Leg, 6 × 37 Wire Rope															
1¼	12	11	9.8	9.2	8.3	7.4	…	…	…	…	…	…	…	…	…
1⅜	15	13	12	11	10	8.9	…	…	…	…	…	…	…	…	…
1½	17	16	14	13	12	10	…	…	…	…	…	…	…	…	…
1¾	24	21	19	18	16	14	…	…	…	…	…	…	…	…	…
2	31	28	25	23	21	18	…	…	…	…	…	…	…	…	…
Two-Leg Bridle or Basket Hitch, 6 × 19 Wire Rope Sling															
¼	1.1	1.0	0.99	…	…	…	0.95	0.88	0.85	0.77	0.72	0.70	0.55	0.51	0.49
⅜	2.4	2.2	2.1	…	…	…	2.1	1.9	1.8	1.7	1.6	1.5	1.2	1.1	1.1
½	4.3	3.9	3.7	…	…	…	3.7	3.4	3.2	3.0	2.8	2.6	2.1	2.0	1.8
⅝	6.7	6.2	5.6	…	…	…	6.2	5.3	4.8	4.7	4.4	4.0	3.3	3.1	2.8
¾	9.5	8.8	7.8	…	…	…	8.2	7.6	6.8	6.7	6.2	5.5	4.8	4.4	3.9
⅞	13	12	10	…	…	…	11	10	8.9	9.1	8.4	7.3	6.4	5.9	5.1
1	17	15	13	…	…	…	14	13	11	12	11	9.4	8.4	7.7	6.7
1⅛	21	19	17	…	…	…	18	16	14	15	13	12	10	9.5	8.4
Two-Leg Bridle or Basket Hitch, 6 × 37 Wire Rope Sling															
1¼	25	22	20	…	…	…	21	19	17	17	16	14	12	11	9.8
1⅜	30	27	24	…	…	…	26	23	20	21	19	17	15	13	12
1½	35	32	28	…	…	…	30	27	24	25	22	20	17	16	14
1¾	46	43	39	…	…	…	41	37	33	34	30	27	24	21	19
2	62	55	49	…	…	…	53	43	43	43	39	35	31	26	25

A—socket or swaged terminal attachment; B—mechanical sleeve attachment; C—hand-tucked splice attachment.

Data taken from *Longshoring Industry*, OSHA Safety and Health Standards Digest, OSHA 2232, 1985.

CRANE CHAIN AND HOOKS

Material for Crane Chains.—The best material for crane and hoisting chains is a good grade of wrought iron, in which the percentage of phosphorus, sulfur, silicon, and other impurities is comparatively low. The tensile strength of the best grades of wrought iron does not exceed 46,000 pounds per square inch, whereas mild steel with about 0.15 per cent carbon has a tensile strength nearly double this amount. The ductility and toughness of wrought iron, however, is greater than that of ordinary commercial steel, and for this reason it is preferable for chains subjected to heavy intermittent strains, because wrought iron will always give warning by bending or stretching, before breaking. Another important reason for using wrought iron in preference to steel is that a perfect weld can be effected more easily. Heat-treated alloy steel is also widely used for chains. This steel contains carbon, 0.30 per cent, max; phosphorus, 0.045 per cent, max; and sulfur, 0.045 per cent, max. The selection and amounts of alloying elements are left to the individual manufacturers.

Strength of Chains.—When calculating the strength of chains it should be observed that the strength of a link subjected to tensile stresses is not equal to twice the strength of an iron bar of the same diameter as the link stock, but is a certain amount less, owing to the bending action caused by the manner in which the load is applied to the link. The strength is also reduced somewhat by the weld. The following empirical formula is commonly used for calculating the breaking load, in pounds, of wrought-iron crane chains:

$$W = 54{,}000D^2$$

in which W = breaking load in pounds and D = diameter of bar (in inches) from which links are made. The working load for chains should not exceed one-third the value of W, and, it is often one-fourth or one-fifth of the breaking load. When a chain is wound around a casting and severe bending stresses are introduced, a greater factor of safety should be used.

Care of Hoisting and Crane Chains.—Chains used for hoisting heavy loads are subject to deterioration, both apparent and invisible. The links wear, and repeated loading causes localized deformations to form cracks that spread until the links fail. Chain wear can be reduced by occasional lubrication. The life of a wrought-iron chain can be prolonged by frequent annealing or normalizing unless it has been so highly or frequently stressed that small cracks have formed. If this condition is present, annealing or normalizing will not "heal" the material, and the links will eventually fracture. To anneal a wrought-iron chain, heat it to cherry-red and allow it to cool slowly. Annealing should be done every six months, and oftener if the chain is subjected to unusually severe service.

Maximum Allowable Wear at Any Point of Link

Chain Size (in.)	Maximum Allowable Wear (in.)	Chain Size (in.)	Maximum Allowable Wear (in.)
1⁄4 (9⁄32)	3⁄64	1	3⁄16
3⁄8	5⁄64	1 1⁄8	7⁄32
1⁄2	7⁄64	1 1⁄4	1⁄4
5⁄8	9⁄64	1 3⁄8	3⁄32
3⁄4	5⁄32	1 1⁄2	5⁄16
7⁄8	11⁄64	1 3⁄4	11⁄32

Source:Longshoring Industry, OSHA 2232, 1985.

Chains should be examined periodically for twists, as a twisted chain will wear rapidly. Any links that have worn excessively should be replaced with new ones, so that every link will do its full share of work during the life of the chain, without exceeding the limit of safety. Chains for hoisting purposes should be made with short links, so that they will wrap closely around the sheaves or drums without bending. The diameter of the winding drums should be not less than 25 or 30 times the diameter of the iron used for the links. The accompanying table lists the maximum allowable wear for various sizes of chains.

Safe Loads for Ropes and Chains.— Safe loads recommended for wire rope or chain slings depend not only upon the strength of the sling but also upon the method of applying it to the load, as shown by the accompanying table giving safe loads as prepared by OSHA. The loads recommended in this table are more conservative than those usually specified, in order to provide ample allowance for some unobserved weakness in the sling, or the possibility of excessive strains due to misjudgment or accident.

The working load limit is defined as the maximum load in pounds that should ever be applied to chain, when the chain is new or in "as new" condition, and when the load is uniformly applied in direct tension to a straight length of chain. This limit is also affected by the number of chains used and their configuration. The accompanying table shows the working load limit for various configurations of heat-treated alloy steel chain using a 4 to 1 design factor, which conforms to ISO practice.

Protection from Sharp Corners: When the load to be lifted has sharp corners or edges, as are often encountered with castings, and with structural steel and other similar objects, pads or wooden protective pieces should be applied at the corners, to prevent the slings from being abraded or otherwise damaged where they come in contact with the load. These precautions are especially important when the slings consist of wire cable or fiber rope, although they should also be used even when slings are made of chain. Wooden cornerpieces are often provided for use in hoisting loads with sharp angles. If pads of burlap or other soft material are used, they should be thick and heavy enough to sustain the pressure, and distribute it over a considerable area, instead of allowing it to be concentrated directly at the edges of the part to be lifted.

Strength of Manila Rope

Dia. (in.)	Circumference (in.)	Weight of 100 feet of Rope[a] (lb)	New Rope Tensile Strength[b] (lb)	Working Load[c] (lb)	Dia. (in.)	Circumference (in.)	Weight of 100 feet of Rope[a] (lb)	New Rope Tensile Strength[b] (lb)	Working Load[c] (lb)
3⁄16	5⁄8	1.50	406	41	1 5⁄16	4	47.8	13,500	1930
1⁄4	3⁄4	2.00	540	54	1 1⁄2	4 1⁄2	60.0	16,700	2380
5⁄16	1	2.90	900	90	1 5⁄8	5	74.5	20,200	2880
3⁄8	1 1⁄8	4.10	1220	122	1 3⁄4	5 1⁄2	89.5	23,800	3400
7⁄16	1 1⁄4	5.25	1580	176	2	6	108	28,000	4000
1⁄2	1 1⁄2	7.50	2380	264	2 1⁄8	6 1⁄2	125	32,400	4620
9⁄16	1 3⁄4	10.4	3100	388	2 1⁄4	7	146	37,000	5300
5⁄8	2	13.3	3960	496	2 1⁄2	7 1⁄2	167	41,800	5950
3⁄4	2 1⁄4	16.7	4860	695	2 5⁄8	8	191	46,800	6700
13⁄16	2 1⁄2	19.5	5850	835	2 7⁄8	8 1⁄2	215	52,000	7450
7⁄8	2 3⁄4	22.4	6950	995	3	9	242	57,500	8200
1	3	27.0	8100	1160	3 1⁄4	10	298	69,500	9950
1 1⁄16	3 1⁄4	31.2	9450	1350	3 1⁄2	11	366	82,000	11,700
1 1⁄8	3 1⁄2	36.0	10,800	1540	4	12	434	94,500	13,500
1 1⁄4	3 3⁄4	41.6	12,200	1740	…	…	…	…	…

[a] Average value is shown; maximum is 5 per cent higher.

[b] Based on tests of new and unused rope of standard construction in accordance with Cordage Institute Standard Test Methods.

[c] These values are for rope in good condition with appropriate splices, in noncritical applications, and under normal service conditions. These values should be reduced where life, limb, or valuable propety are involved, or for exceptional service conditions such as shock loads or sustained loads.

Data from Cordage Institute Rope Specifications for three-strand laid and eight-strand plaited manila rope (standard construction).

Strength of Nylon and Double Braided Nylon Rope

Dia. (in.)	Circum-ference (in.)	Weight of 100 feet of Rope[a] (lb)	New Rope Tensile Strength[b] (lb)	Working Load[c] (lb)	Dia. (in.)	Circum-ference (in.)	Weight of 100 feet of Rope[a] (lb)	New Rope Tensile Strength[a] (lb)	Working Load[c] (lb)
Nylon Rope									
3⁄16	5⁄8	1.00	900	75	1 5⁄16	4	45.0	38,800	4,320
1⁄4	3⁄4	1.50	1,490	124	1 1⁄2	4 1⁄2	55.0	47,800	5,320
5⁄16	1	2.50	2,300	192	1 5⁄8	5	66.5	58,500	6,500
3⁄8	1 1⁄8	3.50	3,340	278	1 3⁄4	5 1⁄2	83.0	70,000	7,800
7⁄16	1 1⁄4	5.00	4,500	410	2	6	95.0	83,000	9,200
1⁄2	1 1⁄2	6.50	5,750	525	2 1⁄8	6 1⁄2	109	95,500	10,600
9⁄16	1 3⁄4	8.15	7,200	720	2 1⁄4	7	129	113,000	12,600
5⁄8	2	10.5	9,350	935	2 1⁄2	7 1⁄2	149	126,000	14,000
3⁄4	2 1⁄4	14.5	12,800	1,420	2 5⁄8	8	168	146,000	16,200
13⁄16	2 1⁄2	17.0	15,300	1,700	2 7⁄8	8 1⁄2	189	162,000	18,000
7⁄8	2 3⁄4	20.0	18,000	2,000	3	9	210	180,000	20,000
1	3	26.4	22,600	2,520	3 1⁄4	10	264	226,000	25,200
1 1⁄16	3 1⁄4	29.0	26,000	2,880	3 1⁄2	11	312	270,000	30,000
1 1⁄8	3 1⁄2	34.0	29,800	3,320	4	12	380	324,000	36,000
1 1⁄4	3 3⁄4	40.0	33,800	3,760	…	…	…	…	…
Double Braided Nylon Rope (Nylon Cover—Nylon Core)									
1⁄4	3⁄4	1.56	1,650	150	1 5⁄16	4	43.1	44,700	5,590
5⁄16	1	2.44	2,570	234	1 3⁄8	4 1⁄4	47.3	49,000	6,130
3⁄8	1 1⁄8	3.52	3,700	336	1 1⁄2	4 1⁄2	56.3	58,300	7,290
7⁄16	1 5⁄16	4.79	5,020	502	1 5⁄8	5	66.0	68,300	8,540
1⁄2	1 1⁄2	6.25	6,550	655	1 3⁄4	5 1⁄2	76.6	79,200	9,900
9⁄16	1 3⁄4	7.91	8,270	919	2	6	100	103,000	12,900
5⁄8	2	9.77	10,200	1,130	2 1⁄8	6 1⁄2	113	117,000	14,600
3⁄4	2 1⁄4	14.1	14,700	1,840	2 1⁄4	7	127	131,000	18,700
13⁄16	2 1⁄2	16.5	17,200	2,150	2 1⁄2	7 1⁄2	156	161,000	23,000
7⁄8	2 3⁄4	19.1	19,900	2,490	2 5⁄8	8	172	177,000	25,300
1	3	25.0	26,000	3,250	3	9	225	231,000	33,000
1 1⁄16	3 1⁄4	28.2	29,300	3,660	3 1⁄4	10	264	271,000	38,700
1 1⁄8	3 1⁄2	31.6	32,800	4,100	3 1⁄2	11	329	338,000	48,300
1 1⁄4	3 3⁄4	39.1	40,600	5,080	4	12	400	410,000	58,600

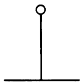
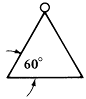
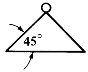
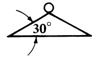
[a] Average value is shown. Maximum for nylon rope is 5 per cent higher; tolerance for double braided nylon rope is ± 5 per cent.

[b] Based on tests of new and unused rope of standard construction in accordance with Cordage Institute Standard Test Methods. For double braided nylon rope these values are minimums and are based on a large number of tests by various manufacturers; these values represent results two standard deviations below the mean. The minimum tensile strength is determined by the formula 1057 × (linear density)$^{0.995}$.

[c] These values are for rope in good condition with appropriate splices, in noncritical applications, and under normal service conditions. These values should be reduced where life, limb, or valuable property are involved, or for exceptional service conditions such as shock loads or sustained loads.

Data from Cordage Institute Specifications for nylon rope (three-strand laid and eight-strand plaited, standard construction) and double braided nylon rope.

Safe Working Loads in Pounds for Manila Rope and Chains

Diameter of Rope, or of Rod or Bar for Chain Links, Inch	Rope or Chain Vertical	Sling at 60°	Sling at 45°	Sling at 30°
				
Manila Rope				
1/4	120	204	170	120
5/16	200	346	282	200
3/8	270	467	380	270
7/16	350	605	493	350
15/32	450	775	635	450
1/2	530	915	798	530
9/16	690	1190	973	690
5/8	880	1520	1240	880
3/4	1080	1870	1520	1080
13/16	1300	2250	1830	1300
7/8	1540	2660	2170	1540
1	1800	3120	2540	1800
1 1/16	2000	3400	2800	2000
1 1/8	2400	4200	3400	2400
1 1/4	2700	4600	3800	2700
1 5/16	3000	5200	4200	3000
1 1/2	3600	6200	5000	3600
1 5/8	4500	7800	6400	4500
1 3/4	5200	9000	7400	5200
2	6200	10,800	8800	6200
2 1/8	7200	12,400	10,200	7200
Crane Chain (Wrought Iron)				
1/4[a]	1060	1835	1500	1060
5/16[a]	1655	2865	2340	1655
3/8	2385	4200	3370	2385
7/16[a]	3250	5600	4600	3250
1/2	4200	7400	6000	4200
9/16[a]	5400	9200	7600	5400
5/8	6600	11,400	9400	6600
3/4	9600	16,600	13,400	9600
7/8	13,000	22,400	18,400	13,000
1	17,000	29,400	24,000	17,000
1 1/8	20,000	34,600	28,400	20,000
1 1/4	24,800	42,600	35,000	24,800
1 3/8	30,000	51,800	42,200	30,000
1 1/2	35,600	61,600	50,400	35,600
1 5/8	41,800	72,400	59,000	41,800
1 3/4	48,400	84,000	68,600	48,400
1 7/8	55,200	95,800	78,200	55,200
2	63,200	109,600	89,600	63,200
Crane Chain (Alloy Steel)				
1/4	3240	5640	4540	3240
3/8	6600	11,400	9300	6600
1/2	11,240	19,500	15,800	11,240
5/8	16,500	28,500	23,300	16,500
3/4	23,000	39,800	32,400	23,000
7/8	28,600	49,800	40,600	28,600
1	38,600	67,000	54,600	38,600
1 1/8	44,400	77,000	63,000	44,400
1 1/4	57,400	99,400	81,000	57,400
1 3/8	67,000	116,000	94,000	67,000
1 1/2	79,400	137,000	112,000	79,400
1 5/8	85,000	147,000	119,000	85,000
1 3/4	95,800	163,000	124,000	95,800

[a] These sizes of wrought chain are no longer manufactured in the United States.

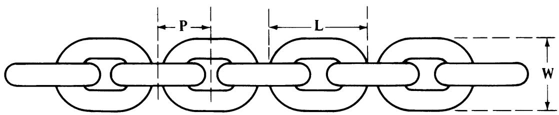
Data from *Longshoring Industry*, OSHA Safety and Health Standards Digest, OSHA 2232, 1985.

Working Load Limit for Heat-Treated Alloy Steel Chain, pounds

Chain Size (in.)	Single Leg	Double Leg			Triple and Quad Leg		
	90°	60°	45°	30°	60°	45°	30°
1/4	3,600	6,200	5,050	3,600	9,300	7,600	5,400
3/8	6,400	11,000	9,000	6,400	16,550	13,500	9,500
1/2	11,400	19,700	16,100	11,400	29,600	24,200	17,100
5/8	17,800	30,800	25,150	17,800	46,250	37,750	26,700
3/4	25,650	44,400	36,250	25,650	66,650	54,400	38,450
7/8	34,900	60,400	49,300	34,900	90,650	74,000	52,350

Source: The Crosby Group.

Loads Lifted by Crane Chains.—To find the approximate weight a chain will lift when rove as a tackle, multiply the safe load given in the table by the number of parts or chains at the movable block, and subtract one-quarter for frictional resistance. To find the size of chain required for lifting a given weight, divide the weight by the number of chains at the movable block, and add one-third for friction; next find in the column headed "Average Safe Working Load" the corresponding load, and then the corresponding size of chain in the column headed "Size." With the heavy chain or where the chain is unusually long, the weight of the chain itself should also be considered.



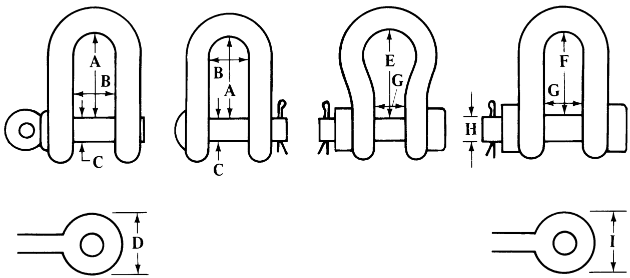
Size	Standard Pitch, *P* Inches	Average Weight per Foot, Pounds	Outside Length, *L* Inches	Outside Width, *W* Inches	Average Safe Working Load, Pounds	Proof Test, Pounds[a]	Approximate Breaking Load, Pounds
1/4	25/32	3/4	1 5/16	7/8	1,200	2,500	5,000
5/16	27/32	1	1 1/2	1 1/16	1,700	3,500	7,000
3/8	31/32	1 1/2	1 3/4	1 1/4	2,500	5,000	10,000
7/16	1 5/32	2	2 1/16	1 3/8	3,500	7,000	14,000
1/2	1 11/32	2 1/2	2 3/8	1 11/16	4,500	9,000	18,000
9/16	1 15/32	3 1/4	2 5/8	1 7/8	5,500	11,000	22,000
5/8	1 23/32	4	3	2 1/16	6,700	14,000	27,000
11/16	1 13/16	5	3 1/4	2 1/4	8,100	17,000	32,500
3/4	1 15/16	6 1/4	3 1/2	2 1/2	10,000	20,000	40,000
13/16	2 1/16	7	3 3/4	2 11/16	10,500	23,000	42,000
7/8	2 3/16	8	4	2 7/8	12,000	26,000	48,000
15/16	2 7/16	9	4 3/8	3 1/16	13,500	29,000	54,000
1	2 1/2	10	4 5/8	3 1/4	15,200	32,000	61,000
1 1/16	2 5/8	12	4 7/8	3 5/16	17,200	35,000	69,000
1 1/8	2 3/4	13	5 1/8	3 3/4	19,500	40,000	78,000
1 3/16	3 1/16	14 1/2	5 9/16	3 7/8	22,000	46,000	88,000
1 1/4	3 1/8	16	5 3/4	4 1/8	23,700	51,000	95,000
1 5/16	3 3/8	17 1/2	6 1/8	4 1/4	26,000	54,000	104,000
1 3/8	3 9/16	19	6 7/16	4 9/16	28,500	58,000	114,000
1 7/16	3 11/16	21 1/2	6 11/16	4 3/4	30,500	62,000	122,000
1 1/2	3 7/8	23	7	5	33,500	67,000	134,000
1 9/16	4	25	7 3/8	5 5/16	35,500	70,500	142,000
1 5/8	4 1/4	28	7 3/4	5 1/2	38,500	77,000	154,000
1 11/16	4 1/2	30	8 1/8	5 11/16	39,500	79,000	158,000
1 3/4	4 3/4	31	8 1/2	5 7/8	41,500	83,000	166,000
1 13/16	5	33	8 7/8	6 1/16	44,500	89,000	178,000

Size	Standard Pitch, *P* Inches	Average Weight per Foot, Pounds	Outside Length, *L* Inches	Outside Width, *W* Inches	Average Safe Working Load, Pounds	Proof Test, Pounds[a]	Approximate Breaking Load, Pounds
1 7/8	5 1/4	35	9 1/4	6 3/8	47,500	95,000	190,000
1 15/16	5 1/2	38	9 5/8	6 9/16	50,500	101,000	202,000
2	5 3/4	40	10	6 3/4	54,000	108,000	216,000
2 1/16	6	43	10 3/8	6 15/16	57,500	115,000	230,000
2 1/8	6 1/4	47	10 3/4	7 1/8	61,000	122,000	244,000
2 3/16	6 1/2	50	11 1/8	7 5/16	64,500	129,000	258,000
2 1/4	6 3/4	53	11 1/2	7 5/8	68,200	136,500	273,000
2 3/8	6 7/8	58 1/2	11 7/8	8	76,000	152,000	304,000
2 1/2	7	65	12 1/4	8 3/8	84,200	168,500	337,000
2 5/8	7 1/8	70	12 5/8	8 3/4	90,500	181,000	362,000
2 3/4	7 1/4	73	13	9 1/8	96,700	193,500	387,000
2 7/8	7 1/2	76	13 1/2	9 1/2	103,000	206,000	412,000
3	7 3/4	86	14	9 7/8	109,000	218,000	436,000

[a] Chains tested to U.S. Government and American Bureau of Shipping requirements.

Additional Tables

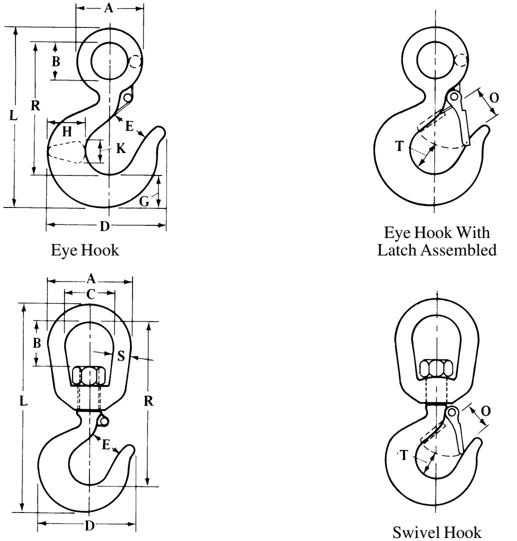
Dimensions of Forged Round Pin, Screw Pin, and Bolt Type Chain Shackles and Bolt Type Anchor Shackles



Working Load Limit (tons)	Nominal Shackle Size	A	B	C	D	E	F	G	H	I
1/2	1/4	7/8	15/16	5/16	11/16	...	...	...	...	...
3/4	5/16	1 1/32	17/32	3/8	13/16	...	...	...	...	...
1	3/8	1 1/4	21/32	7/16	31/32	...	...	...	...	...
1 1/2	7/16	1 7/16	23/32	1/2	1 1/16	...	...	...	...	...
2	1/2	1 5/8	13/16	5/8	1 3/16	1 7/8	1 5/8	13/16	5/8	1 3/16
3 1/4	5/8	2	1 1/16	3/4	1 9/16	2 3/8	2	1 1/16	3/4	1 9/16
4 3/4	3/4	2 3/8	1 1/4	7/8	1 7/8	2 13/16	2 3/8	1 1/4	7/8	1 7/8
6 1/2	7/8	2 13/16	1 7/16	1	2 1/8	3 5/16	2 13/16	1 7/16	1	2 1/8
8 1/2	1	3 3/16	1 11/16	1 1/8	2 3/8	3 3/4	3 3/16	1 11/16	1 1/8	2 3/8
9 1/2	1 1/8	3 9/16	1 13/16	1 1/4	2 5/8	4 1/4	3 9/16	1 13/16	1 1/4	2 5/8
12	1 1/4	3 15/16	2 1/32	1 3/8	3	4 11/16	3 15/16	2 1/32	1 3/8	3
13 1/2	1 3/8	4 3/8	2 1/4	1 1/2	3 5/16	5 3/16	4 3/8	2 1/4	1 1/2	3 5/16
17	1 1/2	4 13/16	2 3/8	1 5/8	3 5/8	5 3/4	4 13/16	2 3/8	1 5/8	3 5/8
25	1 3/4	5 3/4	2 7/8	2	4 1/8	7	5 3/4	2 7/8	2	4 1/8
35	2	6 3/4	3 1/4	2 1/4	5	7 3/4	6 3/4	3 1/4	2 1/4	5

All dimensions are in inches. Load limits are in tons of 2000 pounds.
Source: The Crosby Group.

Dimensions of Crane Hooks



Eye Hook

Eye Hook With Latch Assembled

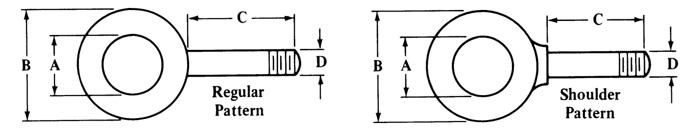
Swivel Hook

Swivel Hook With Latch Assembled

	Capacity of Hook in Tons (tons of 2000 lbs)											
	1.1	1.65	2.2	3.3	4.95	7.7	12.1	16.5	24.2	33	40.7	49.5
	Dimensions for Eye Hooks											
A	1.47	1.75	2.03	2.41	2.94	3.81	4.69	5.38	6.62	7.00	8.50	9.31
B	0.75	0.91	1.12	1.25	1.56	2.00	2.44	2.84	3.50	3.50	4.50	4.94
D	2.88	3.19	3.62	4.09	4.94	6.50	7.56	8.69	11.00	13.62	14.06	15.44
E	0.94	1.03	1.06	1.22	1.50	1.88	2.25	2.50	3.38	4.00	4.25	4.75
G	0.75	0.84	1.00	1.12	1.44	1.81	2.25	2.59	3.00	3.66	4.56	5.06
H	0.81	0.94	1.16	1.31	1.62	2.06	2.62	2.94	3.50	4.62	5.00	5.50
K	0.56	0.62	0.75	0.84	1.12	1.38	1.62	1.94	2.38	3.00	3.75	4.12
L	4.34	4.94	5.56	6.40	7.91	10.09	12.44	13.94	17.09	19.47	24.75	27.38
R	3.22	3.66	4.09	4.69	5.75	7.38	9.06	10.06	12.50	14.06	18.19	20.12
T	0.81	0.81	0.84	1.19	1.38	1.78	2.12	2.56	2.88	3.44	3.88	4.75
O	0.88	0.97	1.00	1.12	1.34	1.69	2.06	2.25	3.00	3.62	3.75	4.25
	Dimensions for Swivel Hooks											
A	2	2.50	3	3	3.50	4.50	5	5.63	7	7	…	…
B	0.94	1.31	1.63	1.56	1.75	2.31	2.38	2.69	4.19	4.19	…	…
C	1.25	1.50	1.75	1.75	2	2.50	2.75	3.13	4	4	…	…
D	2.88	3.19	3.63	4.09	4.94	6.5	7.56	8.69	11	13.63	…	…
E	0.94	1.03	1.06	1.22	1.5	1.88	2.25	2.5	3.38	4	…	…
L	5.56	6.63	7.63	8.13	9.59	12.41	14.50	15.88	21.06	23.22	…	…
R	4.47	5.28	6.02	6.38	7.41	9.59	11.13	12.03	16.56	18.06	…	…
S	0.38	0.50	0.63	0.63	0.75	1	1.13	1.25	1.5	1.5	…	…
T	0.81	0.81	0.84	1.19	1.38	1.78	2.13	2.56	2.88	3.44	…	…
O	0.88	0.97	1	1.13	1.34	1.69	2.06	2.25	3	3.63	…	…

Source: The Crosby Group. All dimensions are in inches. Hooks are made of alloy steel, quenched and tempered. For swivel hooks, the data are for a bail of carbon steel. The ultimate load is four times the working load limit (capacity). The swivel hook is a positioning device and is not intended to rotate under load; special load swiveling hooks must be used in such applications.

Hot Dip Galvanized, Forged Steel Eye-bolts

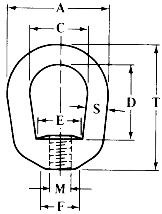


Shank		Eye Diam.		Safe Load[a] (tons)	Shank		Eye Diam.		Safe Load[a] (tons)
D	C	A	B		D	C	A	B	
REGULAR PATTERN									
1/4	2	1/2	1	0.25	3/4	4 1/2	1 1/2	3	2.6
1/4	4	1/2	1	0.25	3/4	6	1 1/2	3	2.6
5/16	2 1/4	5/8	1 1/4	0.4	3/4	8	1 1/2	3	2.6
5/16	4 1/4	5/8	1 1/4	0.4	3/4	10	1 1/2	3	2.6
3/8	2 1/2	3/4	1 1/2	0.6	3/4	10	1 1/2	3	2.6
3/8	4 1/2	3/4	1 1/2	0.6	3/4	10	1 1/2	3	2.6
3/8	6	3/4	1 1/2	0.6	7/8	5	1 3/4	3 1/2	3.6
1/2	3 1/4	1	2	1.1	7/8	8	1 3/4	3 1/2	3.6
1/2	6	1	2	1.1	7/8	10	1 3/4	3 1/2	3.6
1/2	8	1	2	1.1	1	6	2	4	5
1/2	10	1	2	1.1	1	9	2	4	5
1/2	12	1	2	1.1	1	10	2	4	5
5/8	4	1 1/4	2 1/2	1.75	1	10	2	4	5
5/8	6	1 1/4	2 1/2	1.75	1 1/4	8	2 1/2	5	7.6
5/8	8	1 1/4	2 1/2	1.75	1 1/4	10	2 1/2	5	7.6
5/8	10	1 1/4	2 1/2	1.75	1 1/4	10	2 1/2	5	7.6
5/8	12	1 1/4	2 1/2	1.75	…	…	…	…	…
SHOULDER PATTERN									
1/4	2	1/2	7/8	0.25	5/8	6	1 1/4	2 1/4	1.75
1/4	4	1/2	7/8	0.25	3/4	4 1/2	1 1/2	2 3/4	2.6
5/16	2 1/4	5/8	1 1/8	0.4	3/4	6	1 1/2	2 3/4	2.6
5/16	4 1/4	5/8	1 1/8	0.4	7/8	5	1 3/4	3 1/4	3.6
3/8	2 1/2	3/4	1 3/8	0.6	1	6	2	3 3/4	5
3/8	4 1/2	3/4	1 3/8	0.6	1	9	2	3 3/4	5
1/2	3 1/4	1	1 3/4	1.1	1 1/4	8	2 1/2	4 1/2	7.6
1/2	6	1	1 3/4	1.1	1 1/4	12	2 1/2	4 1/2	7.6
5/8	4	1 1/4	2 1/4	1.75	1 1/2	15	3	5 1/2	10.7

[a] The ultimate or breaking load is 5 times the safe working load.

All dimensions are in inches. Safe loads are in tons of 2000 pounds.
Source: The Crosby Group.

Eye Nuts and Lift Eyes

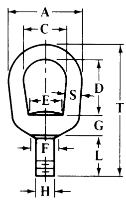


Eye Nuts

The general function of eye nuts is similar to that of eyebolts. Eye nuts are utilized for a variety of applications in either the swivel or tapped design.

M	A	C	D	E	F	S	T	Working Load Limit (lbs)*
1/4	1 1/4	3/4	1 1/16	21/32	1/2	1/4	1 11/16	520
5/16	1 1/4	3/4	1 1/16	21/32	1/2	1/4	1 11/16	850
3/8	1 5/8	1	1 1/4	3/4	9/16	5/16	2 1/16	1,250
7/16	2	1 1/4	1 1/2	1	13/16	3/8	2 1/2	1,700
1/2	2	1 1/4	1 1/2	1	13/16	3/8	2 1/2	2,250
5/8	2 1/2	1 1/2	2	1 3/16	1	1/2	3 3/16	3,600
3/4	3	1 3/4	2 3/8	1 3/8	1 1/8	5/8	3 7/8	5,200
7/8	3 1/2	2	2 5/8	1 5/8	1 5/16	3/4	4 5/16	7,200
1	4	2 1/4	3 1/16	1 7/8	1 9/16	7/8	5	10,000
1 1/8	4	2 1/4	3 1/16	1 7/8	1 9/16	7/8	5	12,300
1 1/4	4 1/2	2 1/2	3 1/2	1 15/16	1 7/8	1	5 3/4	15,500
1 3/8	5	2 3/4	3 3/4	2	2	1 1/8	6 1/4	18,500
1 1/2	5 5/8	3 1/8	4	2 3/8	2 1/4	1 1/4	6 3/4	22,500
2	7	4	6 1/4	4	3 3/8	1 1/2	10	40,000

Lifting Eyes



A	C	D	E	F	G	H	L	S	T	Working Load Limit Threaded (lbs)*
1 1/4	3/4	1 1/16	19/32	1/2	3/8	5/16	11/16	1/4	2 3/8	850
1 5/8	1	1 1/4	3/4	9/16	1/2	3/8	15/16	5/16	3	1,250
2	1 1/4	1 1/2	1	13/16	5/8	1/2	1 1/4	3/8	3 3/4	2,250
2 1/2	1 1/2	2	1 3/16	1	11/16	5/8	1 1/2	1/2	4 11/16	3,600
3	1 3/4	2 3/8	1 3/8	1 1/8	7/8	3/4	1 3/4	5/8	5 5/8	5,200
3 1/2	2	2 5/8	1 5/8	1 5/16	15/16	7/8	2	3/4	6 5/16	7,200
4	2 1/4	3 1/16	1 7/8	1 9/16	1 1/16	1	2 1/16	7/8	7 1/16	10,000
4 1/2	2 1/2	3 1/2	1 15/16	1 7/8	1 1/4	1 1/8	2 1/2	1	8 1/4	12,500
5 5/8	3 1/8	4	2 3/8	2 3/8	1 1/2	1 3/8	2 15/16	1 1/4	9 11/16	18,000

All dimensions are in inches. Data for eye nuts are for hot dip galvanized, quenched, and tempered forged steel. Data for lifting eyes are for quenched and tempered forged steel.

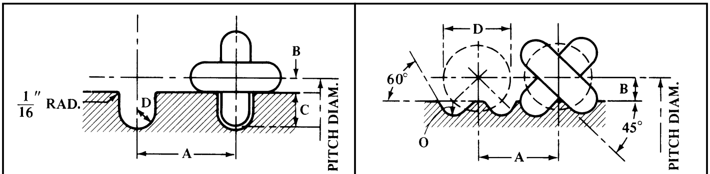
Source: The Crosby Group.

Minimum Sheave- and Drum-Groove Dimensions for Wire Rope Applications

Nominal Rope Diameter	Groove Radius		Nominal Rope Diameter	Groove Radius	
	New	Worn		New	Worn
1/4	0.135	0.129	2 3/8	1.271	1.199
5/16	0.167	0.160	2 1/2	1.338	1.279
3/8	0.201	0.190	2 5/8	1.404	1.339
7/16	0.234	0.220	2 3/4	1.481	1.409
1/2	0.271	0.256	2 7/8	1.544	1.473
9/16	0.303	0.288	3	1.607	1.538
5/8	0.334	0.320	3 1/8	1.664	1.598
3/4	0.401	0.380	3 1/4	1.731	1.658
7/8	0.468	0.440	3 3/8	1.807	1.730
1	0.543	0.513	3 1/2	1.869	1.794
1 1/8	0.605	0.577	3 3/4	1.997	1.918
1 1/4	0.669	0.639	4	2.139	2.050
1 3/8	0.736	0.699	4 1/4	2.264	2.178
1 1/2	0.803	0.759	4 1/2	2.396	2.298
1 5/8	0.876	0.833	4 3/4	2.534	2.434
1 3/4	0.939	0.897	5	2.663	2.557
1 7/8	1.003	0.959	5 1/4	2.804	2.691
2	1.085	1.025	5 1/2	2.929	2.817
2 1/8	1.137	1.079	5 3/4	3.074	2.947
2 1/4	1.210	1.153	6	3.198	3.075

All dimensions are in inches. Data taken from *Wire Rope Users Manual*, 2nd ed., American Iron and Steel Institute, Washington, D. C. The values given in this table are applicable to grooves in sheaves and drums but are not generally suitable for pitch design, since other factors may be involved.

Winding Drum Scores for Chain



Chain Size	*A*	*B*	*C*	*D*	Chain Size	*A*	*B*	*C*	*D*
3/8	1 1/2	3/16	9/16	3/16	3/8	1 1/4	11/32	3/16	1
7/16	1 11/16	7/32	5/8	9/32	7/16	1 7/16	3/8	7/32	1 1/8
1/2	1 7/8	1/4	11/16	5/16	1/2	1 9/16	7/16	1/4	1 1/4
9/16	2 1/16	9/32	3/4	11/32	9/16	1 3/4	15/32	9/32	1 3/8
5/8	2 5/16	5/16	13/16	3/8	5/8	1 7/8	17/32	5/16	1 1/2
11/16	2 1/2	11/32	7/8	13/32	11/16	2 1/16	9/16	11/32	1 5/8
3/4	2 11/16	3/8	15/16	7/16	3/4	2 3/16	5/8	3/8	1 3/4
13/16	2 7/8	13/32	1	15/32	13/16	2 3/8	21/32	13/32	1 7/8
7/8	3 1/8	7/16	1 1/16	1/2	7/8	2 1/2	23/32	7/16	2
15/16	3 5/16	15/32	1 1/8	17/32	15/16	2 11/16	3/4	15/32	2 1/8
1	3 1/2	1/2	1 3/16	9/16	1	2 13/16	13/16	1/2	2 1/4

All dimensions are in inches.