Hi and welcome to Module 6 of An Introduction to Engineering Mechanics. Today's learning objective is to calculate the forces acting on systems of particles or multibody. Problems.

This is a typical multibody problem. this is an example from a, a textbook by Doctors Dave McGill and Wilton King. Although I said in this Coursera course you didn't, there was no required textbook I am going to use several figures and examples From, a textbook called engineering mechanics, statics by, doctors McGill and King, and, I will give, I, I'll put a little note on, below those figures, to sh, to show when I'm using those, but, I know doctors, McGill and King, and they were kind enough. To allow me permission to use material from their textbooks in this course. both of those gentlemen are professors emeritus at Georgia Tech and so it's great that they're supporting this Corsera course. Okay so here's the situation. We have this, this, this person who is exerting a force in this rope. And holding up these systems of pulleys where there's a weight, which would be known, weight 1 here on the right and a weight 2 here which is greater than weight 1. So if we go over here to a model. Got a model of this situation so I'd be the, the giant person if you will, over here on the, on the left hand side. That's holding this cable, as it's attached, and holding this system up with its 2 weights.

So, let's go back over here, and look at the assumptions that we would make in solving this problem. Typically, for these problems, we, we, we make several simplifying assumptions, these, pulley problems, to help us to. Getting a, a, a, a simple but an, a fairly accurate solution. If we find that these assumptions are, are not good, and we need more accurate an, a, more accurate answers, we may need to go back, and, and relook and include some of this some of these forces and things that we're neglecting. So, one of them is that we assume that there's frictionless bearing, bearings in these problems. We assume that there's a, a constant tension throughout the cable. Or the belt or the rope, and we neglect the pulley weights. And so let's look at these assumptions again over here on, on, on my model. And so again, if I'm the, the, the man here the giant person over here on the left. I'm holding this cable and we're saying that the, or this string if you will and we're saying that the tension in this string is, is the same throughout, over to here, and the tension in this String or cable is the same throughout. We're assuming that these bearings are frictionless, okay. We're also neglecting the pulley weights. We're saying that the pulley weights are, are much, insignificant when compared to the weights themselves. And so those, those are some, some good assumptions to simplify the problem and still give us a fairly accurate answer.

So in solving this problem, we take our system which has to be completely in static equilibrium and we apply the equations of equilibrium, and we talked about these last module. we must have a balance of forces in the x direction and a balance of forces in the y direction and so let me draw my x and y directions here. Here. Lets call the left x, and to the y up. And so this whole system has to be an equilibrium, and each piece of the system must be each, an, an equilibrium, to satisfy static equilibrium. And so, what I'm going to do, is I'm going to separate this thing out, and I'm going to look at just 1 little piece here. I'm going to look at this Pulley P1, and I'm going to draw a free body diagram of that. So this is a free body diagram of pulley 1, or pulley P1 if you will. And so I separate that pulley from the rest of the world here, so I'm drawing it Unconstrained. Here is the center. And now I apply the external forces that are acting on that pulley. And so I see that on the r-, on the right hand side here I have this, this chain if you will and so if I cut that chain there's a tension in that chain which is equal to The weight one, and that's going to be pulling down on the right-hand side of this pulley. So, I've got, w1 here. One of our simplifying assumptions was that the tension in that cable is the same throughout, so if I cut it over on this left-hand side and apply a force reaction there, I get w1 down. I have this a connection of the set of the pulley to the ceiling and so I have to cut that and there'd be tension in that, I called that tension 3 and then I have a tension in this other cable which is coming down from. the center of the pulley and if I cut that, that tension, I'll call t2. And I want you to note on t2 that if I follow the tension in t2 all the way around here, that it's actually the tension or the force that the person must exert to hold the system in equilibrium, so let's make a note of that. We'll note that T2 is the force the person exerts. So that's going to be our answer. , So now we apply our equations of equilibrium, as I mentioned. here they are on the right.

We've done 'em in the last module. We don't have any forces in the x direction in this problem, so we don't have to worry about this first equation of equilibrium, so we'll just sum forces in the y direction. And, we'll make sure they're balanced, so we'll set them equal to 0. I have to choose a sign convention for assembling my equations. This is arbitrary, and I'll show you that it doesn't matter when we do this. So, what I'll do to start here, is I'll just choose up, as being positive. And so when I assemble my equation, I have T3 was, is up, so that's going to be positive. And then I have weight one is down, so that's going to be negative and in accordance with my sign convention. T2 is down so that's negative, and the weight one on the other side is down, so it's negative. And all that has to equal zero. And so I end up with T3 = 2 W1. Plus T2. And so that's my equation that results from that force balance. Just to show you that it wouldn't have mattered If I had chosen my sign convention as being down arbitrarily, let's just redo that. So I have some of the forces in the y = 0, I'll choose down is being positive. And I would get, well in this case T3 is up, so it would be negative. -T3. W sub 1 is down, so that's positive in accordance with this sign convention. That's plus W1. T2 is down, so that's plus T2. W1 is down so that's plus W1 = 0. And you'll note, that if I carry T3 to the other side, add the T1s, W1s together that these two equations are exactly the same. And so you get the same result regardless of the sign conven-, convention that you've arbitrarily chose to assemble the equation.

Okay, so, the equation that results though, we have a problem here because we have, how many, equations? Well we have one, equation. And let's count the number of unknowns. Well, we would know

W1. We'd know that weight. Given in the problem, but we don't, don't know, tension T2, which is what we want to find. That's the force the person exerts, and we don't know tension T3. So we have 1 equations, 2 unknowns, and so we have unhappy face. And so what do we do next? And you may want to think about this for a second and then come back. Try to think about what we would do next. Okay, what you thought about that what you need to do in this case is. Okay, every piece of this system has to be in eq. Let's take another piece and draw another free body diagram, so we're going to try. Another free body diagram, and the one I'll choose next is, is pulley p2. So, at the top here I've written my analysis from pulley p1. I, I arrived at the equation t3 = t. 2W1 + T2, so 2 equations, or 1 equation, 2 unknowns. Now lets do a free body diagram of Pulley P2. ,, , And in fact I'd like you to take a few minutes on your own and draw that free body diagram and then come back we'll do it together. Okay, now that you have completed that here is my body pulley P2, here is the centre, I apply my external forces I have, this was t2, and on Pulley 1 in tension, so it was pulling down on Pulley 1, it's going to be pulling up on Pulley 2. This tension in that cable is the same throughout, so it's also, if I cut it on this left-hand side it's going to pull up on the left-hand side t2. In the middle here, if I cut this chain remember the tension in the, in this chain was the same throughout and it was equal to W1. So that's up W1. and then finally I have tension is this rope which is hooked to weight 2 and so that's going to be down. And that's weight too, so that's a good body diagram, and that's what you should have arrived at. Once we've got that, we can assemble our, our equation again. We'll sum, there's no forces in the x, so we'll only sum forces in the y direction. Set it = to 0. I'll choose up as being positive. And so, I've got T2, positive, because it's up, plus W sub 1, positive because it's up, plus T2 - W2 = 0. And if I solve for T2 now, I get T2 = W2 - W1 / 2. And so that's my answer. So if I'm given the values for the weight 1 and weight 2, I, I subtract the 2, and divide by 2. That's going to give me the force that the person must exert. And you'll notice that I get a bonus in this, that I can actually, go up here, and put the result for T2, into this equation, and find out what T3 is.