EXAMPLES

STATICS

EXAMPLE 1.1

Convert 2 km/h to m/s How many ft/s is this?

SOLUTION

Since 1 km = 1000 m and 1 h = 3600 s, the factors of conversion are arranged in the following order, so that a cancellation of the units can be applied:

$$2\text{ km/h} = \frac{2\text{ km}}{\text{h}}\left(\frac{1000\text{ m}}{\text{km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)$$

$$= \frac{2000\text{ m}}{3600\text{ s}} = 0.556\text{ m/s} \qquad \textit{Ans.}$$

From Table 1–2, 1 ft = 0.3048 m. Thus,

$$0.556\text{ m/s} = \left(\frac{0.556\text{ m}}{\text{s}}\right)\left(\frac{1\text{ ft}}{0.3048\text{ m}}\right)$$

$$= 1.82\text{ ft/s} \qquad \textit{Ans.}$$

NOTE: Remember to round off the final answer to three significant figures.

EXAMPLE 1.2

Convert the quantities 300 lb · s and 52 slug/ft^3 to appropriate SI units.

SOLUTION

Using Table 1–2, 1 lb = 4.448 2 N.

$$300\text{ lb}\cdot\text{s} = 300\text{ lb}\cdot\text{s}\left(\frac{4.448\text{ N}}{1\text{ lb}}\right)$$

$$= 1334.5\text{ N}\cdot\text{s} = 1.33\text{ kN}\cdot\text{s} \qquad \textit{Ans.}$$

Since 1 slug = 14.593 8 kg and 1 ft = 0.304 8 m, then

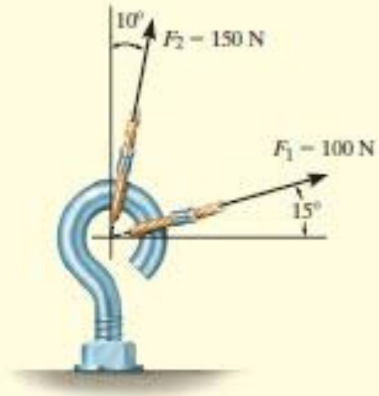
$$52\text{ slug/ft}^3 = \frac{52\text{ slug}}{\text{ft}^3}\left(\frac{14.59\text{ kg}}{1\text{ slug}}\right)\left(\frac{1\text{ ft}}{0.304\,8\text{ m}}\right)^3$$

$$= 26.8(10^3)\text{ kg/m}^3$$

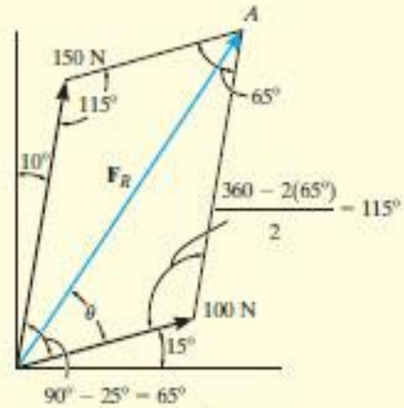
$$= 26.8\text{ Mg/m}^3 \qquad \textit{Ans.}$$

EXAMPLE 2.1

The screw eye in Fig. 2–11*a* is subjected to two forces, $\mathbf{F}_1$ and $\mathbf{F}_2$. Determine the magnitude and direction of the resultant force.



(a)



(b)

SOLUTION

Parallelogram Law. The parallelogram is formed by drawing a line from the head of $\mathbf{F}_1$ that is parallel to $\mathbf{F}_2$, and another line from the head of $\mathbf{F}_2$ that is parallel to $\mathbf{F}_1$. The resultant force $\mathbf{F}_R$ extends to where these lines intersect at point *A*, Fig. 2–11*b*. The two unknowns are the magnitude of $\mathbf{F}_R$ and the angle θ (theta).

Trigonometry. From the parallelogram, the vector triangle is constructed, Fig. 2–11*c*. Using the law of cosines

$$F_R = \sqrt{(100\text{ N})^2 + (150\text{ N})^2 - 2(100\text{ N})(150\text{ N})\cos 115°}$$
$$= \sqrt{10\,000 + 22\,500 - 30\,000(-0.4226)} = 212.6\text{ N}$$
$$= 213\text{ N} \qquad \textit{Ans.}$$

Applying the law of sines to determine θ,

$$\frac{150\text{ N}}{\sin\theta} = \frac{212.6\text{ N}}{\sin 115°} \qquad \sin\theta = \frac{150\text{ N}}{212.6\text{ N}}(\sin 115°)$$
$$\theta = 39.8°$$

Thus, the direction ϕ (phi) of $\mathbf{F}_R$, measured from the horizontal, is

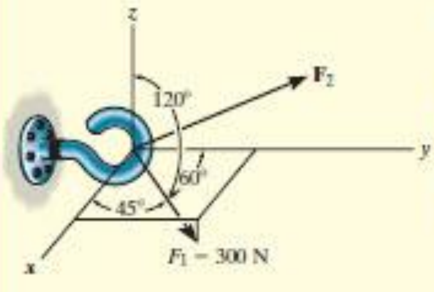
$$\phi = 39.8° + 15.0° = 54.8° \qquad \textit{Ans.}$$

NOTE: The results seem reasonable, since Fig. 2–11*b* shows F_R to have a magnitude larger than its components and a direction that is between them.

(c)

Fig. 2–11

EXAMPLE 2.11



(a)

Two forces act on the hook shown in Fig. 2–32*a*. Specify the magnitude of $\mathbf{F}_2$ and its coordinate direction angles of $\mathbf{F}_2$ that the resultant force $\mathbf{F}_R$ acts along the positive *y* axis and has a magnitude of 800 N.

SOLUTION

To solve this problem, the resultant force $\mathbf{F}_R$ and its two components, $\mathbf{F}_1$ and $\mathbf{F}_2$, will each be expressed in Cartesian vector form. Then, as shown in Fig. 2–33*a*, it is necessary that $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$.

Applying Eq. 2–9,

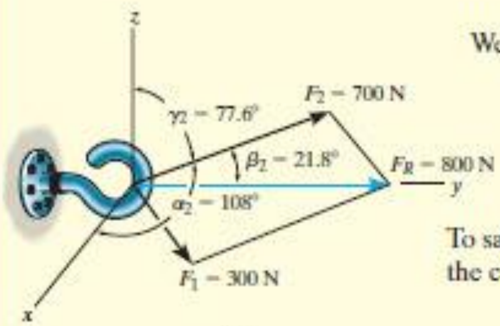
$$\begin{aligned}\mathbf{F}_1 &= F_1 \cos \alpha_1 \mathbf{i} + F_1 \cos \beta_1 \mathbf{j} + F_1 \cos \gamma_1 \mathbf{k}\\ &= 300 \cos 45^\circ\, \mathbf{i} + 300 \cos 60^\circ\, \mathbf{j} + 300 \cos 120^\circ\, \mathbf{k}\\ &= \{212.1\mathbf{i} + 150\mathbf{j} - 150\mathbf{k}\}\ \text{N}\\ \mathbf{F}_2 &= F_{2x}\mathbf{i} + F_{2y}\mathbf{j} + F_{2z}\mathbf{k}\end{aligned}$$

Since $\mathbf{F}_R$ has a magnitude of 800 N and acts in the $+\mathbf{j}$ direction,

$$\mathbf{F}_R = (800\ \text{N})(+\mathbf{j}) = \{800\mathbf{j}\}\ \text{N}$$

We require

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2\\ 800\mathbf{j} &= 212.1\mathbf{i} + 150\mathbf{j} - 150\mathbf{k} + F_{2x}\mathbf{i} + F_{2y}\mathbf{j} + F_{2z}\mathbf{k}\\ 800\mathbf{j} &= (212.1 + F_{2x})\mathbf{i} + (150 + F_{2y})\mathbf{j} + (-150 + F_{2z})\mathbf{k}\end{aligned}$$



(b)

Fig. 2–33

To satisfy this equation the **i**, **j**, **k** components of $\mathbf{F}_R$ must be equal to the corresponding **i**, **j**, **k** components of $(\mathbf{F}_1 + \mathbf{F}_2)$. Hence,

$$\begin{aligned}0 &= 212.1 + F_{2x} & F_{2x} &= -212.1\ \text{N}\\ 800 &= 150 + F_{2y} & F_{2y} &= 650\ \text{N}\\ 0 &= -150 + F_{2z} & F_{2z} &= 150\ \text{N}\end{aligned}$$

The magnitude of $\mathbf{F}_2$ is thus

$$\begin{aligned}F_2 &= \sqrt{(-212.1\ \text{N})^2 + (650\ \text{N})^2 + (150\ \text{N})^2}\\ &= 700\ \text{N}\end{aligned}$$ *Ans.*

We can use Eq. 2–9 to determine $\alpha_2, \beta_2, \gamma_2$.

$$\cos \alpha_2 = \frac{-212.1}{700}; \qquad \alpha_2 = 108^\circ$$ *Ans.*

$$\cos \beta_2 = \frac{650}{700}; \qquad \beta_2 = 21.8^\circ$$ *Ans.*

$$\cos \gamma_2 = \frac{150}{700}; \qquad \gamma_2 = 77.6^\circ$$ *Ans.*

These results are shown in Fig. 2–32*b*.

EXAMPLE 2.12

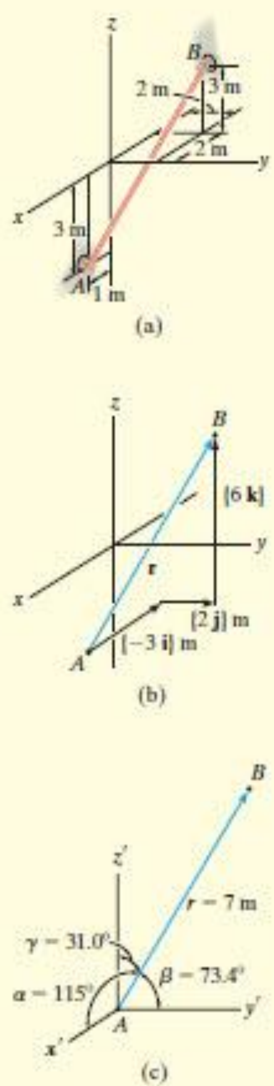


Fig. 2–37

An elastic rubber band is attached to points A and B as shown in Fig. 2–37*a*. Determine its length and its direction measured from A toward B.

SOLUTION

We first establish a position vector from A to B, Fig. 2–37*b*. In accordance with Eq. 2–11, the coordinates of the tail $A(1\text{ m}, 0, -3\text{ m})$ are subtracted from the coordinates of the head $B(-2\text{ m}, 2\text{ m}, 3\text{ m})$, which yields

$$\mathbf{r} = [-2\text{ m} - 1\text{ m}]\mathbf{i} + [2\text{ m} - 0]\mathbf{j} + [3\text{ m} - (-3\text{ m})]\mathbf{k}$$
$$= \{-3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}\}\text{ m}$$

These components of $\mathbf{r}$ can also be determined *directly* by realizing that they represent the direction and distance one must travel along each axis in order to move from A to B, i.e., along the x axis $\{-3\mathbf{i}\}$ m, along the y axis $\{2\mathbf{j}\}$ m, and finally along the z axis $\{6\mathbf{k}\}$ m.

The length of the rubber band is therefore

$$r = \sqrt{(-3\text{ m})^2 + (2\text{ m})^2 + (6\text{ m})^2} = 7\text{ m} \qquad \textit{Ans.}$$

Formulating a unit vector in the direction of $\mathbf{r}$, we have

$$\mathbf{u} = \frac{\mathbf{r}}{r} = -\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{6}{7}\mathbf{k}$$

The components of this unit vector give the coordinate direction angles

$$\alpha = \cos^{-1}\left(-\frac{3}{7}\right) = 115^\circ \qquad \textit{Ans.}$$

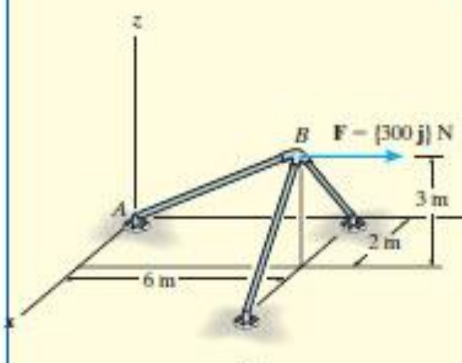
$$\beta = \cos^{-1}\left(\frac{2}{7}\right) = 73.4^\circ \qquad \textit{Ans.}$$

$$\gamma = \cos^{-1}\left(\frac{6}{7}\right) = 31.0^\circ \qquad \textit{Ans.}$$

NOTE: These angles are measured from the *positive axes* of a localized coordinate system placed at the tail of $\mathbf{r}$, as shown in Fig. 2–37*c*.

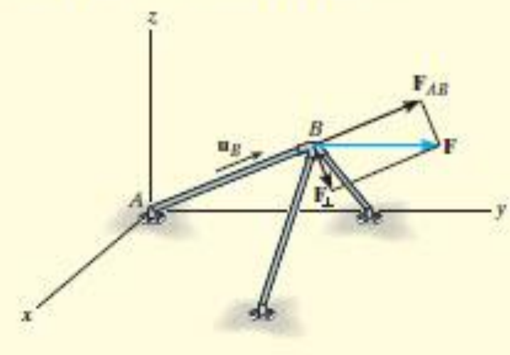
EXAMPLE 2.17

The frame shown in Fig. 2–45*a* is subjected to a horizontal force $\mathbf{F} = \{300\mathbf{j}\}$. Determine the magnitude of the components of this force parallel and perpendicular to member *AB*.



(a)

Fig 2–45



(b)

SOLUTION

The magnitude of the component of $\mathbf{F}$ along *AB* is equal to the dot product of $\mathbf{F}$ and the unit vector $\mathbf{u}_B$, which defines the direction of *AB*, Fig. 2–44*b*. Since

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{2\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}}{\sqrt{(2)^2 + (6)^2 + (3)^2}} = 0.286\mathbf{i} + 0.857\mathbf{j} + 0.429\mathbf{k}$$

then

$$F_{AB} = F\cos\theta = \mathbf{F}\cdot\mathbf{u}_B = (300\mathbf{j})\cdot(0.286\mathbf{i} + 0.857\mathbf{j} + 0.429\mathbf{k})$$
$$= (0)(0.286) + (300)(0.857) + (0)(0.429)$$
$$= 257.1 \text{ N} \qquad \textit{Ans.}$$

Since the result is a positive scalar, $\mathbf{F}_{AB}$ has the same sense of direction as $\mathbf{u}_B$, Fig. 2–45*b*.

Expressing $\mathbf{F}_{AB}$ in Cartesian vector form, we have

$$\mathbf{F}_{AB} = F_{AB}\mathbf{u}_B = (257.1\text{ N})(0.286\mathbf{i} + 0.857\mathbf{j} + 0.429\mathbf{k})$$
$$= \{73.5\mathbf{i} + 220\mathbf{j} + 110\mathbf{k}\}\text{N} \qquad \textit{Ans.}$$

The perpendicular component, Fig. 2–45*b*, is therefore

$$\mathbf{F}_\perp = \mathbf{F} - \mathbf{F}_{AB} = 300\mathbf{j} - (73.5\mathbf{i} + 220\mathbf{j} + 110\mathbf{k})$$
$$= \{-73.5\mathbf{i} + 80\mathbf{j} - 110\mathbf{k}\}\text{ N}$$

Its magnitude can be determined either from this vector or by using the Pythagorean theorem, Fig. 2–45*b*:

$$F_\perp = \sqrt{F^2 - F_{AB}^2} = \sqrt{(300\text{ N})^2 - (257.1\text{ N})^2}$$
$$= 155\text{ N} \qquad \textit{Ans.}$$

EXAMPLE 3.2

Determine the tension in cables *BA* and *BC* necessary to support the 60-kg cylinder in Fig. 3-6*a*.

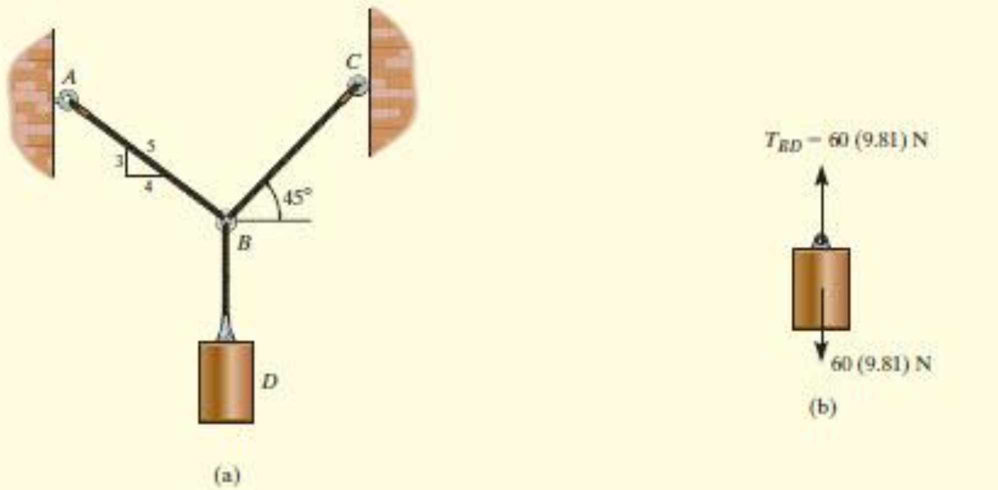


Fig. 3–6

SOLUTION

Free-Body Diagram. Due to equilibrium, the weight of the cylinder causes the tension in cable *BD* to be $T_{BD} = 60(9.81)$ N, Fig. 3-6*b*. The forces in cables *BA* and *BC* can be determined by investigating the equilibrium of ring *B*. Its free-body diagram is shown in Fig. 3-6*c*. The magnitudes of $\mathbf{T}_A$ and $\mathbf{T}_C$ are unknown, but their directions are known.

Equations of Equilibrium. Applying the equations of equilibrium along the *x* and *y* axes, we have

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad T_C \cos 45^\circ - \left(\tfrac{4}{5}\right)T_A = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \quad T_C \sin 45^\circ + \left(\tfrac{3}{5}\right)T_A - 60(9.81)\text{ N} = 0 \qquad (2)$$

Equation (1) can be written as $T_A = 0.8839T_C$. Substituting this into Eq. (2) yields

$$T_C \sin 45^\circ + \left(\tfrac{3}{5}\right)(0.8839T_C) - 60(9.81)\text{ N} = 0$$

So that

$$T_C = 475.66\text{ N} = 476\text{ N} \qquad \textit{Ans.}$$

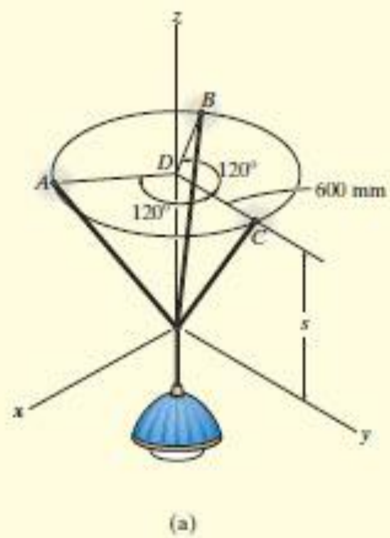
Substituting this result into either Eq. (1) or Eq. (2), we get

$$T_A = 420\text{ N} \qquad \textit{Ans.}$$

NOTE: The accuracy of these results, of course, depends on the accuracy of the data, i.e., measurements of geometry and loads. For most engineering work involving a problem such as this, the data as measured to three significant figures would be sufficient.

EXAMPLE 3.6

The 10-kg lamp in Fig. 3-11*a* is suspended from the three equal-length cords. Determine its smallest vertical distance *s* from the ceiling if the force developed in any cord is not allowed to exceed 50 N.



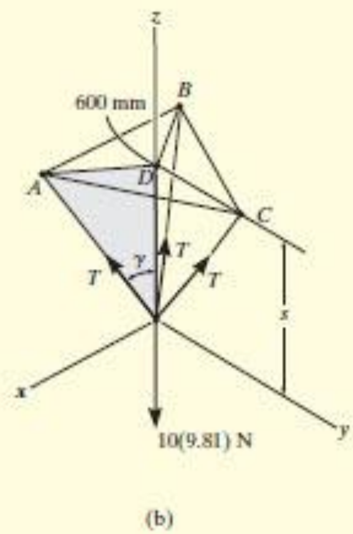


Fig. 3–11

SOLUTION

Free-Body Diagram. Due to symmetry, Fig. 3-11*b*, the distance $DA = DB = DC = 600$ mm. It follows that from $\Sigma F_x = 0$ and $\Sigma F_y = 0$, the tension T in each cord will be the same. Also, the angle between each cord and the z axis is γ.

Equation of Equilibrium. Applying the equilibrium equation along the z axis, with $T = 50$ N, we have

$$\Sigma F_z = 0; \qquad 3[(50\text{ N})\cos\gamma] - 10(9.81)\text{ N} = 0$$

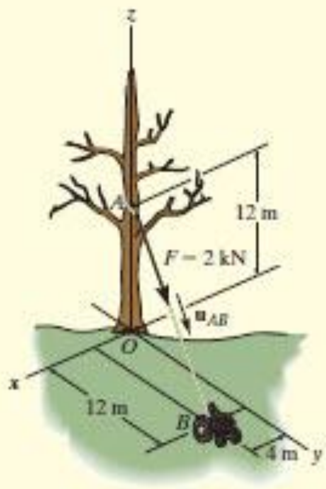
$$\gamma = \cos^{-1}\frac{98.1}{150} = 49.16^\circ$$

From the shaded triangle shown in Fig. 3-11*b*,

$$\tan 49.16^\circ = \frac{600\text{ mm}}{s}$$

$$s = 519\text{ mm} \qquad \textit{Ans.}$$

EXAMPLE 4.3



(a)

Determine the moment produced by the force **F** in Fig. 4–14*a* about point *O*. Express the result as a Cartesian vector.

SOLUTION

As shown in Fig. 4–14*a*, either $\mathbf{r}_A$ or $\mathbf{r}_B$ can be used to determine the moment about point *O*. These position vectors are

$$\mathbf{r}_A = \{12\mathbf{k}\}\ \text{m} \quad \text{and} \quad \mathbf{r}_B = \{4\mathbf{i} + 12\mathbf{j}\}\ \text{m}$$

Force **F** expressed as a Cartesian vector is

$$\mathbf{F} = F\mathbf{u}_{AB} = 2\ \text{kN}\left[\frac{\{4\mathbf{i} + 12\mathbf{j} - 12\mathbf{k}\}\ \text{m}}{\sqrt{(4\ \text{m})^2 + (12\ \text{m})^2 + (-12\ \text{m})^2}}\right]$$

$$= \{0.4588\mathbf{i} + 1.376\mathbf{j} - 1.376\mathbf{k}\}\ \text{kN}$$

Thus

$$\mathbf{M}_O = \mathbf{r}_A \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 12 \\ 0.4588 & 1.376 & -1.376 \end{vmatrix}$$

$$= [0(-1.376) - 12(1.376)]\mathbf{i} - [0(-1.376) - 12(0.4588)]\,\mathbf{j} + [0(1.376) - 0(0.4588)]\mathbf{k}$$

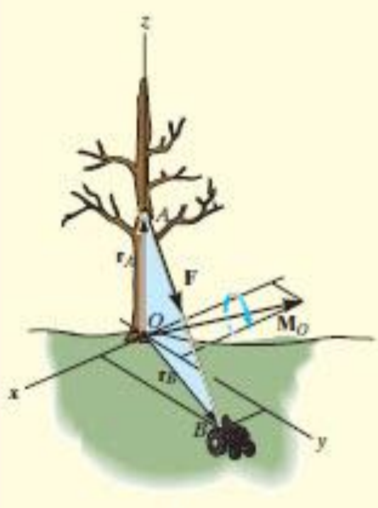
$$= \{-16.5\mathbf{i} + 5.51\mathbf{j}\}\ \text{kN}\cdot\text{m} \qquad \textit{Ans.}$$

or

$$\mathbf{M}_O = \mathbf{r}_B \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 12 & 0 \\ 0.4588 & 1.376 & -1.376 \end{vmatrix}$$

$$= [12(-1.376) - 0(1.376)]\mathbf{i} - [4(-1.376) - 0(0.4588)]\mathbf{j} + [4(1.376) - 12(0.4588)]\mathbf{k}$$

$$= \{-16.5\mathbf{i} + 5.51\mathbf{j}\}\ \text{kN}\cdot\text{m} \qquad \textit{Ans.}$$



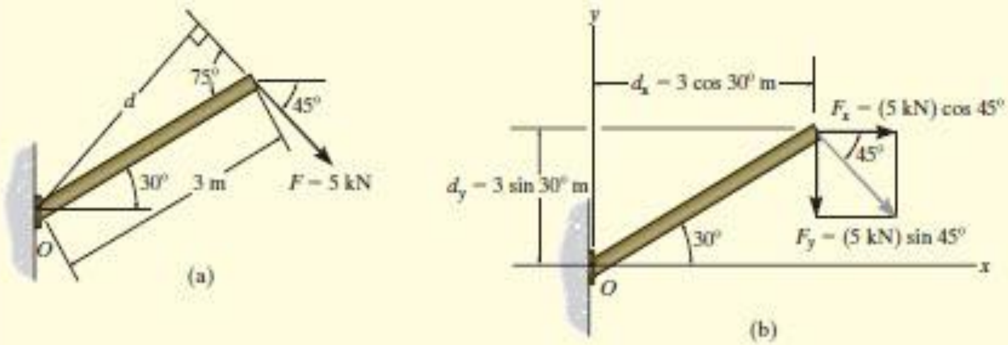
(b)

Fig. 4–14

NOTE: As shown in Fig. 4–14*b*, $\mathbf{M}_O$ acts perpendicular to the plane that contains $\mathbf{F}$, $\mathbf{r}_A$, and $\mathbf{r}_B$. Had this problem been worked using $M_O = Fd$, notice the difficulty that would arise in obtaining the moment arm *d*.

EXAMPLE 4.5

Determine the moment of the force in Fig. 4–18*a* about point O.



SOLUTION I

The moment arm d in Fig. 4–18*a* can be found from trigonometry.

$$d = (3\text{ m})\sin 75^\circ = 2.898\text{ m}$$

Thus,

$$M_O = Fd = (5\text{kN})(2.898\text{ m}) = 14.5\text{ kN}\cdot\text{m} \circlearrowright \qquad \textit{Ans.}$$

Since the force tends to rotate or orbit clockwise about point O, the moment is directed into the page.

SOLUTION II

The x and y components of the force are indicated in Fig. 4–18*b*. Considering counterclockwise moments as positive, and applying the principle of moments, we have

$$\begin{aligned} \circlearrowleft + M_O &= -F_x d_y - F_y d_x \\ &= -(5\cos 45^\circ\text{ kN})(3\sin 30^\circ\text{ m}) - (5\sin 45^\circ\text{ kN})(3\cos 30^\circ\text{ m}) \\ &= -14.5\text{ kN}\cdot\text{m} = 14.5\text{ kN}\cdot\text{m} \circlearrowright \qquad \textit{Ans.} \end{aligned}$$

SOLUTION III

The x and y axes can be set parallel and perpendicular to the rod's axis as shown in Fig. 4-18*c*. Here $\mathbf{F}_x$ produces no moment about point O since its line of action passes through this point. Therefore,

$$\begin{aligned} \circlearrowleft + M_O &= -F_y d_x \\ &= -(5\sin 75^\circ\text{ kN})(3\text{ m}) \\ &= -14.5\text{ kN}\cdot\text{m} = 14.5\text{ kN}\cdot\text{m} \circlearrowright \qquad \textit{Ans.} \end{aligned}$$

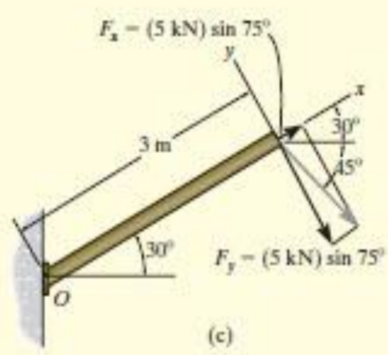


Fig. 4–18

EXAMPLE 4.14

Replace the force and couple system shown in Fig. 4–37*a* by an equivalent resultant force and couple moment acting at point *O*.

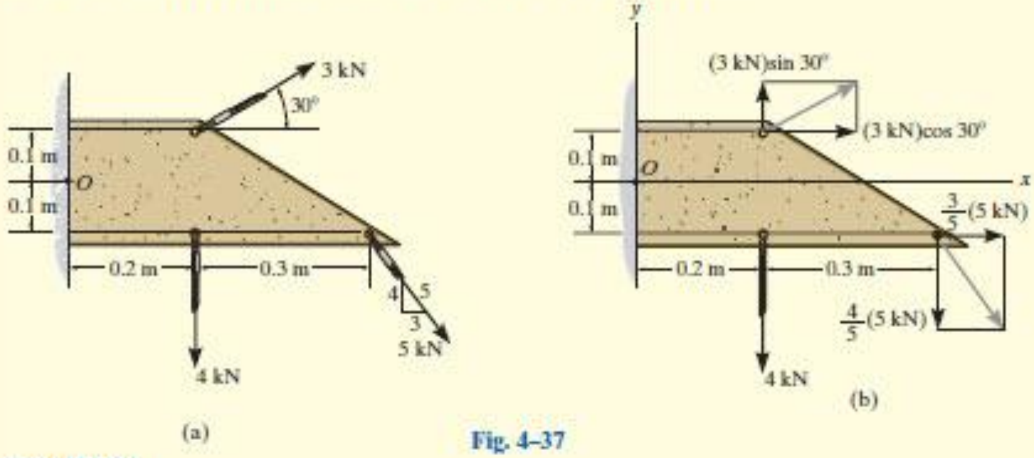


Fig. 4–37

SOLUTION

Force Summation. The 3 kN and 5 kN forces are resolved into their *x* and *y* components as shown in Fig. 4–37*b*. We have

$$\xrightarrow{+} (F_R)_x = \Sigma F_x; \quad (F_R)_x = (3 \text{ kN})\cos 30° + \left(\tfrac{3}{5}\right)(5 \text{ kN}) = 5.598 \text{ kN} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \quad (F_R)_y = (3 \text{ kN})\sin 30° - \left(\tfrac{4}{5}\right)(5 \text{ kN}) - 4 \text{ kN} = -6.50 \text{ kN} = 6.50 \text{ kN}\downarrow$$

Using the Pythagorean theorem, Fig. 4–37*c*, the magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{(5.598 \text{ kN})^2 + (6.50 \text{ kN})^2} = 8.58 \text{ kN} \qquad \textit{Ans.}$$

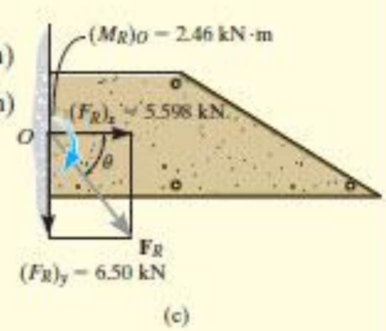
Its direction θ is

$$\theta = \tan^{-1}\left(\frac{(F_R)_y}{(F_R)_x}\right) = \tan^{-1}\left(\frac{6.50 \text{ kN}}{5.598 \text{ kN}}\right) = 49.3° \qquad \textit{Ans.}$$

Moment Summation. The moments of 3 kN and 5 kN about point *O* will be determined using their *x* and *y* components. Referring to Fig. 4–37*b*, we have

$$\circlearrowleft + (M_R)_O = \Sigma M_O;$$

$$\begin{aligned}(M_R)_O = {} & (3 \text{ kN})\sin 30°(0.2 \text{ m}) - (3 \text{ kN})\cos 30°(0.1 \text{ m}) + \left(\tfrac{3}{5}\right)(5 \text{ kN})(0.1 \text{ m}) \\ & - \left(\tfrac{4}{5}\right)(5 \text{ kN})(0.5 \text{ m}) - (4 \text{ kN})(0.2 \text{ m}) \\ = {} & -2.46 \text{ kN}\cdot\text{m} = 2.46 \text{ kN}\cdot\text{m} \circlearrowright \qquad \textit{Ans.}\end{aligned}$$



This clockwise moment is shown in Fig. 4–37*c*.

NOTE: Realize that the resultant force and couple moment in Fig. 4–37*c* will produce the same external effects or reactions at the supports as those produced by the force system, Fig 4–37*a*.

EXAMPLE 5.5

Determine the horizontal and vertical components of reaction on the beam caused by the pin at B and the rocker at A as shown in Fig. 5–12a. Neglect the weight of the beam.

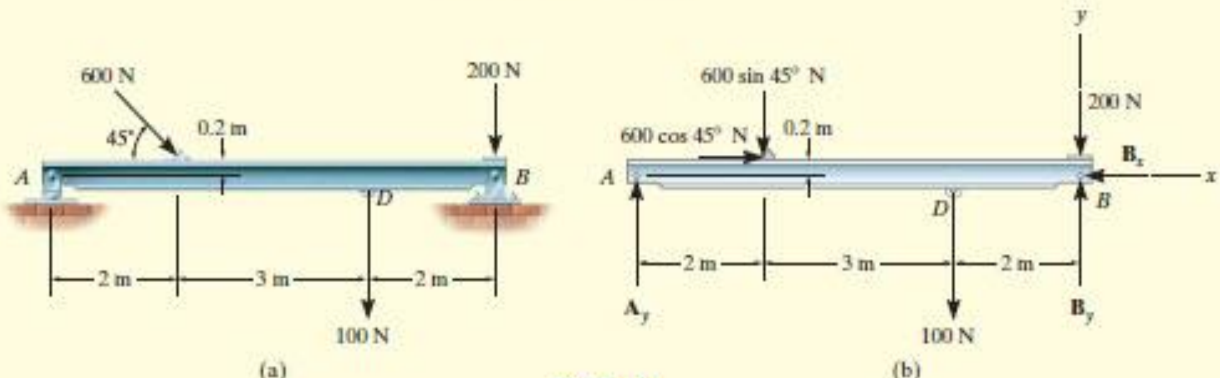


Fig. 5–12

SOLUTION

Free-Body Diagram. Identify each of the forces shown on the free-body diagram of the beam, Fig. 5–12b. (See Example 5.1.) For simplicity, the 600-N force is represented by its x and y components as shown in Fig. 5–12b.

Equations of Equilibrium. Summing forces in the x direction yields

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 600 \cos 45^\circ \text{ N} - B_x = 0$$

$$B_x = 424 \text{ N} \qquad \textit{Ans.}$$

A direct solution for $\mathbf{A}_y$ can be obtained by applying the moment equation $\Sigma M_B = 0$ about point B.

$$\circlearrowleft + \Sigma M_B = 0; \qquad 100 \text{ N}(2 \text{ m}) + (600 \sin 45^\circ \text{ N})(5 \text{ m})$$

$$- (600 \cos 45^\circ \text{ N})(0.2 \text{ m}) - A_y(7 \text{ m}) = 0$$

$$A_y = 319 \text{ N} \qquad \textit{Ans.}$$

Summing forces in the y direction, using this result, gives

$$+\uparrow \Sigma F_y = 0; \qquad 319 \text{ N} - 600 \sin 45^\circ \text{ N} - 100 \text{ N} - 200 \text{ N} + B_y = 0$$

$$B_y = 405 \text{ N} \qquad \textit{Ans.}$$

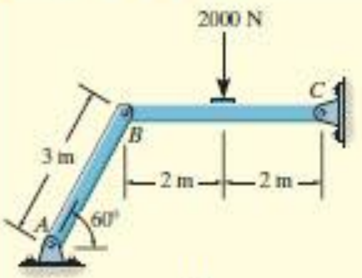
NOTE: We can check this result by summing moments about point A.

$$\circlearrowleft + \Sigma M_A = 0; \qquad -(600 \sin 45^\circ \text{ N})(2 \text{ m}) - (600 \cos 45^\circ \text{ N})(0.2 \text{ m})$$

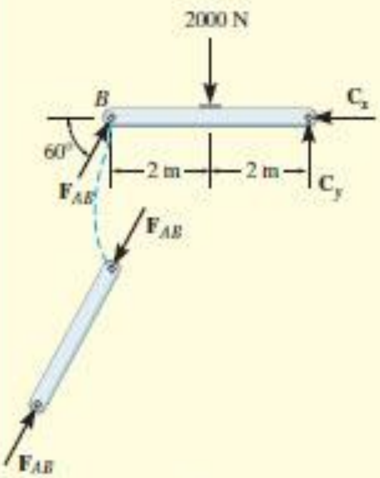
$$-(100 \text{ N})(5 \text{ m}) - (200 \text{ N})(7 \text{ m}) + B_y(7 \text{ m}) = 0$$

$$B_y = 405 \text{ N} \qquad \textit{Ans.}$$

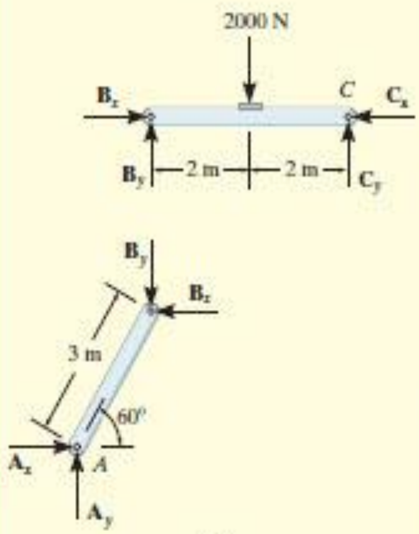
EXAMPLE 6.14



(a)



(b)



(c)

Fig. 6–26

Determine the horizontal and vertical components of force which the pin at C exerts on member BC of the frame in Fig. 6–26*a*.

SOLUTION I

Free-Body Diagrams. By inspection it can be seen that AB is a two-force member. The free-body diagrams are shown in Fig. 6–26*b*.

Equations of Equilibrium. The *three unknowns* can be determined by applying the three equations of equilibrium to member CB.

$$\circlearrowleft + \Sigma M_C = 0;\quad 2000\text{ N}(2\text{ m}) - (F_{AB}\sin 60^\circ)(4\text{ m}) = 0 \quad F_{AB} = 1154.7\text{ N}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad 1154.7\cos 60^\circ\text{ N} - C_x = 0 \qquad C_x = 577\text{ N} \qquad \textit{Ans.}$$

$$+\uparrow \Sigma F_y = 0;\quad 1154.7\sin 60^\circ\text{ N} - 2000\text{ N} + C_y = 0 \quad C_y = 1000\text{ N} \quad \textit{Ans.}$$

SOLUTION II

Free-Body Diagrams. If one does not recognize that AB is a two-force member, then more work is involved in solving this problem. The free-body diagrams are shown in Fig. 6–26*c*.

Equations of Equilibrium. The *six unknowns* are determined by applying the three equations of equilibrium to each member.

Member AB

$$\circlearrowleft + \Sigma M_A = 0;\quad B_x(3\sin 60^\circ\text{ m}) - B_y(3\cos 60^\circ\text{ m}) = 0 \qquad (1)$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad A_x - B_x = 0 \qquad (2)$$

$$+\uparrow \Sigma F_y = 0;\quad A_y - B_y = 0 \qquad (3)$$

Member BC

$$\circlearrowleft + \Sigma M_C = 0;\quad 2000\text{ N}(2\text{ m}) - B_y(4\text{ m}) = 0 \qquad (4)$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad B_x - C_x = 0 \qquad (5)$$

$$+\uparrow \Sigma F_y = 0;\quad B_y - 2000\text{ N} + C_y = 0 \qquad (6)$$

The results for C_x and C_y can be determined by solving these equations in the following sequence: 4, 1, 5, then 6. The results are

$$B_y = 1000\text{ N}$$

$$B_x = 577\text{ N}$$

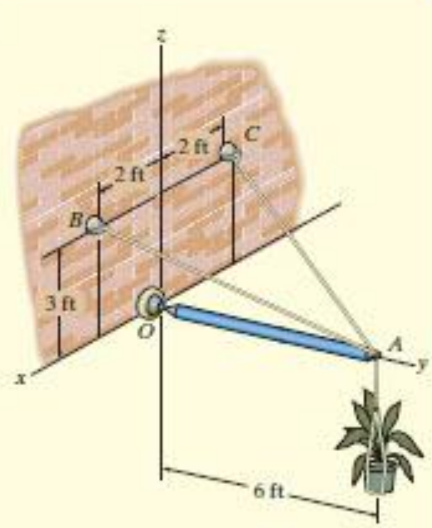
$$C_x = 577\text{ N} \qquad \textit{Ans.}$$

$$C_y = 1000\text{ N} \qquad \textit{Ans.}$$

By comparison, Solution I is simpler since the requirement that F_{AB} in Fig. 6–26*b* be equal, opposite, and collinear at the ends of member AB automatically satisfies Eqs. 1, 2, and 3 above and therefore eliminates the need to write these equations. *As a result, save yourself some time and effort by always identifying the two-force members before starting the analysis!*

EXAMPLE 5.17

The boom is used to support the 75-lb flowerpot in Fig. 5–30*a*. Determine the tension developed in wires *AB* and *AC*.



(a)

Fig. 5–30

SOLUTION

Free-Body Diagram. The free-body diagram of the boom is shown in Fig. 5–30*b*.

Equations of Equilibrium. We will use a vector analysis.

$$\mathbf{F}_{AB} = F_{AB}\left(\frac{\mathbf{r}_{AB}}{r_{AB}}\right) = F_{AB}\left(\frac{\{2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}\}\text{ ft}}{\sqrt{(2\text{ ft})^2 + (-6\text{ ft})^2 + (3\text{ ft})^2}}\right)$$

$$= \tfrac{2}{7}F_{AB}\mathbf{i} - \tfrac{6}{7}F_{AB}\mathbf{j} + \tfrac{3}{7}F_{AB}\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC}\left(\frac{\mathbf{r}_{AC}}{r_{AC}}\right) = F_{AC}\left(\frac{\{-2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}\}\text{ ft}}{\sqrt{(-2\text{ ft})^2 + (-6\text{ ft})^2 + (3\text{ ft})^2}}\right)$$

$$= -\tfrac{2}{7}F_{AC}\mathbf{i} - \tfrac{6}{7}F_{AC}\mathbf{j} + \tfrac{3}{7}F_{AC}\mathbf{k}$$

We can eliminate the force reaction at *O* by writing the moment equation of equilibrium about point *O*.

$$\Sigma\mathbf{M}_O = \mathbf{0}; \qquad \mathbf{r}_A \times (\mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{W}) = \mathbf{0}$$

$$(6\mathbf{j}) \times \left[\left(\tfrac{2}{7}F_{AB}\mathbf{i} - \tfrac{6}{7}F_{AB}\mathbf{j} + \tfrac{3}{7}F_{AB}\mathbf{k}\right) + \left(-\tfrac{2}{7}F_{AC}\mathbf{i} - \tfrac{6}{7}F_{AC}\mathbf{j} + \tfrac{3}{7}F_{AC}\mathbf{k}\right) + (-75\mathbf{k})\right] = \mathbf{0}$$

$$\left(\tfrac{18}{7}F_{AB} + \tfrac{18}{7}F_{AC} - 450\right)\mathbf{i} + \left(-\tfrac{12}{7}F_{AB} + \tfrac{12}{7}F_{AC}\right)\mathbf{k} = \mathbf{0}$$

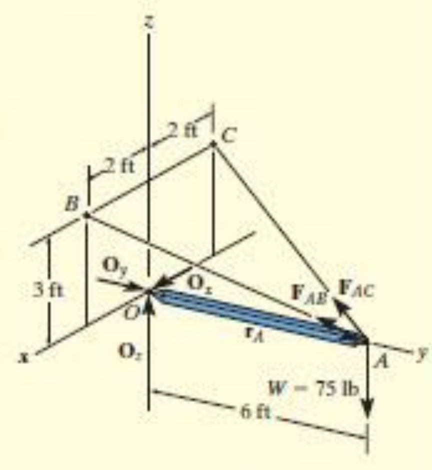
$$\Sigma M_x = 0; \qquad \tfrac{18}{7}F_{AB} + \tfrac{18}{7}F_{AC} - 450 = 0 \qquad (1)$$

$$\Sigma M_y = 0; \qquad 0 = 0$$

$$\Sigma M_z = 0; \qquad -\tfrac{12}{7}F_{AB} + \tfrac{12}{7}F_{AC} = 0 \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously,

$$F_{AB} = F_{AC} = 87.5\text{ lb} \qquad \textit{Ans.}$$



(b)

EXAMPLE 6.1

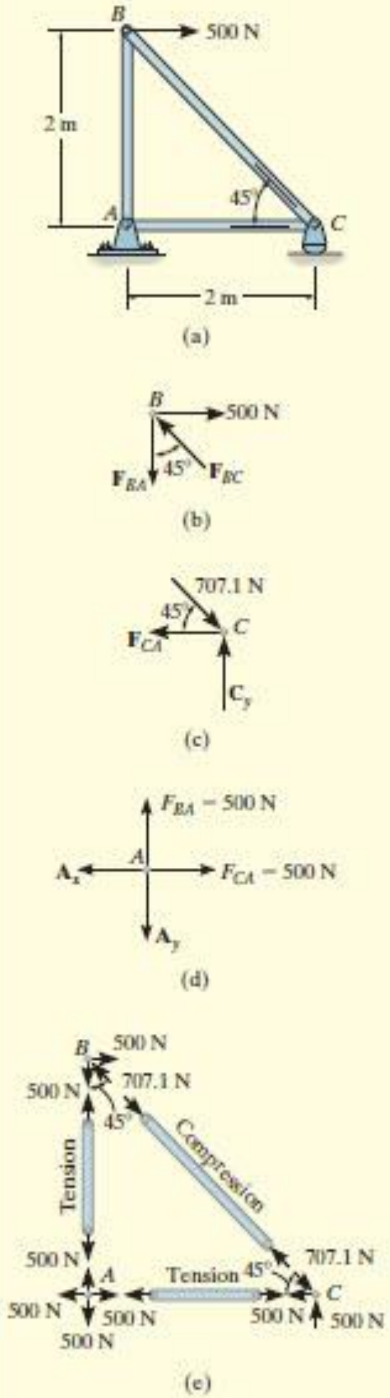


Fig. 6–8

Determine the force in each member of the truss shown in Fig. 6–8*a* and indicate whether the members are in tension or compression.

SOLUTION

Since we should have no more than two unknown forces at the joint and at least one known force acting there, we will begin our analysis at joint *B*.

Joint B. The free-body diagram of the joint at *B* is shown in Fig. 6–8*b*. Applying the equations of equilibrium, we have

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 500 \text{ N} - F_{BC} \sin 45^\circ = 0 \quad F_{BC} = 707.1 \text{ N (C)} \quad \textit{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \quad F_{BC} \cos 45^\circ - F_{BA} = 0 \quad F_{BA} = 500 \text{ N (T)} \quad \textit{Ans.}$$

Since the force in member *BC* has been calculated, we can proceed to analyze joint *C* to determine the force in member *CA* and the support reaction at the rocker.

Joint C. From the free-body diagram of joint *C*, Fig. 6–8*c*, we have

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad -F_{CA} + 707.1 \cos 45^\circ \text{ N} = 0 \quad F_{CA} = 500 \text{ N (T)} \quad \textit{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \quad C_y - 707.1 \sin 45^\circ \text{ N} = 0 \quad C_y = 500 \text{ N} \quad \textit{Ans.}$$

Joint A. Although it is not necessary, we can determine the components of the support reactions at joint *A* using the results of F_{CA} and F_{BA}. From the free-body diagram, Fig. 6–8*d*, we have

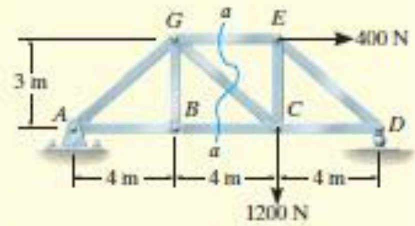
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 500 \text{ N} - A_x = 0 \quad A_x = 500 \text{ N}$$

$$+\uparrow \Sigma F_y = 0; \quad 500 \text{ N} - A_y = 0 \quad A_y = 500 \text{ N}$$

NOTE: The results of the analysis are summarized in Fig. 6–8*e*. Note that the free-body diagram of each joint (or pin) shows the effects of all the connected members and external forces applied to the joint, whereas the free-body diagram of each member shows only the effects of the end joints on the member.

EXAMPLE 6.5

Determine the force in members *GE*, *GC*, and *BC* of the truss shown in Fig. 6–16*a*. Indicate whether the members are in tension or compression.



(a)

SOLUTION

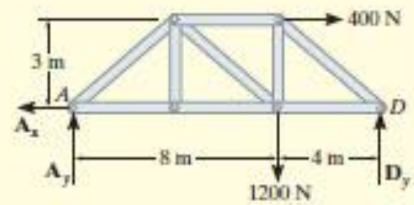
Section *aa* in Fig. 6–16*a* has been chosen since it cuts through the *three* members whose forces are to be determined. In order to use the method of sections, however, it is *first* necessary to determine the external reactions at *A* or *D*. Why? A free-body diagram of the entire truss is shown in Fig. 6–16*b*. Applying the equations of equilibrium, we have

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 400\text{ N} - A_x = 0 \qquad A_x = 400\text{ N}$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad -1200\text{ N}(8\text{ m}) - 400\text{ N}(3\text{ m}) + D_y(12\text{ m}) = 0$$

$$D_y = 900\text{ N}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y - 1200\text{ N} + 900\text{ N} = 0 \qquad A_y = 300\text{ N}$$



(b)

Free-Body Diagram. For the analysis the free-body diagram of the left portion of the sectioned truss will be used, since it involves the least number of forces, Fig. 6–16*c*.

Equations of Equilibrium. Summing moments about point *G* eliminates $\mathbf{F}_{GE}$ and $\mathbf{F}_{GC}$ and yields a direct solution for F_{BC}.

$$\circlearrowleft + \Sigma M_G = 0; \qquad -300\text{ N}(4\text{ m}) - 400\text{ N}(3\text{ m}) + F_{BC}(3\text{ m}) = 0$$

$$F_{BC} = 800\text{ N} \quad (\text{T}) \qquad \textit{Ans.}$$

In the same manner, by summing moments about point *C* we obtain a direct solution for F_{GE}.

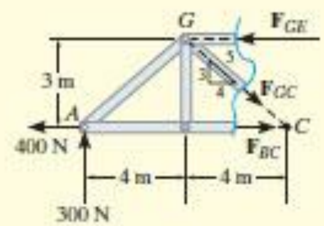
$$\circlearrowleft + \Sigma M_C = 0; \qquad -300\text{ N}(8\text{ m}) + F_{GE}(3\text{ m}) = 0$$

$$F_{GE} = 800\text{ N} \quad (\text{C}) \qquad \textit{Ans.}$$

Since $\mathbf{F}_{BC}$ and $\mathbf{F}_{GE}$ have no vertical components, summing forces in the *y* direction directly yields F_{GC}, i.e.,

$$+\uparrow \Sigma F_y = 0; \qquad 300\text{ N} - \tfrac{3}{5}F_{GC} = 0$$

$$F_{GC} = 500\text{ N} \quad (\text{T}) \qquad \textit{Ans.}$$



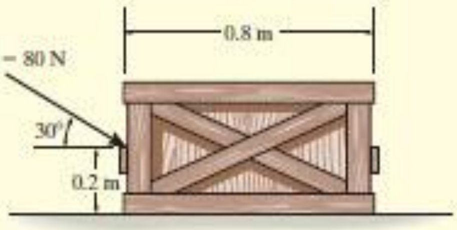
(c)

Fig. 6–16

NOTE: Here it is possible to tell, by inspection, the proper direction for each unknown member force. For example, $\Sigma M_C = 0$ requires $\mathbf{F}_{GE}$ to be *compressive* because it must balance the moment of the 300-N force about *C*.

EXAMPLE 8.1

The uniform crate shown in Fig. 8–7*a* has a mass of 20 kg. If a force $P = 80$ N is applied to the crate, determine if it remains in equilibrium. The coefficient of static friction is $\mu_s = 0.3$.

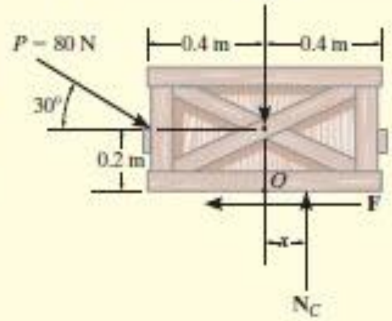


(a)

Fig. 8–7

SOLUTION

Free-Body Diagram. As shown in Fig. 8–7*b*, the *resultant* normal force $\mathbf{N}_C$ must act a distance x from the crate's center line in order to counteract the tipping effect caused by $\mathbf{P}$. There are *three unknowns*, F, N_C, and x, which can be determined strictly from the *three* equations of equilibrium.



(b)

Equations of Equilibrium.

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 80 \cos 30^\circ \text{ N} - F = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad -80 \sin 30^\circ \text{ N} + N_C - 196.2 \text{ N} = 0$$

$$\circlearrowleft + \Sigma M_O = 0; \qquad 80 \sin 30^\circ \text{ N}(0.4 \text{ m}) - 80 \cos 30^\circ \text{ N}(0.2 \text{ m}) + N_C(x) = 0$$

Solving,

$$F = 69.3 \text{ N}$$

$$N_C = 236 \text{ N}$$

$$x = -0.00908 \text{ m} = -9.08 \text{ mm}$$

Since x is negative it indicates the *resultant* normal force acts (slightly) to the *left* of the crate's center line. No tipping will occur since $x < 0.4$ m. Also, the *maximum* frictional force which can be developed at the surface of contact is $F_{max} = \mu_s N_C = 0.3(236 \text{ N}) = 70.8$ N. Since $F = 69.3 \text{ N} < 70.8$ N, the crate will *not slip*, although it is very close to doing so.

EXAMPLE 8.3

The uniform 10-kg ladder in Fig. 8–9*a* rests against the smooth wall at *B*, and the end *A* rests on the rough horizontal plane for which the coefficient of static friction is $\mu_s = 0.3$. Determine the angle of inclination θ of the ladder and the normal reaction at *B* if the ladder is on the verge of slipping.

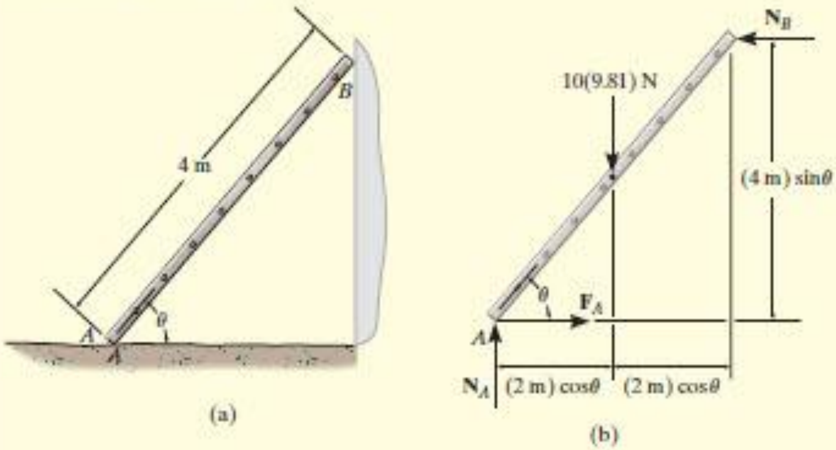


Fig. 8–9

SOLUTION

Free-Body Diagram. As shown on the free-body diagram, Fig. 8–9*b*, the frictional force $\mathbf{F}_A$ must act to the right since impending motion at *A* is to the left.

Equations of Equilibrium and Friction. Since the ladder is on the verge of slipping, then $F_A = \mu_s N_A = 0.3N_A$. By inspection, N_A can be obtained directly.

$+\uparrow \Sigma F_y = 0; \qquad N_A - 10(9.81)\text{ N} = 0 \qquad N_A = 98.1\text{ N}$

Using this result, $F_A = 0.3(98.1\text{ N}) = 29.43\text{ N}$. Now N_B can be found.

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 29.43\text{ N} - N_B = 0$

$$N_B = 29.43\text{ N} = 29.4\text{ N} \qquad \textit{Ans.}$$

Finally, the angle θ can be determined by summing moments about point *A*.

$\circlearrowleft + \Sigma M_A = 0; \qquad (29.43\text{ N})(4\text{ m})\sin\theta - [10(9.81)\text{ N}](2\text{ m})\cos\theta = 0$

$$\frac{\sin\theta}{\cos\theta} = \tan\theta = 1.6667$$

$$\theta = 59.04^\circ = 59.0^\circ \qquad \textit{Ans.}$$

EXAMPLE 12.3

During a test a rocket travels upward at 75 m/s, and when it is 40 m from the ground its engine fails. Determine the maximum height s_B reached by the rocket and its speed just before it hits the ground. While in motion the rocket is subjected to a constant downward acceleration of 9.81 m/s² due to gravity. Neglect the effect of air resistance.

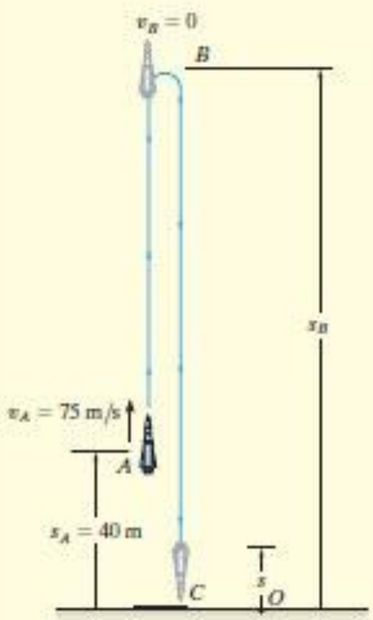


Fig. 12–4

SOLUTION

Coordinate System. The origin O for the position coordinate s is taken at ground level with positive upward, Fig. 12–4.

Maximum Height. Since the rocket is traveling *upward*, $v_A = +75$m/s when $t = 0$. At the maximum height $s = s_B$ the velocity $v_B = 0$. For the entire motion, the acceleration is $a_c = -9.81$ m/s² (negative since it acts in the *opposite* sense to positive velocity or positive displacement). Since a_c is *constant* the rocket's position may be related to its velocity at the two points A and B on the path by using Eq. 12–6, namely,

$$(+\uparrow) \qquad v_B^2 = v_A^2 + 2a_c(s_B - s_A)$$

$$0 = (75 \text{ m/s})^2 + 2(-9.81 \text{ m/s}^2)(s_B - 40 \text{ m})$$

$$s_B = 327 \text{ m} \qquad \textit{Ans.}$$

Velocity. To obtain the velocity of the rocket just before it hits the ground, we can apply Eq. 12–6 between points B and C, Fig. 12–4.

$$(+\uparrow) \qquad v_C^2 = v_B^2 + 2a_c(s_C - s_B)$$

$$= 0 + 2(-9.81 \text{ m/s}^2)(0 - 327 \text{ m})$$

$$v_C = -80.1 \text{ m/s} = 80.1 \text{ m/s} \downarrow \qquad \textit{Ans.}$$

The negative root was chosen since the rocket is moving downward.

Similarly, Eq. 12–6 may also be applied between points A and C, i.e.,

$$(+\uparrow) \qquad v_C^2 = v_A^2 + 2a_c(s_C - s_A)$$

$$= (75 \text{ m/s})^2 + 2(-9.81 \text{ m/s}^2)(0 - 40 \text{ m})$$

$$v_C = -80.1 \text{ m/s} = 80.1 \text{ m/s} \downarrow \qquad \textit{Ans.}$$

NOTE: It should be realized that the rocket is subjected to a *deceleration* from A to B of 9.81 m/s², and then from B to C it is *accelerated* at this rate. Furthermore, even though the rocket momentarily comes to *rest* at B ($v_B = 0$) the acceleration at B is still 9.81 m/s² downward!

EXAMPLE 12.10

For a short time, the path of the plane in Fig. 12–19*a* is described by $y = (0.001x^2)$ m. If the plane is rising with a constant velocity of 10 m/s, determine the magnitudes of the velocity and acceleration of the plane when it is at $y = 100$ m.

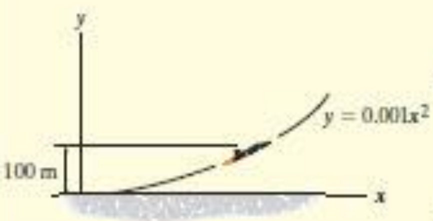
SOLUTION

When $y = 100$ m, then $100 = 0.001x^2$ or $x = 316.2$ m. Also, since $v_y = 10$ m/s, then

$$y = v_y t; \qquad 100\text{ m} = (10\text{ m/s})\,t \qquad t = 10\text{ s}$$

Velocity. Using the chain rule (see Appendix C) to find the relationship between the velocity components, we have

$$v_y = \dot{y} = \frac{d}{dt}(0.001x^2) = (0.002x)\dot{x} = 0.002xv_x \qquad (1)$$



(a)

Thus

$$10\text{ m/s} = 0.002(316.2\text{ m})(v_x)$$
$$v_x = 15.81\text{ m/s}$$

The magnitude of the velocity is therefore

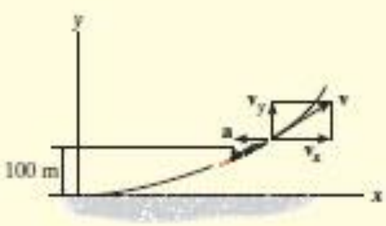
$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(15.81\text{ m/s})^2 + (10\text{ m/s})^2} = 18.7\text{ m/s} \qquad \textit{Ans.}$$

Acceleration. Using the chain rule, the time derivative of Eq. (1) gives the relation between the acceleration components.

$$a_y = \dot{v}_y = 0.002\dot{x}v_x + 0.002x\dot{v}_x = 0.002(v_x^2 + xa_x)$$

When $x = 316.2$ m, $v_x = 15.81$ m/s, $\dot{v}_y = a_y = 0$,

$$0 = 0.002((15.81\text{ m/s})^2 + 316.2\text{ m}(a_x))$$
$$a_x = -0.791\text{ m/s}^2$$



(b)

Fig. 12–19

The magnitude of the plane's acceleration is therefore

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(-0.791\text{ m/s}^2)^2 + (0\text{ m/s}^2)^2}$$
$$= 0.791\text{ m/s}^2 \qquad \textit{Ans.}$$

These results are shown in Fig. 12–19*b*.

EXAMPLE 12.14

When the skier reaches point A along the parabolic path in Fig. 12–27a, he has a speed of 6 m/s which is increasing at 2 m/s². Determine the direction of his velocity and the direction and magnitude of his acceleration at this instant. Neglect the size of the skier in the calculation.

SOLUTION

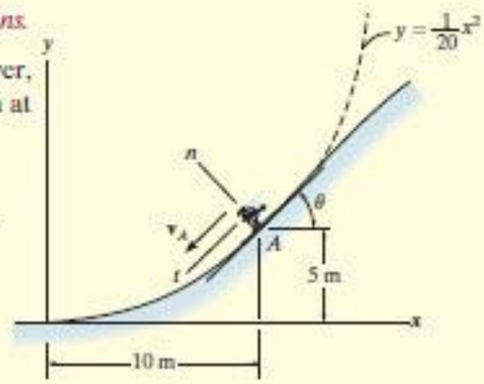
Coordinate System. Although the path has been expressed in terms of its x and y coordinates, we can still establish the origin of the n, t axes at the fixed point A on the path and determine the components of **v** and **a** along these axes, Fig. 12–27a.

Velocity. By definition, the velocity is always directed tangent to the path. Since $y = \frac{1}{20}x^2$, $dy/dx = \frac{1}{10}x$, then at $x = 10$ m, $dy/dx = 1$. Hence, at A, **v** makes an angle of $\theta = \tan^{-1}1 = 45^\circ$ with the x axis, Fig. 12–27a. Therefore,

$$v_A = 6 \text{ m/s} \quad 45^\circ \searrow \qquad \textit{Ans.}$$

The acceleration is determined from $\mathbf{a} = \dot{v}\mathbf{u}_t + (v^2/\rho)\mathbf{u}_n$. However, it is first necessary to determine the radius of curvature of the path at A (10 m, 5 m). Since $d^2y/dx^2 = \frac{1}{10}$, then

$$\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|} = \left.\frac{\left[1 + \left(\frac{1}{10}x\right)^2\right]^{3/2}}{\left|\frac{1}{10}\right|}\right|_{x=10\text{ m}} = 28.28 \text{ m}$$



(a)

The acceleration becomes

$$\mathbf{a}_A = \dot{v}\mathbf{u}_t + \frac{v^2}{\rho}\mathbf{u}_n$$
$$= 2\mathbf{u}_t + \frac{(6 \text{ m/s})^2}{28.28 \text{ m}}\mathbf{u}_n$$
$$= \{2\mathbf{u}_t + 1.273\mathbf{u}_n\} \text{ m/s}^2$$

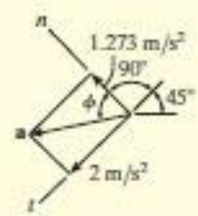
As shown in Fig. 12–27b,

$$a = \sqrt{(2 \text{ m/s}^2)^2 + (1.273 \text{ m/s}^2)^2} = 2.37 \text{ m/s}^2$$

$$\phi = \tan^{-1}\frac{2}{1.273} = 57.5^\circ$$

Thus, $45^\circ + 90^\circ + 57.5^\circ - 180^\circ = 12.5^\circ$ so that,

$$a = 2.37 \text{ m/s}^2 \quad 12.5^\circ \searrow \qquad \textit{Ans.}$$



(b)

Fig. 12–27

NOTE: By using n, t coordinates, we were able to readily solve this problem through the use of Eq. 12–18, since it accounts for the separate changes in the magnitude and direction of **v**.

12.9 Absolute Dependent Motion Analysis of Two Particles

In some types of problems the motion of one particle will *depend* on the corresponding motion of another particle. This dependency commonly occurs if the particles, here represented by blocks, are interconnected by inextensible cords which are wrapped around pulleys. For example, the movement of block A downward along the inclined plane in Fig. 12–36 will cause a corresponding movement of block B up the other incline. We can show this mathematically by first specifying the location of the blocks using *position coordinates* s_A and s_B. Note that each of the coordinate axes is (1) measured from a *fixed* point (O) or *fixed* datum line, (2) measured along each inclined plane *in the direction of motion* of each block, and (3) has a positive sense from C to A and D to B. If the total cord length is l_T, the two position coordinates are related by the equation

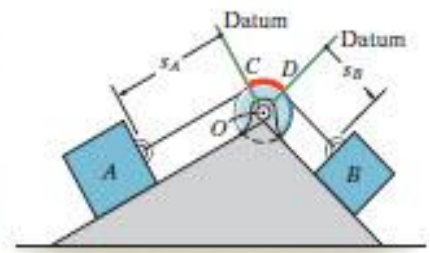


Fig. 12–36

$$s_A + l_{CD} + s_B = l_T$$

Here l_{CD} is the length of the cord passing over arc CD. Taking the time derivative of this expression, realizing that l_{CD} and l_T *remain constant*, while s_A and s_B measure the segments of the cord that change in length. We have

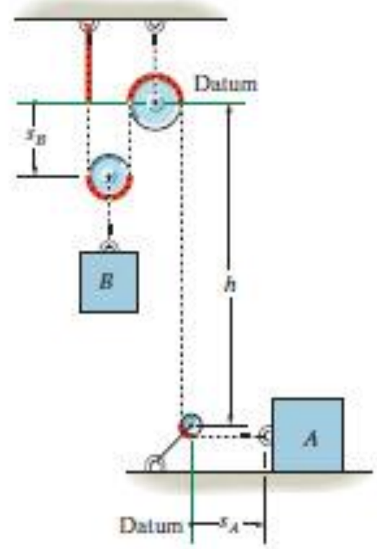
$$\frac{ds_A}{dt} + \frac{ds_B}{dt} = 0 \qquad \text{or} \qquad v_B = -v_A$$

The negative sign indicates that when block A has a velocity downward, i.e., in the direction of positive s_A, it causes a corresponding upward velocity of block B; i.e., B moves in the negative s_B direction.

In a similar manner, time differentiation of the velocities yields the relation between the accelerations, i.e.,

$$a_B = -a_A$$

A more complicated example is shown in Fig. 12–37*a*. In this case, the position of block A is specified by s_A, and the position of the *end* of the cord from which block B is suspended is defined by s_B. As above, we have chosen position coordinates which (1) have their origin at fixed points or datums, (2) are measured in the direction of motion of each block, and (3) are positive to the right for s_A and positive downward for s_B. During the motion, the length of the red colored segments of the cord in Fig. 12–37*a remains constant*. If l represents the total length of cord minus these segments, then the position coordinates can be related by the equation



(a)

Fig. 12–37

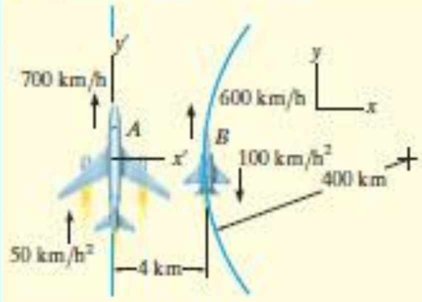
$$2s_B + h + s_A = l$$

Since l and h are constant during the motion, the two time derivatives yield

$$2v_B = -v_A \qquad 2a_B = -a_A$$

Hence, when B moves downward ($+s_B$), A moves to the left ($-s_A$) with twice the motion.

EXAMPLE 12.26



(a)

Plane A in Fig. 12–44a is flying along a straight-line path, whereas plane B is flying along a circular path having a radius of curvature of $\rho_B = 400$ km. Determine the velocity and acceleration of B as measured by the pilot of A.

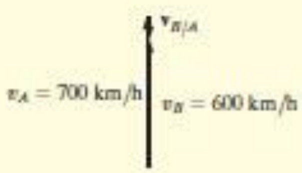
SOLUTION

Velocity. The origin of the x and y axes are located at an arbitrary fixed point. Since the motion relative to plane A is to be determined, the *translating frame of reference* x', y' is attached to it, Fig. 12–44a. Applying the relative-velocity equation in scalar form since the velocity vectors of both planes are parallel at the instant shown, we have

$$(+\uparrow) \qquad v_B = v_A + v_{B/A}$$

$$600 \text{ km/h} = 700 \text{ km/h} + v_{B/A}$$

$$v_{B/A} = -100 \text{ km/h} = 100 \text{ km/h} \downarrow \qquad \textit{Ans.}$$



(b)

The vector addition is shown in Fig. 12–44b.

Acceleration. Plane B has both tangential and normal components of acceleration since it is flying along a *curved path*. From Eq. 12–20, the magnitude of the normal component is

$$(a_B)_n = \frac{v_B^2}{\rho} = \frac{(600 \text{ km/h})^2}{400 \text{ km}} = 900 \text{ km/h}^2$$

Applying the relative-acceleration equation gives

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

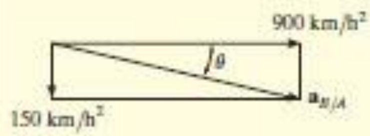
$$900\mathbf{i} - 100\mathbf{j} = 50\mathbf{j} + \mathbf{a}_{B/A}$$

Thus,

$$\mathbf{a}_{B/A} = \{900\mathbf{i} - 150\mathbf{j}\} \text{ km/h}^2$$

From Fig. 12–44c, the magnitude and direction of $\mathbf{a}_{B/A}$ are therefore

$$a_{B/A} = 912 \text{ km/h}^2 \quad \theta = \tan^{-1}\frac{150}{900} = 9.46^\circ \qquad \textit{Ans.}$$



(c)

Fig. 12–44

NOTE: The solution to this problem was possible using a translating frame of reference, since the pilot in plane A is "translating." Observation of the motion of plane A with respect to the pilot of plane B, however, must be obtained using a *rotating* set of axes attached to plane B. (This assumes, of course, that the pilot of B is fixed in the rotating frame, so he does not turn his eyes to follow the motion of A.) The analysis for this case is given in Example 16.21.

EXAMPLE 16.2

The motor shown in the photo is used to turn a wheel and attached blower contained within the housing. The details of the design are shown in Fig. 16–6*a*. If the pulley *A* connected to the motor begins to rotate from rest with a constant angular acceleration of $\alpha_A = 2\ \text{rad/s}^2$, determine the magnitudes of the velocity and acceleration of point *P* on the wheel, after the pulley has turned two revolutions. Assume the transmission belt does not slip on the pulley and wheel.

SOLUTION

Angular Motion. First we will convert the two revolutions to radians. Since there are 2π rad in one revolution, then

$$\theta_A = 2\ \text{rev}\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right) = 12.57\ \text{rad}$$

Since α_A is constant, the angular velocity of pulley *A* is therefore

$$(\circlearrowright +) \qquad \omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$$

$$\omega_A^2 = 0 + 2(2\ \text{rad/s}^2)(12.57\ \text{rad} - 0)$$

$$\omega_A = 7.090\ \text{rad/s}$$

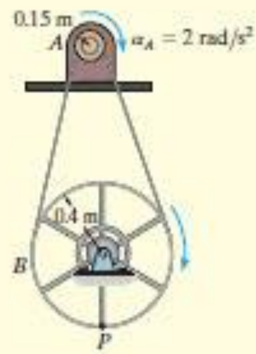
The belt has the same speed and tangential component of acceleration as it passes over the pulley and wheel. Thus,

$$v = \omega_A r_A = \omega_B r_B;\ 7.090\ \text{rad/s}\ (0.15\ \text{m}) = \omega_B(0.4\ \text{m})$$

$$\omega_B = 2.659\ \text{rad/s}$$

$$a_t = \alpha_A r_A = \alpha_B r_B;\ 2\ \text{rad/s}^2\ (0.15\ \text{m}) = \alpha_B(0.4\ \text{m})$$

$$\alpha_B = 0.750\ \text{rad/s}^2$$



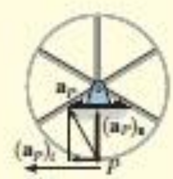
(a)

Motion of *P*. As shown on the kinematic diagram in Fig. 16–6*b*, we have

$$v_P = \omega_B r_B = 2.659\ \text{rad/s}\ (0.4\ \text{m}) = 1.06\ \text{m/s} \qquad \textit{Ans.}$$

$$(a_P)_t = \alpha_B r_B = 0.750\ \text{rad/s}^2\ (0.4\ \text{m}) = 0.3\ \text{m/s}^2$$

$$(a_P)_n = \omega_B^2 r_B = (2.659\ \text{rad/s})^2(0.4\ \text{m}) = 2.827\ \text{m/s}^2$$



(b)

Thus

$$a_P = \sqrt{(0.3\ \text{m/s}^2)^2 + (2.827\ \text{m/s}^2)^2} = 2.84\ \text{m/s}^2 \qquad \textit{Ans.}$$

Fig. 16–6

EXAMPLE 16.4

At a given instant, the cylinder of radius r, shown in Fig. 16–8, has an angular velocity $\boldsymbol{\omega}$ and angular acceleration $\boldsymbol{\alpha}$. Determine the velocity and acceleration of its center G if the cylinder rolls without slipping.

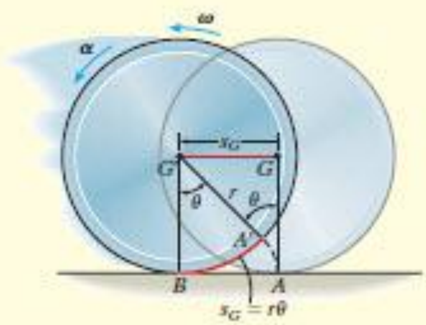


Fig. 16–8

SOLUTION

Position Coordinate Equation. The cylinder undergoes general plane motion since it simultaneously translates and rotates. By inspection, point G moves in a *straight line* to the left, from G to G', as the cylinder rolls, Fig. 16–8. Consequently its new position G' will be specified by the *horizontal* position coordinate s_G, which is measured from G to G'. Also, as the cylinder rolls (without slipping), the arc length $A'B$ on the rim which was in contact with the ground from A to B, is equivalent to s_G. Consequently, the motion requires the radial line GA to rotate θ to the position $G'A'$. Since the arc $A'B = r\theta$, then G travels a distance

$$s_G = r\theta$$

Time Derivatives. Taking successive time derivatives of this equation, realizing that r is constant, $\omega = d\theta/dt$, and $\alpha = d\omega/dt$, gives the necessary relationships:

$$s_G = r\theta$$

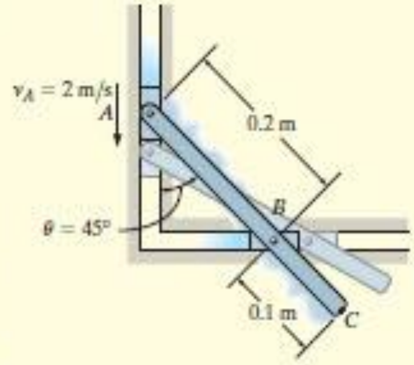
$$v_G = r\omega \qquad \textit{Ans.}$$

$$a_G = r\alpha \qquad \textit{Ans.}$$

NOTE: Remember that these relationships are valid only if the cylinder (disk, wheel, ball, etc.) rolls *without* slipping.

EXAMPLE 16.6

The link shown in Fig. 16–13*a* is guided by two blocks at A and B, which move in the fixed slots. If the velocity of A is 2 m/s downward, determine the velocity of B at the instant $\theta = 45^\circ$.



(a)

SOLUTION (VECTOR ANALYSIS)

Kinematic Diagram. Since points A and B are restricted to move along the fixed slots and $\mathbf{v}_A$ is directed downward, the velocity $\mathbf{v}_B$ must be directed horizontally to the right, Fig. 16–13*b*. This motion causes the link to rotate counterclockwise; that is, by the right-hand rule the angular velocity $\boldsymbol{\omega}$ is directed outward, perpendicular to the plane of motion. Knowing the magnitude and direction of $\mathbf{v}_A$ and the lines of action of $\mathbf{v}_B$ and $\boldsymbol{\omega}$, it is possible to apply the velocity equation $\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A}$ to points A and B in order to solve for the two unknown magnitudes v_B and ω. Since $\mathbf{r}_{B/A}$ is needed, it is also shown in Fig. 16–13*b*.

Velocity Equation. Expressing each of the vectors in Fig. 16–13*b* in terms of their **i**, **j**, **k** components and applying Eq. 16–16 to A, the base point, and B, we have

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A}$$

$$v_B\mathbf{i} = -2\mathbf{j} + [\omega\mathbf{k} \times (0.2 \sin 45^\circ\mathbf{i} - 0.2 \cos 45^\circ\mathbf{j})]$$

$$v_B\mathbf{i} = -2\mathbf{j} + 0.2\omega \sin 45^\circ\mathbf{j} + 0.2\omega \cos 45^\circ\mathbf{i}$$

Equating the **i** and **j** components gives

$$v_B = 0.2\omega \cos 45^\circ \qquad 0 = -2 + 0.2\omega \sin 45^\circ$$

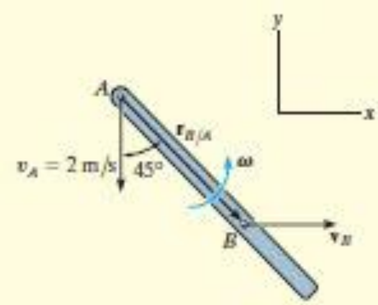
Thus,

$$\omega = 14.1 \text{ rad/s} \circlearrowleft$$

$$v_B = 2 \text{ m/s} \rightarrow \qquad \textit{Ans.}$$

Since both results are *positive*, the *directions* of $\mathbf{v}_B$ and $\boldsymbol{\omega}$ are indeed *correct* as shown in Fig. 16–13*b*. It should be emphasized that these results are *valid only* at the instant $\theta = 45^\circ$. A recalculation for $\theta = 44^\circ$ yields $v_B = 2.07$ m/s and $\omega = 14.4$ rad/s; whereas when $\theta = 46^\circ$, $v_B = 1.93$ m/s and $\omega = 13.9$ rad/s, etc.

NOTE: Once the velocity of a point (A) on the link and the angular velocity are *known*, the velocity of any other point on the link can be determined. As an exercise, see if you can apply Eq. 16–16 to points A and C or to points B and C and show that when $\theta = 45^\circ$, $v_C = 3.16$ m/s, directed at an angle of 18.4° up from the horizontal.



(b)

Fig. 16–13

EXAMPLE 16.19

At the instant $\theta = 60°$, the rod in Fig. 16–33 has an angular velocity of 3 rad/s and an angular acceleration of 2 rad/s². At this same instant, collar C travels outward along the rod such that when $x = 0.2$ m the velocity is 2 m/s and the acceleration is 3 m/s², both measured relative to the rod. Determine the Coriolis acceleration and the velocity and acceleration of the collar at this instant.

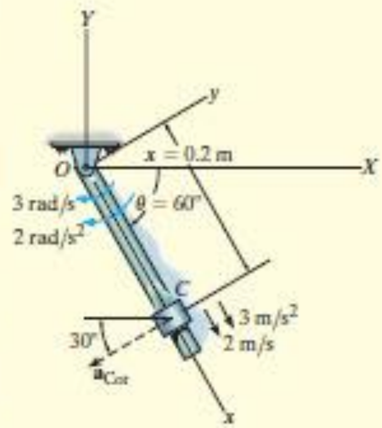


Fig. 16–33

SOLUTION

Coordinate Axes. The origin of both coordinate systems is located at point O, Fig. 16–33. Since motion of the collar is reported relative to the rod, the moving x, y, z frame of reference is *attached* to the rod.

Kinematic Equations.

$$\mathbf{v}_C = \mathbf{v}_O + \boldsymbol{\Omega} \times \mathbf{r}_{C/O} + (\mathbf{v}_{C/O})_{xyz} \quad (1)$$

$$\mathbf{a}_C = \mathbf{a}_O + \dot{\boldsymbol{\Omega}} \times \mathbf{r}_{C/O} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{C/O}) + 2\boldsymbol{\Omega} \times (\mathbf{v}_{C/O})_{xyz} + (\mathbf{a}_{C/O})_{xyz} \quad (2)$$

It will be simpler to express the data in terms of **i**, **j**, **k** component vectors rather than **I**, **J**, **K** components. Hence,

Motion of moving reference	Motion of C with respect to moving reference
$\mathbf{v}_O = \mathbf{0}$	$\mathbf{r}_{C/O} = \{0.2\mathbf{i}\}$ m
$\mathbf{a}_O = \mathbf{0}$	$(\mathbf{v}_{C/O})_{xyz} = \{2\mathbf{i}\}$ m/s
$\boldsymbol{\Omega} = \{-3\mathbf{k}\}$ rad/s	$(\mathbf{a}_{C/O})_{xyz} = \{3\mathbf{i}\}$ m/s²
$\dot{\boldsymbol{\Omega}} = \{-2\mathbf{k}\}$ rad/s²	

The Coriolis acceleration is defined as

$$\mathbf{a}_{Cor} = 2\boldsymbol{\Omega} \times (\mathbf{v}_{C/O})_{xyz} = 2(-3\mathbf{k}) \times (2\mathbf{i}) = \{-12\mathbf{j}\} \text{ m/s}^2 \quad \textit{Ans.}$$

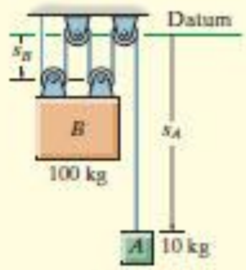
This vector is shown dashed in Fig. 16–33. If desired, it may be resolved into **I**, **J** components acting along the X and Y axes, respectively.

The velocity and acceleration of the collar are determined by substituting the data into Eqs. 1 and 2 and evaluating the cross products, which yields

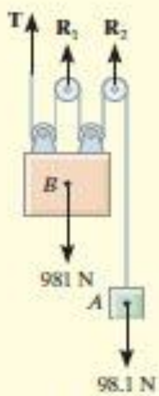
$$\begin{aligned}\mathbf{v}_C &= \mathbf{v}_O + \boldsymbol{\Omega} \times \mathbf{r}_{C/O} + (\mathbf{v}_{C/O})_{xyz}\\ &= \mathbf{0} + (-3\mathbf{k}) \times (0.2\mathbf{i}) + 2\mathbf{i}\\ &= \{2\mathbf{i} - 0.6\mathbf{j}\} \text{ m/s} \quad \textit{Ans.}\end{aligned}$$

$$\begin{aligned}\mathbf{a}_C &= \mathbf{a}_O + \dot{\boldsymbol{\Omega}} \times \mathbf{r}_{C/O} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{C/O}) + 2\boldsymbol{\Omega} \times (\mathbf{v}_{C/O})_{xyz} + (\mathbf{a}_{C/O})_{xyz}\\ &= \mathbf{0} + (-2\mathbf{k}) \times (0.2\mathbf{i}) + (-3\mathbf{k}) \times [(-3\mathbf{k}) \times (0.2\mathbf{i})] + 2(-3\mathbf{k}) \times (2\mathbf{i}) + 3\mathbf{i}\\ &= \mathbf{0} - 0.4\mathbf{j} - 1.80\mathbf{i} - 12\mathbf{j} + 3\mathbf{i}\\ &= \{1.20\mathbf{i} - 12.4\mathbf{j}\} \text{ m/s}^2 \quad \textit{Ans.}\end{aligned}$$

EXAMPLE 14.6



(a)



(b)

Fig. 14–14

Blocks A and B shown in Fig. 14–14a have a mass of 10 kg and 100 kg, respectively. Determine the distance B travels when it is released from rest to the point where its speed becomes 2 m/s.

SOLUTION

This problem may be solved by considering the blocks separately and applying the principle of work and energy to each block. However, the work of the (unknown) cable tension can be eliminated from the analysis by considering blocks A and B together as a *single system*.

Work (Free-Body Diagram). As shown on the free-body diagram of the system, Fig. 14–14b, the cable force $\mathbf{T}$ and reactions $\mathbf{R}_1$ and $\mathbf{R}_2$ do *no work*, since these forces represent the reactions at the supports and consequently they do not move while the blocks are displaced. The weights both do positive work if we *assume* both move downward, in the positive sense of direction of s_A and s_B.

Principle of Work and Energy. Realizing the blocks are released from rest, we have

$$\Sigma T_1 + \Sigma U_{1-2} = \Sigma T_2$$

$$\left\{\tfrac{1}{2}m_A(v_A)_1^2 + \tfrac{1}{2}m_B(v_B)_1^2\right\} + \{W_A\,\Delta s_A + W_B\,\Delta s_B\} = \left\{\tfrac{1}{2}m_A(v_A)_2^2 + \tfrac{1}{2}m_B(v_B)_2^2\right\}$$

$$\{0 + 0\} + \{98.1\text{ N}\,(\Delta s_A) + 981\text{ N}\,(\Delta s_B)\} = \left\{\tfrac{1}{2}(10\text{ kg})(v_A)_2^2 + \tfrac{1}{2}(100\text{ kg})(2\text{ m/s})^2\right\} \qquad (1)$$

Kinematics. Using the methods of kinematics discussed in Sec. 12.9, it may be seen from Fig. 14–14a that the total length l of all the vertical segments of cable may be expressed in terms of the position coordinates s_A and s_B as

$$s_A + 4s_B = l$$

Hence, a change in position yields the displacement equation

$$\Delta s_A + 4\,\Delta s_B = 0$$

$$\Delta s_A = -4\,\Delta s_B$$

Here we see that a downward displacement of one block produces an upward displacement of the other block. Note that Δs_A and Δs_B must have the *same* sign convention in both Eqs. 1 and 2. Taking the time derivative yields

$$v_A = -4v_B = -4(2\text{ m/s}) = -8\text{ m/s} \qquad (2)$$

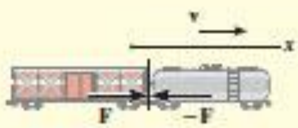
Retaining the negative sign in Eq. 2 and substituting into Eq. 1 yields

$$\Delta s_B = 0.883\text{ m}\downarrow \qquad \textit{Ans.}$$

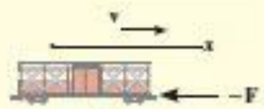
EXAMPLE 15.4

The 15-Mg boxcar A is coasting at 1.5 m/s on the horizontal track when it encounters a 12-Mg tank car B coasting at 0.75 m/s toward it as shown in Fig. 15–8a. If the cars collide and couple together, determine (a) the speed of both cars just after the coupling, and (b) the average force between them if the coupling takes place in 0.8 s.

(a)



(b)



(c)

Fig. 15–8

SOLUTION

Part (a) Free-Body Diagram.* Here we have considered *both* cars as a single system, Fig. 15–8b. By inspection, momentum is conserved in the x direction since the coupling force $\mathbf{F}$ is *internal* to the system and will therefore cancel out. It is assumed both cars, when coupled, move at $\mathbf{v}_2$ in the positive x direction.

Conservation of Linear Momentum.

$$(\overset{+}{\rightarrow}) \qquad m_A(v_A)_1 + m_B(v_B)_1 = (m_A + m_B)v_2$$

$$(15\,000\text{ kg})(1.5\text{ m/s}) - 12\,000\text{ kg}(0.75\text{ m/s}) = (27\,000\text{ kg})v_2$$

$$v_2 = 0.5\text{ m/s}\rightarrow \qquad \textit{Ans.}$$

Part (b). The average (impulsive) coupling force, $\mathbf{F}_{avg}$, can be determined by applying the principle of linear momentum to *either one* of the cars.

Free-Body Diagram. As shown in Fig. 15–8c, by isolating the boxcar the coupling force is *external* to the car.

Principle of Impulse and Momentum. Since $\int F\,dt = F_{avg}\,\Delta t = F_{avg}(0.8\text{ s})$, we have

$$(\overset{+}{\rightarrow}) \qquad m_A(v_A)_1 + \Sigma\int F\,dt = m_A v_2$$

$$(15\,000\text{ kg})(1.5\text{ m/s}) - F_{avg}(0.8\text{ s}) = (15\,000\text{ kg})(0.5\text{ m/s})$$

$$F_{avg} = 18.8\text{ kN} \qquad \textit{Ans.}$$

NOTE: Solution was possible here since the boxcar's final velocity was obtained in Part (a). Try solving for F_{avg} by applying the principle of impulse and momentum to the tank car.

*Only horizontal forces are shown on the free-body diagram.

EXAMPLE 17.4

The pendulum in Fig. 17–7 is suspended from the pin at O and consists of two thin rods, each having a weight of 10 lb. Determine the moment of inertia of the pendulum about an axis passing through (a) point O, and (b) the mass center G of the pendulum.

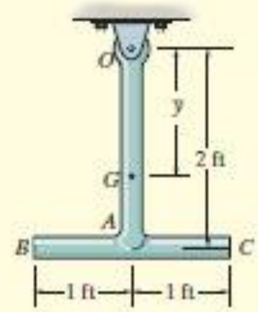


Fig. 17–7

SOLUTION

Part (a). Using the table on the inside back cover, the moment of inertia of rod OA about an axis perpendicular to the page and passing through point O of the rod is $I_O = \frac{1}{3}ml^2$. Hence,

$$(I_{OA})_O = \frac{1}{3}ml^2 = \frac{1}{3}\left(\frac{10\text{ lb}}{32.2\text{ ft/s}^2}\right)(2\text{ ft})^2 = 0.414\text{ slug}\cdot\text{ft}^2$$

This same value can be obtained using $I_G = \frac{1}{12}ml^2$ and the parallel-axis theorem.

$$(I_{OA})_O = \frac{1}{12}ml^2 + md^2 = \frac{1}{12}\left(\frac{10\text{ lb}}{32.2\text{ ft/s}^2}\right)(2\text{ ft})^2 + \left(\frac{10\text{ lb}}{32.2\text{ ft/s}^2}\right)(1\text{ ft})^2$$
$$= 0.414\text{ slug}\cdot\text{ft}^2$$

For rod BC we have

$$(I_{BC})_O = \frac{1}{12}ml^2 + md^2 = \frac{1}{12}\left(\frac{10\text{ lb}}{32.2\text{ ft/s}^2}\right)(2\text{ ft})^2 + \left(\frac{10\text{ lb}}{32.2\text{ ft/s}^2}\right)(2\text{ ft})^2$$
$$= 1.346\text{ slug}\cdot\text{ft}^2$$

The moment of inertia of the pendulum about O is therefore

$$I_O = 0.414 + 1.346 = 1.76\text{ slug}\cdot\text{ft}^2 \qquad \textit{Ans.}$$

Part (b). The mass center G will be located relative to point O. Assuming this distance to be $\bar{y}$, Fig. 17–7, and using the formula for determining the mass center, we have

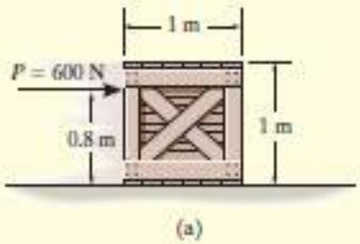
$$\bar{y} = \frac{\Sigma\tilde{y}m}{\Sigma m} = \frac{1(10/32.2) + 2(10/32.2)}{(10/32.2) + (10/32.2)} = 1.50\text{ ft}$$

The moment of inertia I_G may be found in the same manner as I_O, which requires successive applications of the parallel-axis theorem to transfer the moments of inertia of rods OA and BC to G. A more direct solution, however, involves using the result for I_O, i.e.,

$$I_O = I_G + md^2;\quad 1.76\text{ slug}\cdot\text{ft}^2 = I_G + \left(\frac{20\text{ lb}}{32.2\text{ ft/s}^2}\right)(1.50\text{ ft})^2$$
$$I_G = 0.362\text{ slug}\cdot\text{ft}^2 \qquad \textit{Ans.}$$

EXAMPLE 17.7

A uniform 50-kg crate rests on a horizontal surface for which the coefficient of kinetic friction is $\mu_k = 0.2$. Determine the acceleration if a force of $P = 600$ N is applied to the crate as shown in Fig. 17–12*a*.



(a)

SOLUTION

Free-Body Diagram. The force **P** can cause the crate either to slide or to tip over. As shown in Fig. 17–12*b*, it is assumed that the crate slides, so that $F = \mu_k N_C = 0.2N_C$. Also, the resultant normal force $\mathbf{N}_C$ acts at O, a distance x (where $0 < x \leq 0.5$ m) from the crate's center line.* The three unknowns are N_C, x, and a_G.

Equations of Motion.

$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \quad 600\text{ N} - 0.2N_C = (50\text{ kg})a_G \qquad (1)$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_C - 490.5\text{ N} = 0 \qquad (2)$$

$$\circlearrowleft + \Sigma M_G = 0; \quad -600\text{ N}(0.3\text{ m}) + N_C(x) - 0.2N_C(0.5\text{ m}) = 0 \qquad (3)$$

Solving,

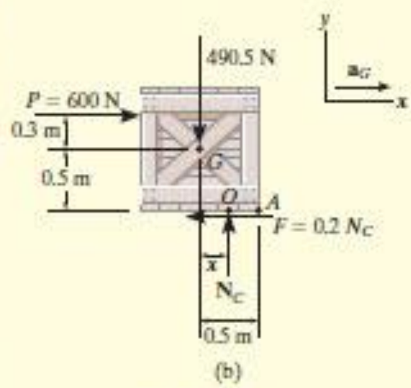
$$N_C = 490.5\text{ N}$$

$$x = 0.467\text{ m}$$

$$a_G = 10.0\text{ m/s}^2 \rightarrow \qquad \textit{Ans.}$$

Since $x = 0.467\text{ m} < 0.5\text{ m}$, indeed the crate slides as originally assumed.

NOTE: If the solution had given a value of $x > 0.5$ m, the problem would have to be reworked since tipping occurs. If this were the case, $\mathbf{N}_C$ would act at the *corner point A* and $F \leq 0.2N_C$.



(b)

Fig. 17–12

*The line of action of $\mathbf{N}_C$ does not necessarily pass through the mass center G ($x = 0$), since $\mathbf{N}_C$ must counteract the tendency for tipping caused by **P**. See Sec. 8.1 of *Engineering Mechanics: Statics*.

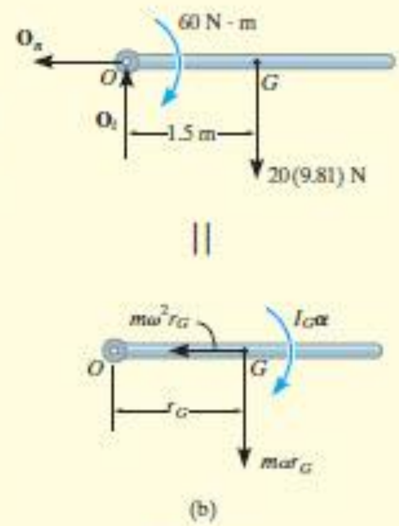
EXAMPLE 17.10

At the instant shown in Fig. 17–16*a*, the 20-kg slender rod has an angular velocity of $\omega = 5$ rad/s. Determine the angular acceleration and the horizontal and vertical components of reaction of the pin on the rod at this instant.

(a)

SOLUTION

Free-Body and Kinetic Diagrams. Fig. 17–16*b*. As shown on the kinetic diagram, point G moves around a circular path and so it has two components of acceleration. It is important that the tangential component $a_t = \alpha r_G$ act downward since it must be in accordance with the rotational sense of $\boldsymbol{\alpha}$. The three unknowns are O_n, O_t, and α.



(b)

Fig. 17–16

Equation of Motion.

$\overset{+}{\leftarrow} \Sigma F_n = m\omega^2 r_G;\qquad O_n = (20\text{ kg})(5\text{ rad/s})^2(1.5\text{ m})$

$+\downarrow \Sigma F_t = m\alpha r_G;\qquad -O_t + 20(9.81)\text{N} = (20\text{ kg})(\alpha)(1.5\text{ m})$

$\circlearrowright + \Sigma M_G = I_G\alpha;\quad O_t(1.5\text{ m}) + 60\text{ N}\cdot\text{m} = \left[\frac{1}{12}(20\text{ kg})(3\text{ m})^2\right]\alpha$

Solving

$$O_n = 750\text{ N}\qquad O_t = 19.05\text{ N}\qquad \alpha = 5.90\text{ rad/s}^2\qquad \textit{Ans.}$$

A more direct solution to this problem would be to sum moments about point O to eliminate $\mathbf{O}_n$ and $\mathbf{O}_t$ and obtain a *direct solution* for α. Here,

$$\circlearrowright + \Sigma M_O = \Sigma(\mathcal{M}_k)_O;\quad 60\text{ N}\cdot\text{m} + 20(9.81)\text{ N}(1.5\text{ m}) =$$

$$\left[\tfrac{1}{12}(20\text{ kg})(3\text{ m})^2\right]\alpha + [20\text{ kg}(\alpha)(1.5\text{ m})](1.5\text{ m})$$

$$\alpha = 5.90\text{ rad/s}^2\qquad \textit{Ans.}$$

Also, since $I_O = \frac{1}{3}ml^2$ for a slender rod, we can apply

$$\circlearrowright + \Sigma M_O = I_O\alpha;\ 60\text{ N}\cdot\text{m} + 20(9.81)\text{ N}(1.5\text{ m}) = \left[\tfrac{1}{3}(20\text{ kg})(3\text{ m})^2\right]\alpha$$

$$\alpha = 5.90\text{ rad/s}^2\qquad \textit{Ans.}$$

NOTE: By comparison, the last equation provides the simplest solution for α and *does not* require use of the kinetic diagram.

EXAMPLE 18.1

The bar shown in Fig. 18–11*a* has a mass of 10 kg and is subjected to a couple moment of $M = 50\ \text{N}\cdot\text{m}$ and a force of $P = 80$ N, which is always applied perpendicular to the end of the bar. Also, the spring has an unstretched length of 0.5 m and remains in the vertical position due to the roller guide at B. Determine the total work done by all the forces acting on the bar when it has rotated downward from $\theta = 0°$ to $\theta = 90°$.

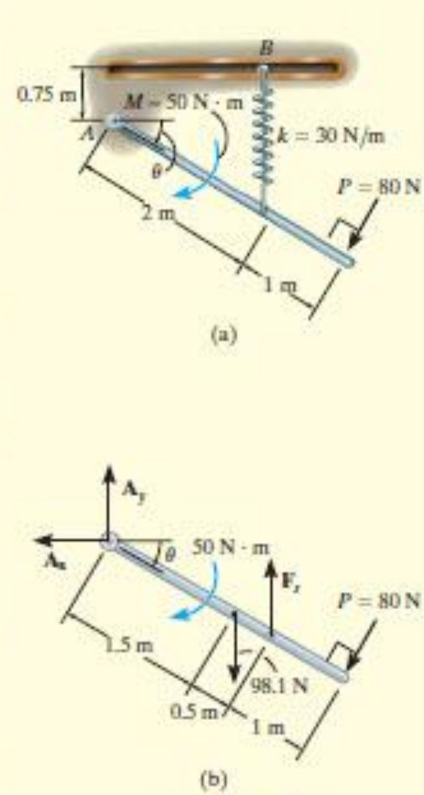


Fig. 18–11

SOLUTION

First the free-body diagram of the bar is drawn in order to account for all the forces that act on it, Fig. 18–11*b*.

Weight *W*. Since the weight $10(9.81)\ \text{N} = 98.1\ \text{N}$ is displaced downward 1.5 m, the work is

$$U_W = 98.1\ \text{N}(1.5\ \text{m}) = 147.2\ \text{J}$$

Why is the work positive?

Couple Moment *M*. The couple moment rotates through an angle of $\theta = \pi/2$ rad. Hence,

$$U_M = 50\ \text{N}\cdot\text{m}(\pi/2) = 78.5\ \text{J}$$

Spring Force F_s. When $\theta = 0°$ the spring is stretched $(0.75\ \text{m} - 0.5\ \text{m}) = 0.25$ m, and when $\theta = 90°$, the stretch is $(2\ \text{m} + 0.75\ \text{m}) - 0.5\ \text{m} = 2.25$ m. Thus,

$$U_s = -\left[\tfrac{1}{2}(30\ \text{N/m})(2.25\ \text{m})^2 - \tfrac{1}{2}(30\ \text{N/m})(0.25\ \text{m})^2\right] = -75.0\ \text{J}$$

By inspection the spring does negative work on the bar since $\mathbf{F}_s$ acts in the opposite direction to displacement. This checks with the result.

Force *P*. As the bar moves downward, the force is displaced through a distance of $(\pi/2)(3\ \text{m}) = 4.712$ m. The work is positive. Why?

$$U_P = 80\ \text{N}(4.712\ \text{m}) = 377.0\ \text{J}$$

Pin Reactions. Forces $\mathbf{A}_x$ and $\mathbf{A}_y$ do no work since they are not displaced.

Total Work. The work of all the forces when the bar is displaced is thus

$$U = 147.2\ \text{J} + 78.5\ \text{J} - 75.0\ \text{J} + 377.0\ \text{J} = 528\ \text{J} \qquad \textit{Ans.}$$

EXAMPLE 18.5

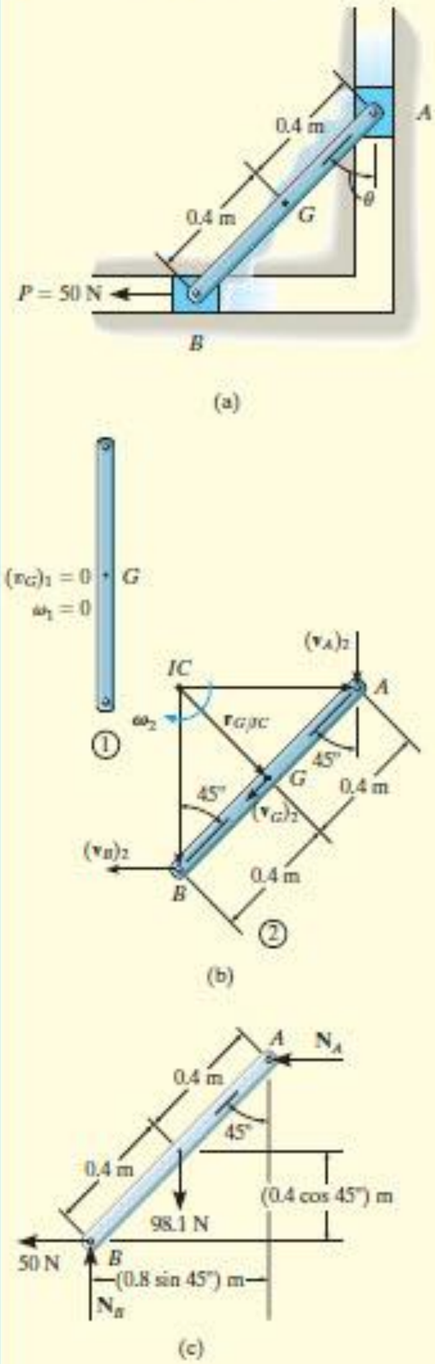


Fig. 18–15

The 10-kg rod shown in Fig. 18–15*a* is constrained so that its ends move along the grooved slots. The rod is initially at rest when $\theta = 0°$. If the slider block at B is acted upon by a horizontal force $P = 50$ N, determine the angular velocity of the rod at the instant $\theta = 45°$. Neglect friction and the mass of blocks A and B.

SOLUTION

Why can the principle of work and energy be used to solve this problem?

Kinetic Energy (Kinematic Diagrams). Two kinematic diagrams of the rod, when it is in the initial position 1 and final position 2, are shown in Fig. 18–15*b*. When the rod is in position 1, $T_1 = 0$ since $(\mathbf{v}_G)_1 = \boldsymbol{\omega}_1 = \mathbf{0}$. In position 2 the angular velocity is $\boldsymbol{\omega}_2$ and the velocity of the mass center is $(\mathbf{v}_G)_2$. Hence, the kinetic energy is

$$T_2 = \tfrac{1}{2}m(v_G)_2^2 + \tfrac{1}{2}I_G\omega_2^2$$

$$= \tfrac{1}{2}(10\text{ kg})(v_G)_2^2 + \tfrac{1}{2}\left[\tfrac{1}{12}(10\text{ kg})(0.8\text{ m})^2\right]\omega_2^2$$

$$= 5(v_G)_2^2 + 0.2667(\omega_2)^2$$

The two unknowns $(v_G)_2$ and ω_2 can be related from the instantaneous center of zero velocity for the rod. Fig. 18–15*b*. It is seen that as A moves downward with a velocity $(\mathbf{v}_A)_2$, B moves horizontally to the left with a velocity $(\mathbf{v}_B)_2$. Knowing these directions, the IC is located as shown in the figure. Hence,

$$(v_G)_2 = r_{G/IC}\omega_2 = (0.4\tan 45°\text{ m})\omega_2$$

$$= 0.4\omega_2$$

Therefore,

$$T_2 = 0.8\omega_2^2 + 0.2667\omega_2^2 = 1.0667\omega_2^2$$

Of course, we can also determine this result using $T_2 = \tfrac{1}{2}I_{IC}\omega_2^2$.

Work (Free-Body Diagram). Fig. 18–15*c*. The normal forces $\mathbf{N}_A$ and $\mathbf{N}_B$ do no work as the rod is displaced. Why? The 98.1-N weight is displaced a vertical distance of $\Delta y = (0.4 - 0.4\cos 45°)$ m; whereas the 50-N force moves a horizontal distance of $s = (0.8\sin 45°)$ m. Both of these forces do positive work. Why?

Principle of Work and Energy.

$$\{T_1\} + \{\Sigma U_{1-2}\} = \{T_2\}$$

$$\{T_1\} + \{W\,\Delta y + Ps\} = \{T_2\}$$

$$\{0\} + \{98.1\text{ N}(0.4\text{ m} - 0.4\cos 45°\text{ m}) + 50\text{ N}(0.8\sin 45°\text{ m})\}$$

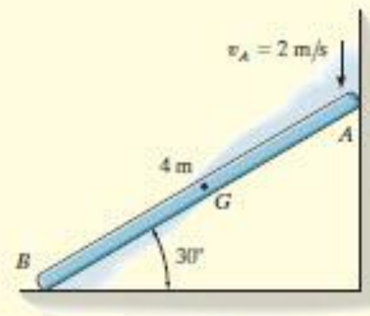
$$= \{1.0667\omega_2^2\text{ J}\}$$

Solving for ω_2 gives

$$\omega_2 = 6.11\text{ rad/s} \circlearrowright \qquad \textit{Ans.}$$

EXAMPLE 19.1

At a given instant the 5-kg slender bar has the motion shown in Fig. 19–3*a*. Determine its angular momentum about point G and about the IC at this instant.



(a)

SOLUTION

Bar. The bar undergoes *general plane motion*. The IC is established in Fig. 19–3*b*, so that

$$\omega = \frac{2 \text{ m/s}}{4 \text{ m} \cos 30^\circ} = 0.5774 \text{ rad/s}$$

$$v_G = (0.5774 \text{ rad/s})(2 \text{ m}) = 1.155 \text{ m/s}$$

Thus,

$$(\circlearrowleft +)\ H_G = I_G\omega = \left[\tfrac{1}{12}(5 \text{ kg})(4 \text{ m})^2\right](0.5774 \text{ rad/s}) = 3.85 \text{ kg}\cdot\text{m}^2/\text{s} \circlearrowright \quad \textit{Ans.}$$

Adding $I_G\omega$ and the moment of mv_G about the IC yields

$$(\circlearrowleft +)\ H_{IC} = I_G\omega + d(mv_G)$$

$$= \left[\tfrac{1}{12}(5 \text{ kg})(4 \text{ m})^2\right](0.5774 \text{ rad/s}) + (2 \text{ m})(5 \text{ kg})(1.155 \text{ m/s})$$

$$= 15.4 \text{ kg}\cdot\text{m}^2/\text{s} \circlearrowright \quad \textit{Ans.}$$

We can also use

$$(\circlearrowleft +)\ H_{IC} = I_{IC}\omega$$

$$= \left[\tfrac{1}{12}(5 \text{ kg})(4 \text{ m})^2 + (5 \text{ kg})(2 \text{ m})^2\right](0.5774 \text{ rad/s})$$

$$= 15.4 \text{ kg}\cdot\text{m}^2/\text{s} \circlearrowright \quad \textit{Ans.}$$

(b)

Fig. 19–3