


Transmissions

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Wednesday Section
2/16/2005



Strategies -> Concepts -> Modules

- Strategies (What are you going to do?)
 - Basic movements on table, how you will score
 - Analysis of times to move, physics independent of your final machine design
 - Graphically – Lines of Motion
- Concepts (How are you going to do it?)
 - Different methods of moving around and scoring
 - Analysis of how concepts interact with the table
 - Graphically – Basic blocky solid models, sketch models
- Modules (How are you going to build it?)
 - Detailed design of what you are doing
 - Analysis of how your machine will work, including power budget, actuator analysis, etc as well as checking for proper strength
 - Graphically – More detailed solid model
- Components (What will you build it with?)
 - Selecting screws, gears, etc
 - Analysis of individual components to prevent failure (eg. bolt analysis, FEA, contact stresses, etc)
 - Graphically – Detailed solid model which allows for direct generation of drawings

Weighted Selection Chart

- Objectively select your strategies, concepts, etc
- Assign weights to each Functional Requirement
- Score each Idea on 0 to 10 scale
- Sum up scores and highest total wins
- Can you combine high scoring ideas to improve one?

	FR 1 (10%)	FR 2 (30%)	FR 3 (60%)	Total
Idea 1	5	5	5	5
Idea 2	1	2	10	<u>6.7</u>
Idea 3	9	6	3	4.5

Types of Transmissions

- Linkages
- Belts, Pulleys and Winches
- Wheels
- Screws
- Gears

Belt Basics

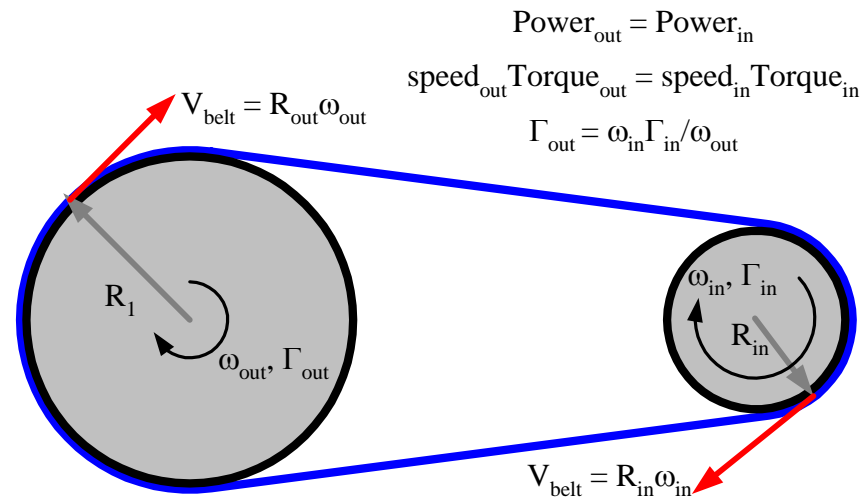
- Basics – Power, Torque, Velocity
- More Details – Spreadsheets!

Toothed Belt:

$$F = \frac{2\Gamma}{D}$$

Flat Belt:

$$F = \frac{T\mu D}{2}$$



Wheel Basics

- Wheels are Linkages!
 - Instant center at contact point on ground or obstacle
- Force Applied
 - Due to traction
 - Ideal maximum is friction
 - Real 2.007 maximum is the motor torque

$$F_{traction} = \mu * N = \frac{2\Gamma}{D}$$

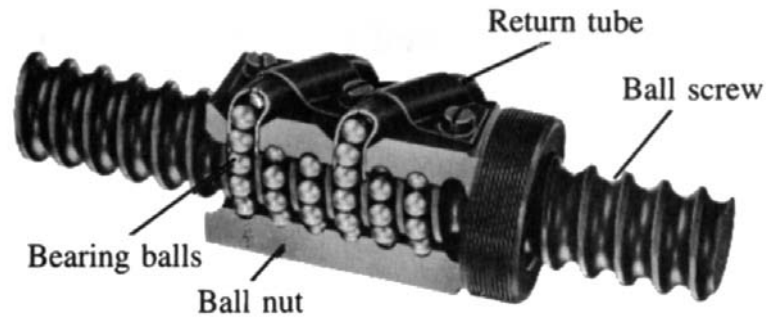
$$V_{vehicle} = \frac{2\omega_{motor}}{D_{wheel}}$$

What do lead screws and gears do?

- Lead or Ball screws – convert rotary motion from a motor to linear motion along the screw
- Gears – can convert rotary motion to linear or rotary motion at the same or a different angle
- Transmit power through changes in force and velocity



Lead and Ball Screws



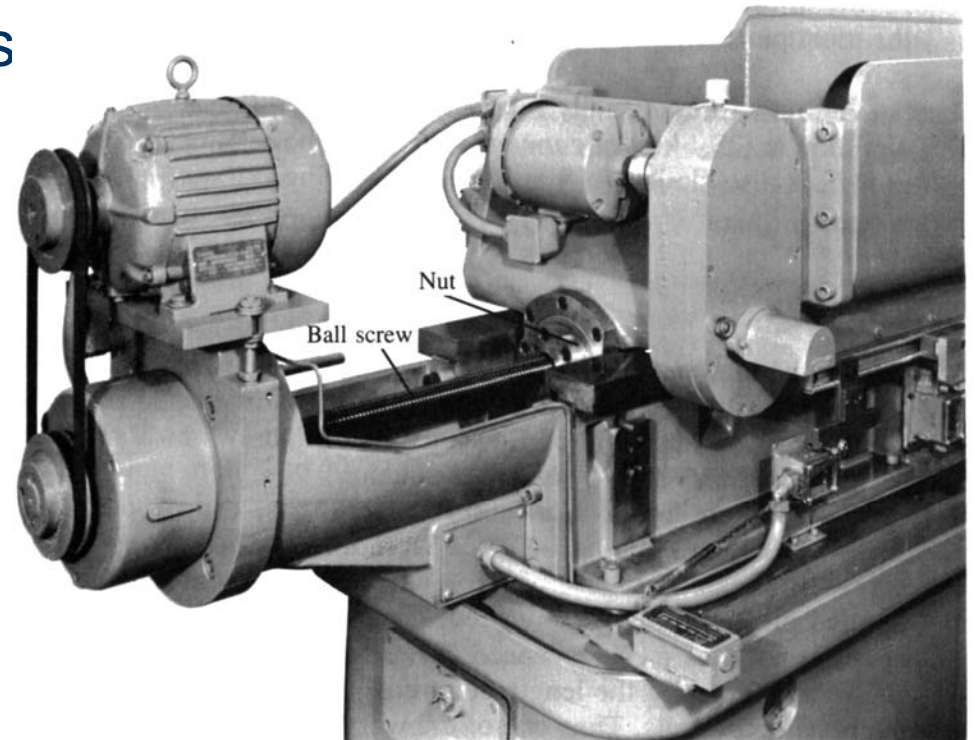
- Lead screw
 - Basically a screw and nut
 - Uses principle of a wedge to drive the nut
 - Lots of friction = low efficiency (30%)
- Ball Screws
 - Same idea as lead screw – replace thread to thread contact with balls
 - Lots less friction = higher efficiency (90%+)

Leads and Ball Screws

- Used in lots of machines – look at a lathe or milling machine
- Critical equation based on conservation of energy:

$$F = \eta \times \frac{2\pi \cdot \Gamma}{lead}$$

- Velocity of carriage: $v = \omega \cdot lead$

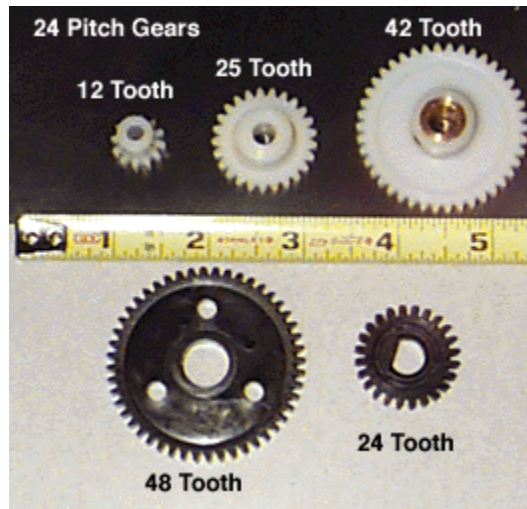


Gear Types

- Spur Gears, Bevel Gears, and the Rack – see your kit



Bevel Gears



Spur Gears



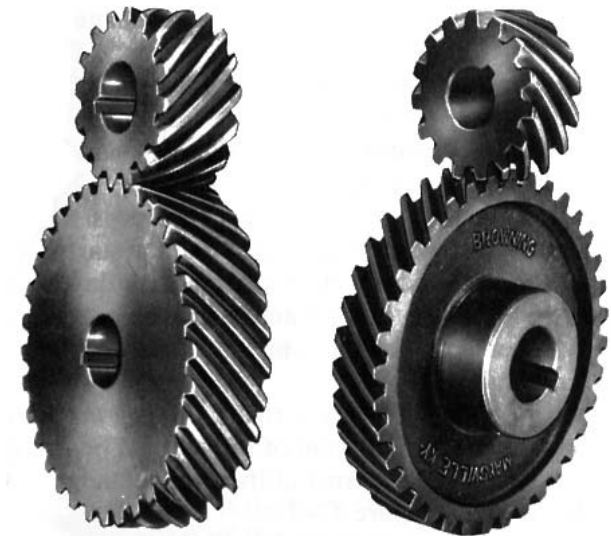
Rack and Pinion

Gear Types

- Helical and Worm Gears aren't in your kit



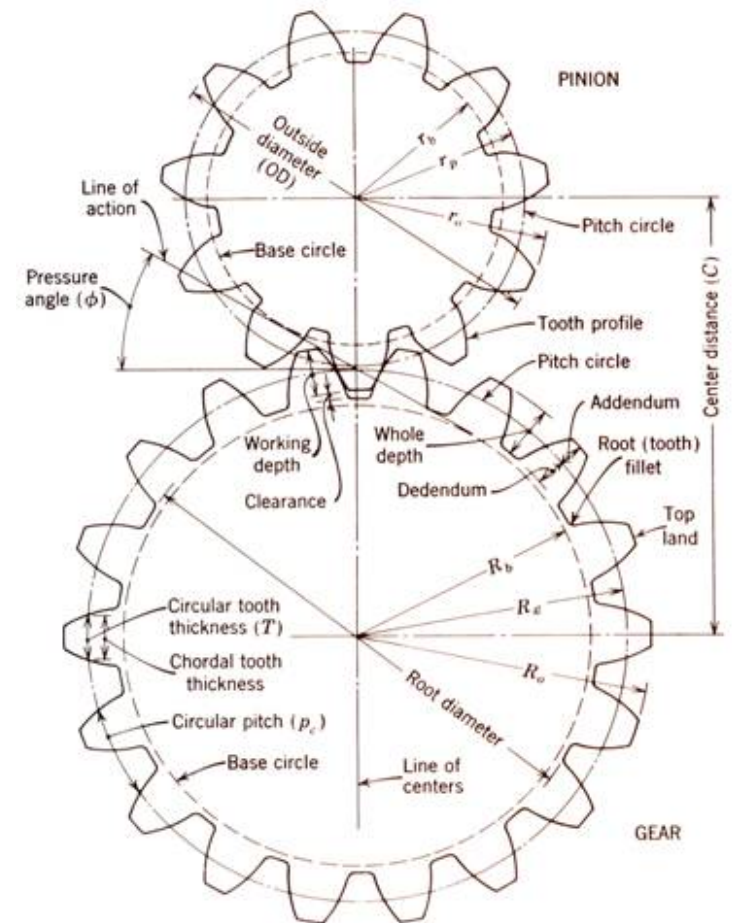
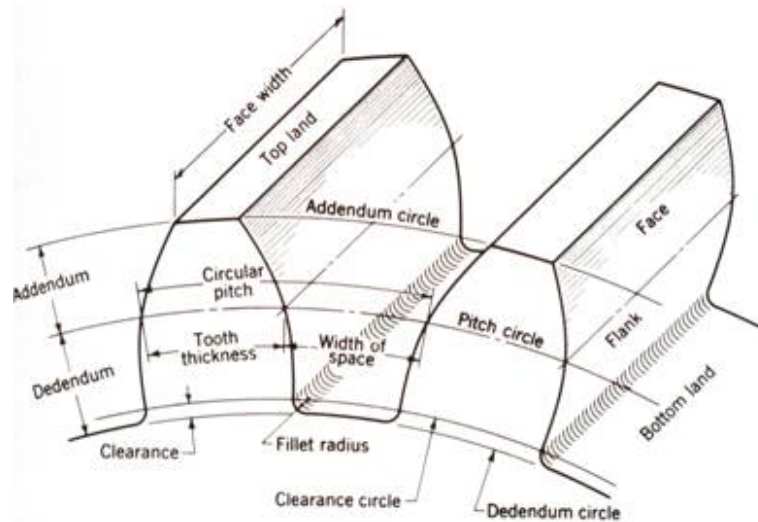
Worm Gears



Helical Gears

Gear Calculations

- Lotsa Lotsa variables!



Gears: Basic Metric Calculations

- Main Variables for First Order:

- N – Number of Teeth
- D – Pitch Diameter
- mod – Module Number
- C – Center Distance

$$D = N * \text{mod}$$

$$C = 0.5 \cdot (D_{\text{pinion}} + D_{\text{gear}})$$

between Two Gears

- Module number for two meshing gears must always be the same!

$$\text{mod}_1 = \text{mod}_2$$

$$\frac{N_1}{N_2} = \frac{D_1}{D_2}$$

Gear Trains: Basic Calculations

- Constraint 1: Tangential Velocity is the Same
 - Gears can't slip so the velocity at the contact point must be moving at the same speed
- Constraint 2: Power is Conserved or Lost!!
 - Remember conservation of energy? Power is transferred over gears but cannot be amplified, only lost in friction.

$$v_1 = v_2$$
$$\omega_1 \cdot \frac{D_1}{2} = \omega_2 \cdot \frac{D_2}{2}$$
$$\frac{\omega_1}{\omega_2} = \frac{D_2}{D_1} = \frac{N_2}{N_1}$$

$$\eta \cdot P_1 = P_2$$
$$\eta \cdot \Gamma_1 \cdot \omega_1 = \Gamma_2 \cdot \omega_2$$
$$\eta \cdot \frac{\Gamma_1}{\Gamma_2} = \frac{\omega_2}{\omega_1} = \frac{N_1}{N_2}$$

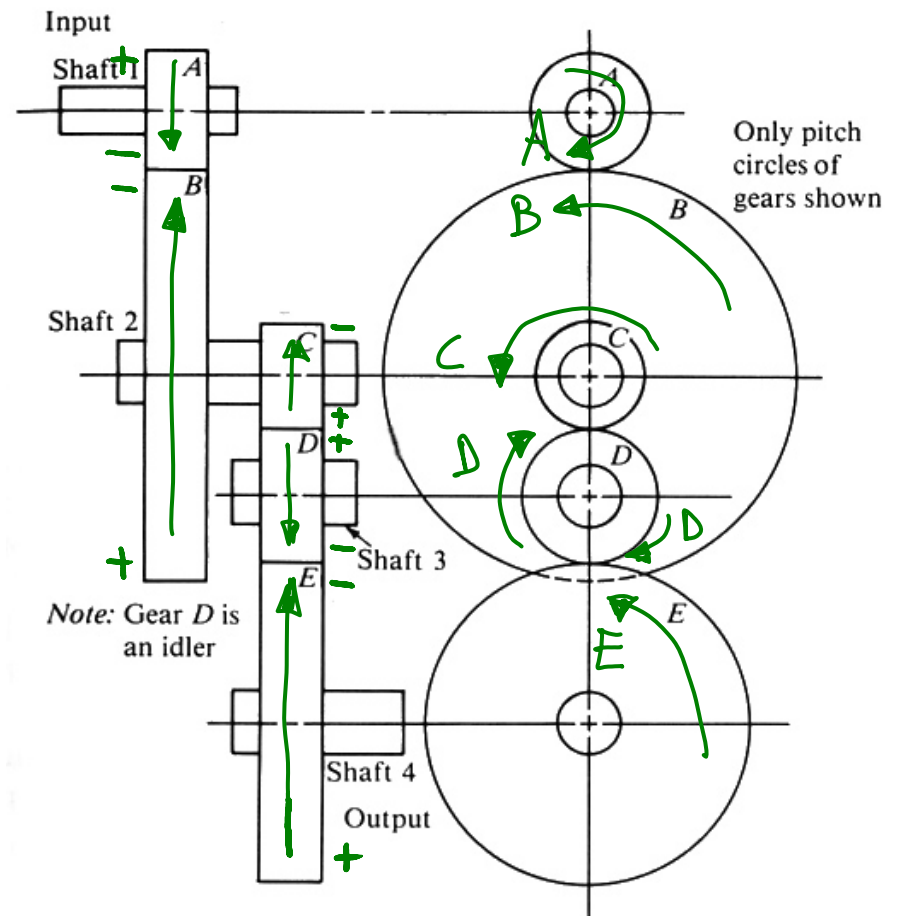
Ideal Gear Train Summary

$$TR = \frac{N_1}{N_2} = \frac{D_1}{D_2} = \frac{\Gamma_1}{\Gamma_2} = \frac{\omega_2}{\omega_1}$$

Include efficiency times torque for non-ideal system!

But what about the minus signs?

- To get proper signs:
 - Follow through with signs or arrows as shown in lecture notes
 - For simple systems, do it graphically with a “virtual belt”



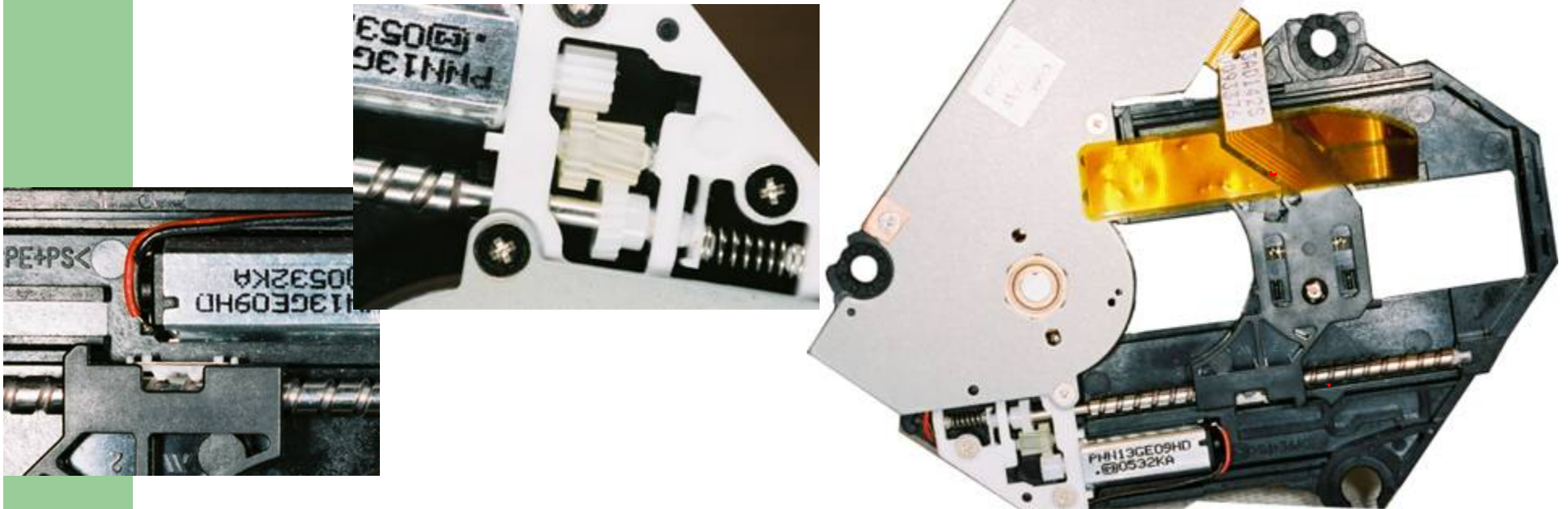
Gears: Selection of Parameters

- To account for other variables, use spreadsheet *spurgears.xls* for conservative estimations of spur gear tooth stress
- It is VERY POSSIBLE to strip gear teeth with your 2.007 motors! You will have to think of ways to prevent a single gear's teeth from being stripped!
- For long life in real products, service factors and many other critical geometry checks need to be performed
 - Consult the *Machinery's Handbook*, or a gear design handbook or AGMA standards
 - Proper tooth design involves more careful assessment of the tooth geometry and loads using the *Lewis Form Factor*
 - Improper lubrication is often the greatest cause of gear failure

	A	B	C
1	Spreadsheet to estimate gear tooth strength		
2	Production gears must be designed using the Lewis Form Factor or FEA		
3	Written 1/18/01 by Alex Slocum		
4	Inputs		
5	Torque, T (in-lb, n-m)	8.8	1.0
6	Pressure angle, f (deg, rad)	20	0.34906585
7	Pitch, P	24	
8	Number of teeth on pinion, Np	12	
9	Number of teeth on gear, Ng	48	
10	Center distance tolerance, Ctol (inches)	0.005	
11	Face width, w (inches)	0.188	
12	Pinion yield stress, sigp (psi)	6000	
13	Gear yield stress, sigg (psi)	6000	
14	Stress concentration factor at tooth root, scf	1	
15			
16	Outputs		
17	Gear ratio, mg	4	
18	Pinion pitch diameter, Dp (inches)	0.500	
19	Gear pitch diameter, Dg (inches)	2.000	
20	Center distance, C (inches)	1.255	
21	Tooth thickness, tt (inches)	0.0654	
22	Addendum, a (inches)	0.0417	
23	Dedendum, b (inches)	0.0520	
24	Pinion tooth force, Fp (lbs)	8.85	
25	Gear tooth force, Fg (lbs)	2.21	
26			
27	Tooth section parameters		
28	Chordal area, Ac (inches^2)	0.0123	
29	First Moment, Q (inches^3)	2.01E-04	
30	Moment of Inertia, I (inches^4)	4.39E-06	
31	Distance Neutral axis to outer fiber, cc (inches)	0.0327	
32			
33	Pinion tooth stresses (stress ratio must be less than 1)		stress ratio
34	Shear of the tooth (F/A) (psi)	719	0.21
35	Bending shear stress (FQ/wl) (psi)	2157	0.62
36	Bending stress (F(b+a)/I) (psi)	6174	1.03
37			

Case Study Exercise – CD Drive

- Cheap portable CD drive uses a tiny DC motor, gear train, and lead screw to move the optical pickup unit (OPU)



What do we know about the setup?

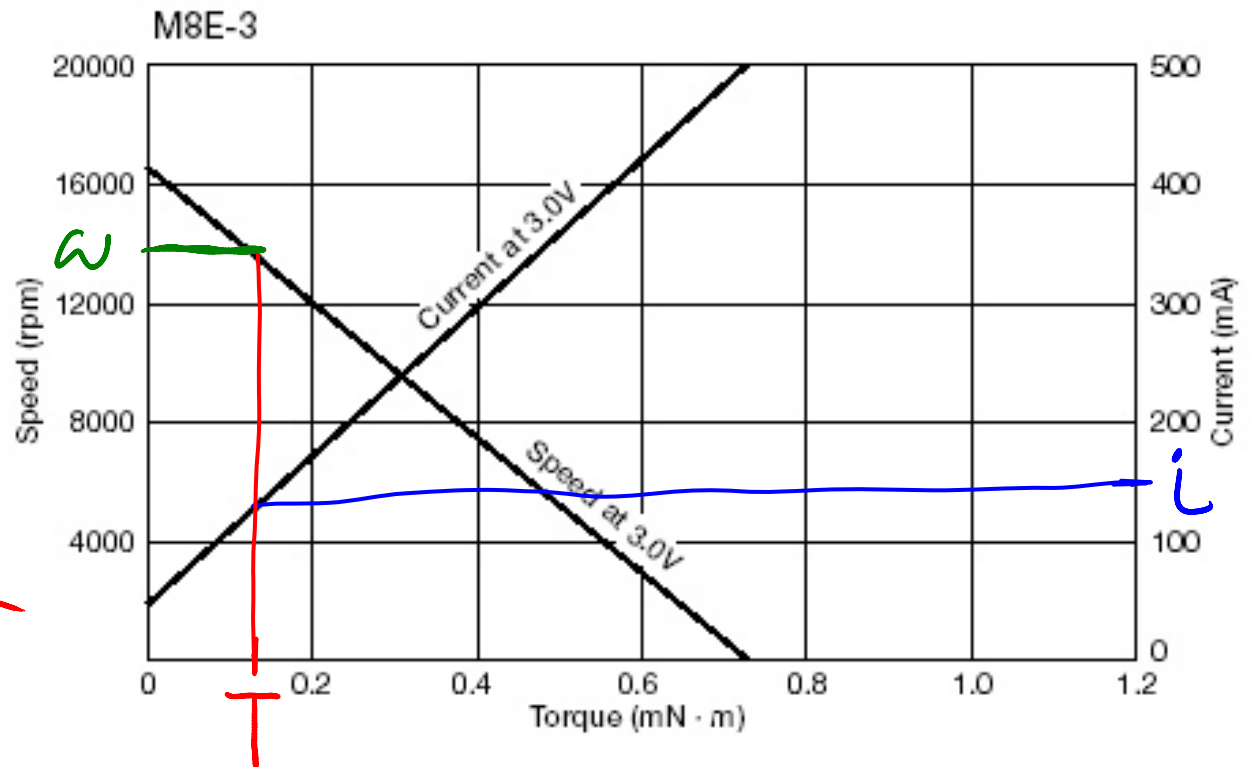
$$V = 3.0 \text{ V}$$

Rated Speed =
13,500 rpm

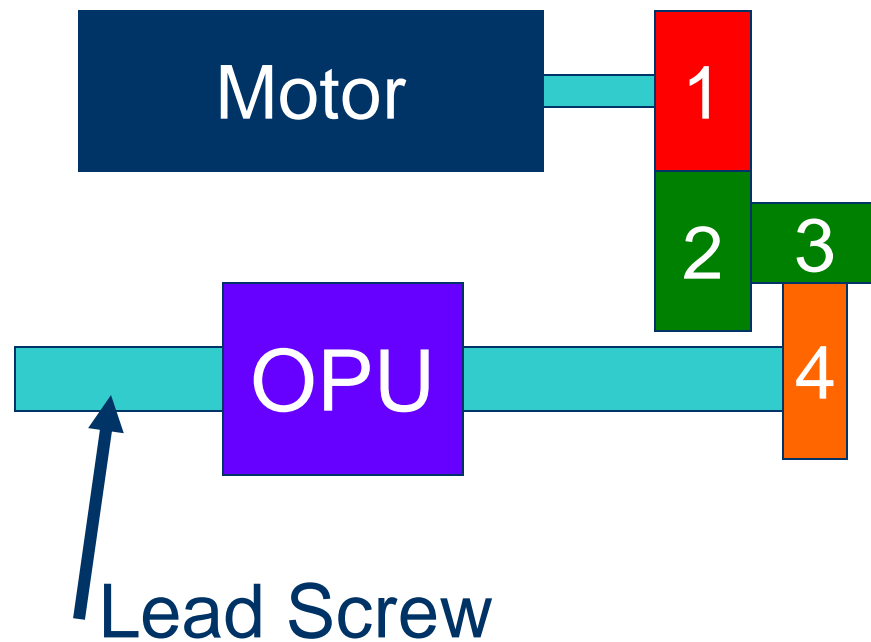
From Chart:

$$T = 0.147 \text{ mN}\cdot\text{m}$$

$$i = 155 \text{ mA}$$



Gear Train Information



$$N1 = 13$$

$$N2 = 19$$

$$N3 = 12$$

$$N4 = 18$$

$$\text{Lead} = 1\text{mm/rev}$$

Gear Train Calcs.



- What is the Gear Train Ratio?

Gear Train Calcs.

- What is the output velocity of the train?

Gear Train Calcs.



- What is the output torque of the train?

Lead Screw Calcs.

- What is the output force of the screw?

Lead Screw Calcs.

- What is the output velocity of the screw?

The Motor in Action



Output Power



- What is the output power of the screw?

System Efficiency



- What is system efficiency (2 ways)?

Gear Calculations

Note Title

2/28/2003

Electrical Power

$$\begin{aligned} P_{\text{Elec}} &= IV \\ &= 155 \text{ mA} \cdot 3 \text{ V} \\ &= 465 \text{ watts} \end{aligned}$$

Mechanical Power

$$\begin{aligned} P_{\text{mech}} &= T \omega \\ &= 0.147 \text{ mNm} \cdot 13500 \frac{\text{rev}}{\text{min}} \cdot \frac{1 \text{ min}}{60 \text{ s}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} \\ &= 0.207 \text{ watts} \end{aligned}$$

Efficiency

$$\eta = \frac{P_{\text{mech}}}{P_{\text{Elec}}} = \frac{0.207}{0.465} = 45\%$$

Gear Train Ratio

$$TR = \frac{N_1}{N_2} \frac{N_3}{N_4} = \frac{13}{19} \cdot \frac{12}{18} = 0.456$$

Output Velocity

$$\omega_2 = \frac{N_1}{N_2} \omega_1 \quad \omega_2 = \omega_3 \quad \omega_4 = \frac{N_3}{N_4} \omega_3$$

$$\begin{aligned} \omega_4 &= \frac{N_1}{N_2} \cdot \frac{N_3}{N_4} \omega_1 = TR \cdot \omega_1 \\ &= 6157.9 \text{ rpm} \end{aligned}$$

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Output Torque

$$\Gamma_4 = \eta_{12} \eta_{34} \cdot \frac{1}{TR} \Gamma_1 \quad \text{let } \eta_{12} = \eta_{34} = 95\%$$

$$= .95 \cdot .95 \cdot \frac{1}{.456} \cdot 0.147 \text{ mN} \cdot \text{m}$$

$$\Gamma_4 = 0.291 \text{ mN} \cdot \text{m}$$

Force on Lead Screw

$$F = \frac{2\pi \Gamma}{\text{lead}} \quad \text{let } \eta = 30\%$$

$$= \frac{2\pi \cdot .3 \cdot .291 \text{ mN} \cdot \text{m}}{.001 \text{ mm/rev}}$$

$$F = 548 \text{ mN}$$

Velocity of Screw

$$v = \omega \cdot \text{lead} \\ = 6157.9 \text{ rpm} \cdot 1 \text{ mm/rev} \cdot \frac{1 \text{ min}}{60 \text{ s}}$$

$$v = 102.6 \text{ mm/sec}$$

Power out on Shaft

$$P_{\text{out}} = F \cdot v = 548 \text{ mN} \cdot 102.6 \text{ mm/sec} \\ = .056 \text{ W}$$

Efficiency - 2 Ways

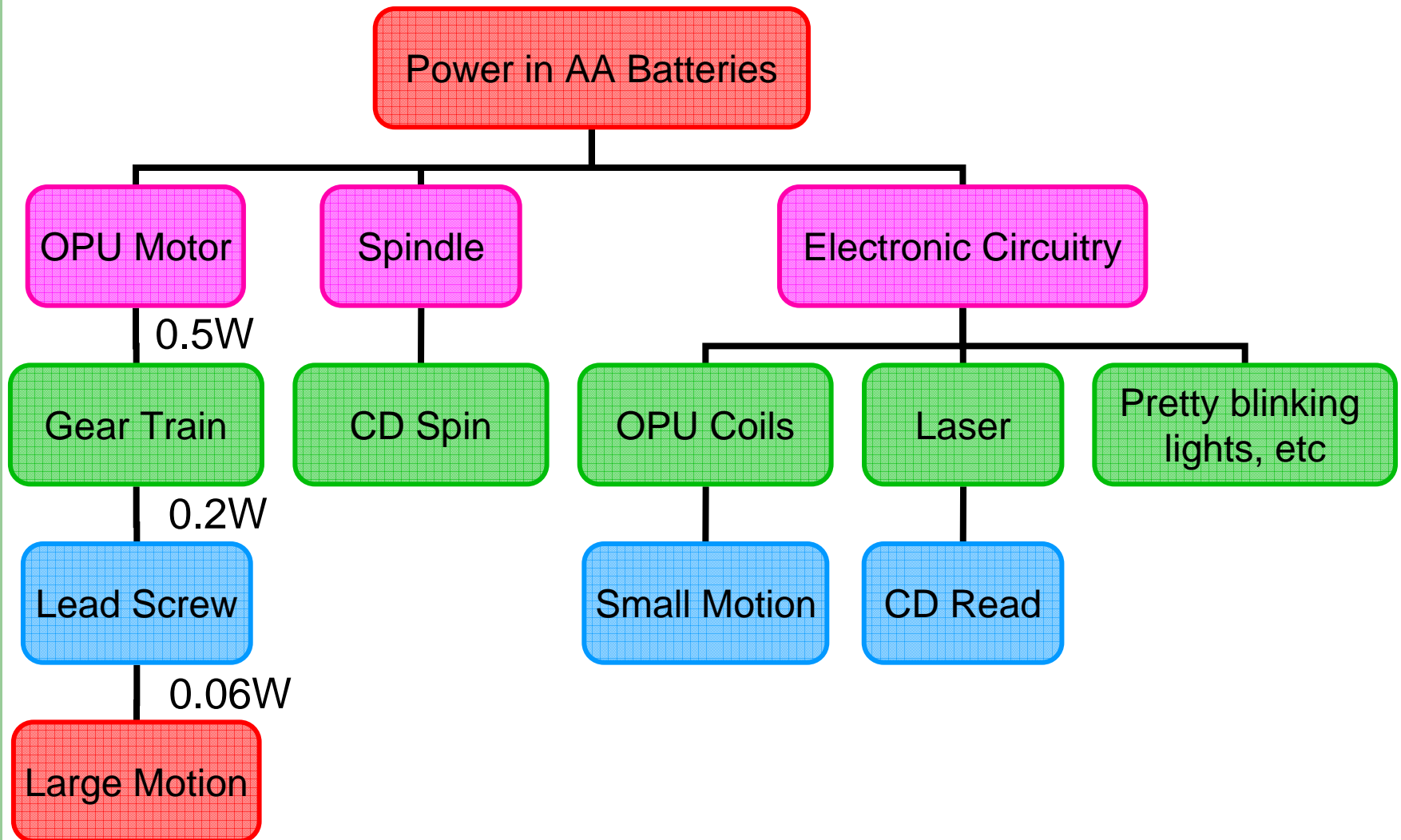
$$\eta = \frac{P_{\text{out}}}{P_{\text{elec}}} = \frac{.056}{.465} = 12\%$$

$$\eta = \eta_{\text{mech}} \eta_{\text{gear1}} \eta_{\text{gear2}} \eta_{\text{thread}} = 12\%$$

Power and Energy Budgets

- How much power are you using at one time?
 - $P_{\text{total}} = P_{\text{motor}} + P_{\text{spring}} + P_{\text{solenoid}} + P_{\text{piston}}$
 - $P_{\text{battery}} \geq P_{\text{motor}} + P_{\text{solenoid}} + P_{\text{piston}}$
- How much energy are you using?
 - Energy cells > total energy required by system
 - Energy = Power * Time

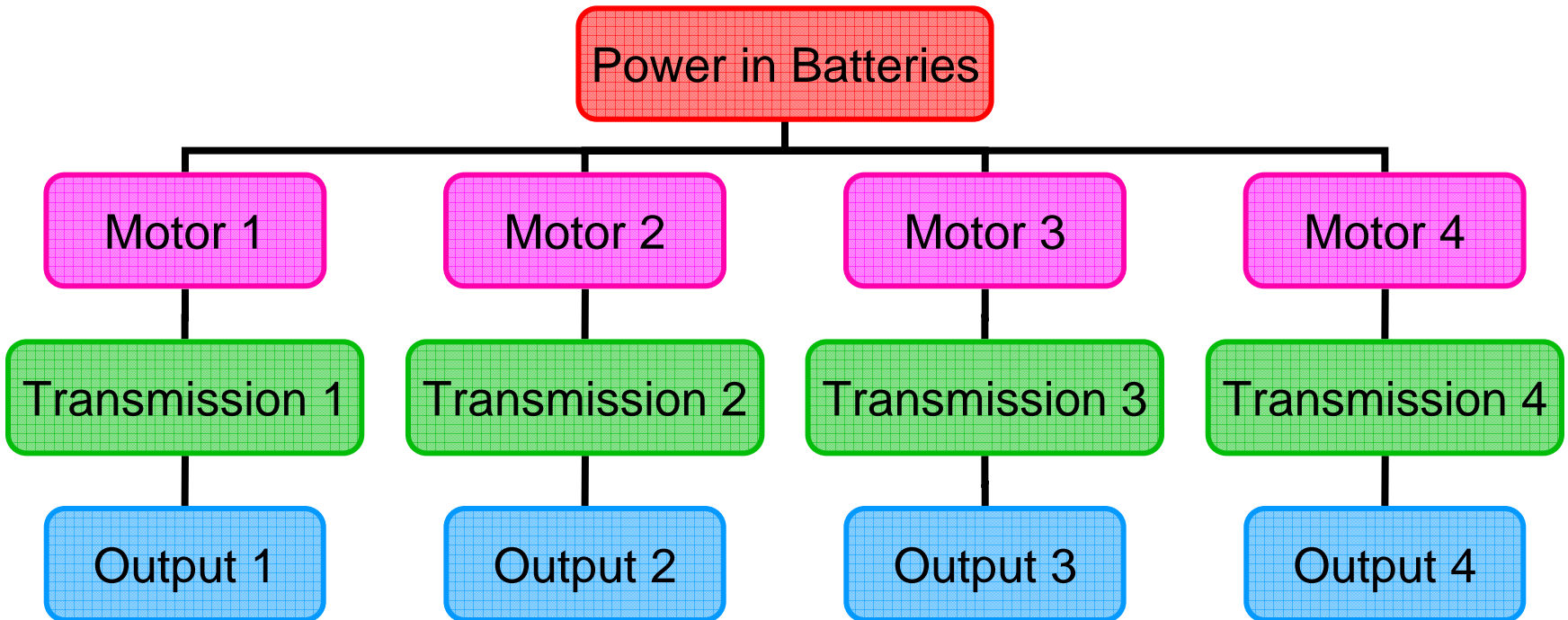
CD Drive Power Budget



Power Elements

- Motors
 - Torque * angular speed or force * linear speed
- Spring
 - Torsional
 - Force * distance/time
 - Extension
 - Force * distance/time
- Solenoid
 - Force * stroke/time
- Batteries
 - Current * voltage
- Piston
 - Force * distance/time

Power Budget Structure



To do a complete power budget, you should be able to fill in force, torque, velocity, power and energy in each of the above blocks, as required for your design. Also, you may have additional blocks for triggers which you will have to consider.