8.5.4 Gleason Zerol Spiral Bevel Gears

When the spiral angle $\beta_m = 0$, the bevel gear is called a Zerol bevel gear. The calculation equations of **Table 8-2** for Gleason straight bevel gears are applicable. They also should take care again of the rule of hands; left and right of a pair must be matched. **Figure 8-12** is left-hand Zerol bevel gear.

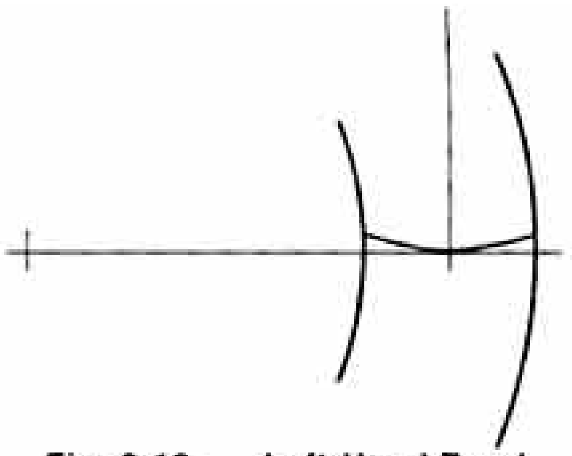


Fig. 8-12 Left-Hand Zerol Bevel Gear

Table 8-6 The Calculations of Spiral Bevel Gears of the Gleason System

No.	Item	Symbol	Formula	Example	
				Pinion	Gear
1	Shaft Angle	Σ		90°	
2	Outside Radial Module	m		3	
3	Normal Pressure Angle	α_n		20°	
4	Spiral Angle	β_m		35°	
5	Number of Teeth and Spiral Hand	$z_1,\ z_2$		20 (L)	40 (R)
6	Radial Pressure Angle	α_t	$\tan^{-1}\left(\frac{\tan\alpha_n}{\cos\beta_m}\right)$	23.95680	
7	Pitch Diameter	d	zm	60	120
8	Pitch Cone Angle	δ_1 δ_2	$\tan^{-1}\left(\frac{\sin\Sigma}{\frac{z_2}{z_1}+\cos\Sigma}\right)$ $\Sigma-\delta_1$	26.56505°	63.43495°
9	Cone Distance	R_e	$\frac{d_2}{2\sin\delta_2}$	67.08204	
10	Face Width	b	It should be less than $R_e/3$ or $10m$	20	
11	Addendum	h_{a1} h_{a2}	$1.700m-h_{a2}$ $0.460m+\frac{0.390m}{\left(\frac{z_2\cos\delta_1}{z_1\cos\delta_2}\right)}$	3.4275	1.6725
12	Dedendum	h_f	$1.888m-h_a$	2.2365	3.9915
13	Dedendum Angle	θ_f	$\tan^{-1}(h_f/R_e)$	1.90952°	3.40519°
14	Addendum Angle	θ_{a1} θ_{a2}	θ_{f2} θ_{f1}	3.40519°	1.90952°
15	Outer Cone Angle	δ_a	$\delta+\theta_a$	29.97024°	65.34447°
16	Root Cone Angle	δ_f	$\delta-\theta_f$	24.65553°	60.02976°
17	Outside Diameter	d_a	$d+2h_a\cos\delta$	66.1313	121.4959
18	Pitch Apex to Crown	X	$R_e\cos\delta-h_a\sin\delta$	58.4672	28.5041
19	Axial Face Width	X_b	$\frac{b\cos\delta_a}{\cos\theta_a}$	17.3563	8.3479
20	Inner Outside Diameter	d_i	$d_a-\frac{2b\sin\delta_a}{\cos\theta_a}$	46.1140	85.1224

SECTION 9 WORM MESH

The worm mesh is another gear type used for connecting skew shafts, usually 90º. See **Figure 9-1**. Worm meshes are characterized by high velocity ratios. Also, they offer the advantage of higher load capacity associated with their line contact in contrast to the point contact of the crossed-helical mesh.

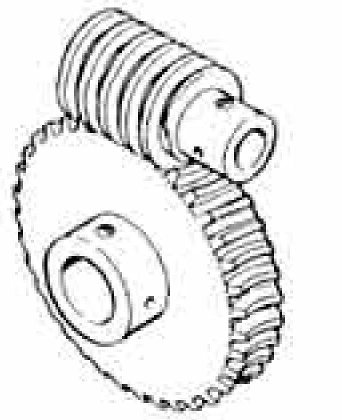


Fig. 9-1 Typical Worm Mesh

9.1 Worm Mesh Geometry

Although the worm tooth form can be of a the most popular is equivalent to a V-type screw thread, as in **Figure 9-1**. The mating worm gear teeth have a helical lead. (**Note**: The name "worm wheel" is often used interchangeably with "worm gear".) A central section of the mesh, taken through the worm's axis and perpendicular to the worm gear's axis, as shown in **Figure 9-2**, reveals a rack-type tooth of the worm, and a curved involute tooth form for the worm gear. However, the involute features are only true for the central section. Sections on either side of the worm axis reveal non-symmetric and non-involute tooth profiles. Thus, a worm gear mesh is not a true involute mesh. Also, for conjugate action, the center distance of the mesh must be an exact duplicate of that used in generating the worm gear.

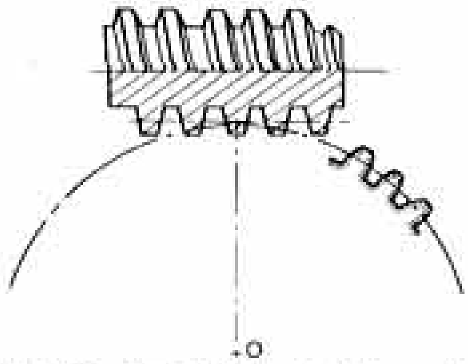


Fig. 9-2 Central Section of a Worm and Worm Gear

To increase the length-of-action, the worm gear is made of a throated shape to wrap around the worm.

9.1.1 Worm Tooth Proportions

Worm tooth dimensions, such as addendum, dedendum, pressure angle, etc., follow the same standards as those for spur and helical gears. The standard values apply to the central section of the mesh. See **Figure 9-3a**. A high pressure angle is favored and in some applications values as high as 25º and 30º are used.

9.1.2 Number of Threads

The worm can be considered resembling a helical gear with a high helix angle. For extremely high helix angles, there is one continuous tooth or thread. For slightly smaller angles, there can be two, three or even more threads. Thus, a worm is characterized by the number of threads, Z_W

9.1.3 Pitch Diameters, Lead and Lead Angle

Referring to **Figure 9-3**:

Pitch diameter of worm $= d_W = \frac{Z_W P_n}{\pi \sin\gamma}$ (**9-1**)

Pitch diameter of worm gear $= d_g = \frac{Z_g P_n}{\pi \cos\gamma}$ (**9-2**)

where:

Z_W = number of threads of worm; Z_g = number of teeth in worm gear

L = lead of worm $= z_w p_x = \frac{z_w p_n}{\cos\gamma}$

γ = lead angle $= \tan^{-1}\left(\frac{z_w m}{d_w}\right) = \sin^{-1}\left(\frac{z_w p_n}{\pi d_w}\right)$

$P_n = P_x \cos\gamma$

9.1.4 Center Distance

$$C = \frac{d_w + D_g}{2} = \frac{p_n}{2\pi}\left(\frac{z_g}{\cos\gamma} + \frac{z_w}{\sin\gamma}\right) \qquad (9\text{-}3)$$

9.2 Cylindrical Worm Gear Calculations

Cylindrical worms may be considered cylindrical type gears with screw threads. Generally, the mesh has a 90º shaft angle. The number of threads in the worm is equivalent to the number of teeth in a gear of a screw type gear mesh. Thus, a one-thread worm is equivalent to a one-tooth gear; and two-threads equivalent to two-teeth, etc. Referring to **Figure 9-4**, for a lead angle γ, measured on the pitch cylinder, each rotation of the worm makes the thread advance one lead.

There are four worm tooth profiles in JIS B 1723, as defined below.

Type I Worm: This worm tooth profile is trapezoid in the radial or axial plane.

Type II Worm: This tooth profile is trapezoid viewed in the normal surface.

Type III Worm:This worm is formed by a cutter in which the tooth profile is trapezoid

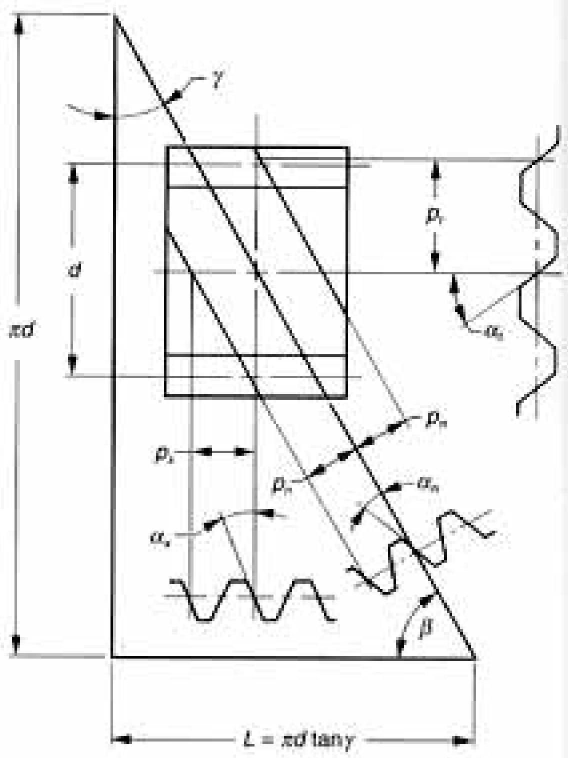
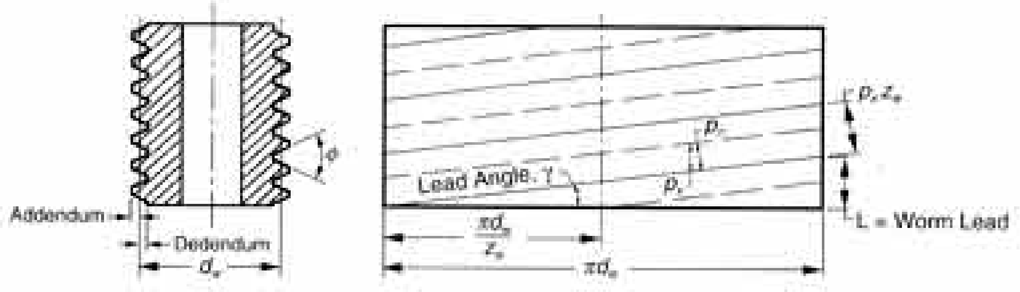


Fig. 9-4 Cylindrical Worm (Right-Hand)



(a) Tooth Proportion of Central Section
(b) Development of Worm's Pitch Cylinder; Two Thread Example: $z_w = 2$
Fig. 9-3 Worm Tooth Proportions and Geometric Relationships

form viewed from the radial surface or axial plane set at the lead angle. Examples are milling and grinding profile cutters.

Type IV Worm: This tooth profile is involute as viewed from the radial surface or at the lead angle. It is an involute helicoid, and is known by that name.

Type lll worm is the most popular. In this type, the normal pressure angle α_n has the tendency to become smaller than that of the cutter, α_c

Per JIS, Type lll worm uses a radial module m_t, and cutter pressure angle α_c = 20º as the module and pressure angle. A special worm hob is required to cut a Type lll worm gear.

Standard values of radial module, m_t, are presented in **Table 9-1**.

Table 9-1 Radial Module of Cylindrical Worm Gears

1	1.25	1.60	2.00	2.50	3.15	4.00	5.00
6.30	8.00	10.00	12.50	16.00	20.00	25.00	-

Because the worm mesh couples nonparallel and nonintersecting axes, the radial surface of the worm, or radial cross section, is the same as the normal surface of the worm gear. Similarly, the normal surface of the worm is the radial surface of the worm gear. The common surface of the worm and worm gear is the normal surface. Using the normal module, m_n is most popular. Then, an ordinary hob can be used to cut the worm gear.

Table 9-2 presents the relationships among worm and worm gear radial surfaces, normal surfaces, axial surfaces, module, pressure angle, pitch and lead.

Table 9-2 The Relations of Cross Sections of Worm Gears

Worm		
Axial Surface	**Normal Surface**	**Radial Surface**
$m_x = \frac{m_n}{\cos\gamma}$	m_n	$m_t = \frac{m_n}{\sin\gamma}$
$\alpha_x = \tan^{-1}\left(\frac{\tan\alpha_n}{\cos\gamma}\right)$	α_n	$\alpha_t = \tan^{-1}\left(\frac{\tan\alpha_n}{\sin\gamma}\right)$
$p_x = \pi m_x$	$p_n = \pi m_n$	$p_t = \pi m_t$
$L = \pi m_x z_w$	$L = \frac{\pi m_n z_w}{\cos\gamma}$	$L = \pi m_t z_w \tan\gamma$
Radial Surface	**Normal Surface**	**Axial Surface**
Worm Gear		

NOTE: The Radial Surface is the plane perpendicular to the axis.

Reference to **Figure 9-4** can help the understanding of the relationships in **Table 9-2**. They are similar to the relations in **Formulas (6-11)** and **(6-12)** that the helix angle β be substituted by (90º - γ) We can consider that a worm with lead angle γ is almost the same as a screw gear with helix angle (90º - γ).

9.2.1 Axial Module Worm Gears

Table 9-3 presents the equations, for dimensions shown in **Figure 9-5**, for worm gears with axial module, m_x and normal pressure angle α_n = 20º.

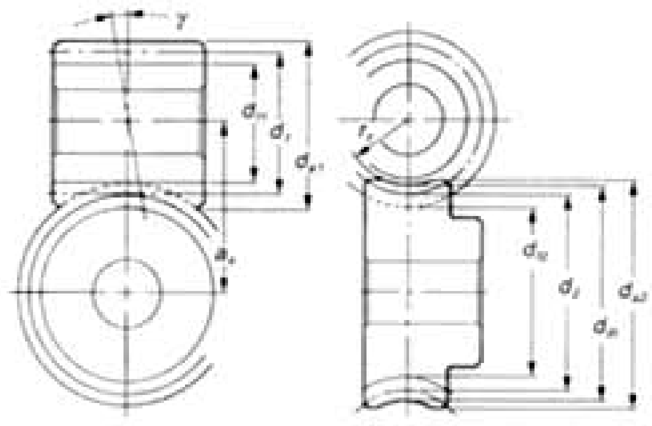


Fig. 9-5 Dimensions of Cylindrical Worm Gears

9.2.2 Normal Module System Worm Gears

The equations for normal module system worm gears are based on a normal module, m_n and normal pressure angle, an = 20º See **Table 9-4**.

9.3 Crowning Of The Worm Gear Tooth

Crowning is critically important to worm gears (worm wheels) Not only can it eliminate abnormal tooth contact due to incorrect assembly, but it also provides for the forming of an oil film, which enhances the lubrication effect of the mesh. This can favorably impact endurance and transmission efficiency of the worm mesh There are four methods of crowning worm gears:

1. Cut Worm Gear With A Hob Cutter Of Greater Pitch Diameter Than The Worm.

A crownless worm gear results when it is made by using a hob that has an identical pitch diameter as that of the worm. This crownless worm gear is very difficult to assemble correctly. Proper tooth contact and a complete oil film are usually not possible.

However, it is relatively easy to obtain a crowned worm gear by cutting it with a hob whose pitch diameter is slightly larger than that of the worm. This is shown in **Figure 9-6**. This creates teeth contact in the center region with space for oil film formation.

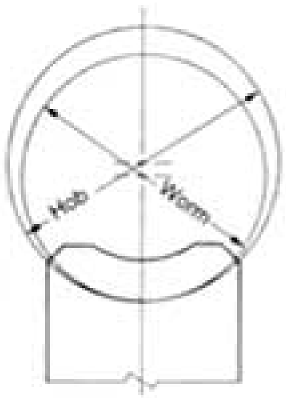


Fig. 9-6 The Method of Using a Greater Diameter Hob

2. Recut With Hob Center Distance Adjustment.

The first step is to cut the worm gear at standard center distance. This results in no crowning. Then the worm gear is finished with the same hob by recutting with the hob axis shifted parallel to the worm gear axis by ±Δh. This results in a crowning effect, shown in **Figure 9-7**.

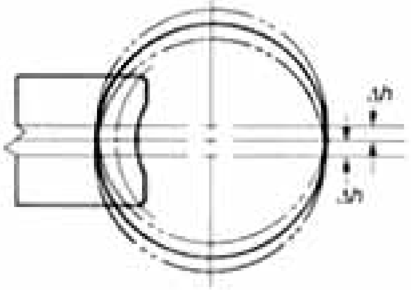


Fig. 9-7 Offsetting Up or Down

3. Hob Axis Inclining Δθ From Standard Position.

In standard cutting, the hob axis is oriented at the proper angle to the worm gear axis. After that, the hob axis is shifted slightly left and then right, Δθ, in a plane parallel to the worm gear axis, to cut a crown effect on the worm gear tooth. This is shown in **Figure 9-8.**

Only method **1** is popular. Methods **2** and **3** are seldom used.

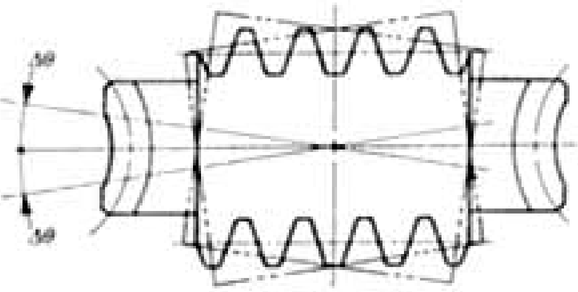


Fig. 9-8 Inclining Right or Left

Table 9-3 The Calculations of Axial Module System Worm Gears (See Figure 9-5)

No.	Item	Symbol	Formula	Example	
				Worm	Wheel
1	Axial Module	m_x		3	
2	Normal Pressure Angle	α_n		20°	
3	No. of Threads, No. of Teeth	z_w, z_2		∇	30 (R)
4	Standard Pitch Diameter	d_1 d_2	$Q\,m_x$ **Note 1** $z_2 m_x$	44.000	90.000
5	Lead Angle	γ	$\tan^{-1}\left(\frac{m_x z_w}{d_1}\right)$	7.76517°	
6	Coefficient of Profile Shift	x_{a2}		–	0
7	Center Distance	a_x	$\frac{d_1 + d_2}{2} + x_{a2} m_x$	67.000	
8	Addendum	h_{a1} h_{a2}	$1.00 m_x$ $(1.00 + x_{a2}) m_x$	3.000	3.000
9	Whole Depth	h	$2.25 m_x$	6.750	
10	Outside Diameter	d_{a1} d_{a2}	$d_1 + 2h_{a1}$ $d_2 + 2h_{a2} + m_x$ **Note 2**	50.000	99.000
11	Throat Diameter	d_{th}	$d_2 + 2h_{a2}$	–	96.000
12	Throat Surface Radius	r_i	$\frac{d_1}{2} - h_{a1}$	–	19.000
13	Root Diameter	d_{f1} d_{f2}	$d_{a1} - 2h$ $d_{th} - 2h$	36.500	82.500

∇ Double-Threaded Right-Hand Worm

Note 1: Diameter Factor, Q, means pitch diameter of worm, d_1, over axial module, m_x

$$Q = \frac{d_1}{m_x}$$

Note 2: There are several calculation methods of worm outside diameter d_{a2} besides those in **Table 9-3**.

Note 3: The length of worm with teeth, b_1, would be sufficient if:

$$b_1 = \pi\, m_x(4.5 + 0.02 z_2)$$

Note 4: Working blank width of worm gear $b_e = 2m_x\,(Q + 1)^{1/2}$. So the actual blank width of $b \geq b_e + 1.5 m_x$ would be enough.

Table 9-4 The Calculations of Normal Module System Worm Gears

No.	Item	Symbol	Formula	Example	
				Worm	Worm Gear
1	Normal Module	m_n		3	
2	Normal Pressure Angle	α_n		20°	
3	No. of Threads, No. of Teeth	z_w, z_2		∇	30 (R)
4	Pitch Diameter of Worm	d_1		44.000	–
5	Lead Angle	γ	$\sin^{-1}\left(\frac{m_n z_w}{d_1}\right)$	7.83748°	
6	Pitch Diameter of Worm Gear	d_2	$\frac{z_2 m_n}{\cos\gamma}$	–	90.8486
7	Coefficient of Profile Shift	x_{n2}		–	–0.1414
8	Center Distance	a_x	$\frac{d_1 + d_2}{2} + x_{n2} m_n$	67.000	
9	Addendum	h_{a1} h_{a2}	$1.00 m_n$ $(1.00 + x_{n2}) m_n$	3.000	2.5758
10	Whole Depth	h	$2.25 m_n$	6.75	
11	Outside Diameter	d_{a1} d_{a2}	$d_1 + 2h_{a1}$ $d_2 + 2h_{a2} + m_n$	50.000	99.000
12	Throat Diameter	d_{th}	$d_2 + 2h_{a2}$	–	96.000
13	Throat Surface Radius	r_i	$\frac{d_1}{2} - h_{a1}$	–	19.000
14	Root Diameter	d_{f1} d_{f2}	$d_{a1} - 2h$ $d_{th} - 2h$	36.500	82.500

∇ Double-Threaded Right-Hand Worm

Note: All notes are the same as those of **Table 9-3**.

4. Use A Worm With A Larger Pressure Angle Than The Worm Gear.

This is a very complex method, both theoretically and practically. Usually, the crowning is done to the worm gear, but in this method the modification is on the worm. That is, to change the pressure angle and pitch of the worm without changing the pitch line parallel to the axis, in accordance with the relationships shown in **Equations 9-4**:

$$p_x \cos\alpha_x = p_x' \cos\alpha_x' \qquad \textbf{(9-4)}$$

In order to raise the pressure angle from before change, α_x' to after change, α_x, it is necessary to increase the axial pitch, P_x' to a new value, P_x per **Equation (9-4)**. The amount of crowning is represented as the space between the worm and worm gear at the meshing point A in **Figure 9-9**. This amount may be approximated by the following equation:

Amount of Crowning

$$= k \frac{P_x - P_x'}{P_x'} \frac{d_1}{2} \qquad (9\text{-}5)$$

where:

d_1 = Pitch diameter of worm

k = Factor from **Table 9-5** and **Figure 9-10**

P_x = Axial pitch after change

p_x'= Axial pitch before change

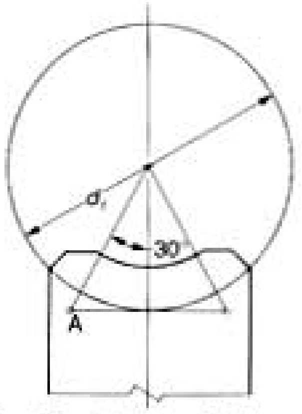


Fig. 9-9 Position A is the Point of Determinig Crowning Amount

Table 9-5 The Value of Factor *k*

α_x	14.5°	17.5°	20°	22.5°
k	0.55	0.46	0.41	0.375

An example of calculating worm crowning is shown in **Table 9-6**.

Because the theory and equations of these methods are so complicated, they are beyond the scope of this treatment. Usually, all stock worm gears are produced with crowning.

Table 9-6 The Calculation of Worm Crowning

No.	Item	Symbol	Formula	Example
	Before Crowning			
1	Axial Module	m_x'		3
2	Normal Pressure Angle	α_n'		20°
3	Number of Threads of Worm	z_w		2
4	Pitch Diameter of Worm	d_1		44.000
5	Lead Angle	γ'	$\tan^{-1}\left(\frac{m_x' z_w}{d_1}\right)$	7.765166°
6	Axial Pressure Angle	α_x'	$\tan^{-1}\left(\frac{\tan\alpha_n'}{\cos\gamma'}\right)$	20.170236°
7	Axial Pitch	p_x'	$\pi m_x'$	9.424778
8	Lead	L'	$\pi m_x' z_w$	18.849556
9	Amount of Crowning	C_R'	*	0.04
10	Factor (k)	k	From **Table 9-5**	0.41
	After Crowning			
11	Axial Pitch	p_x	$p_x'\left(\frac{2C_R}{kd_1}+1\right)$	9.466573
12	Axial Pressure Angle	α_x	$\cos^{-1}\left(\frac{p_x'}{p_x}\cos\alpha_x'\right)$	20.847973°
13	Axial Module	m_x	$\frac{p_x}{\pi}$	3.013304
14	Lead Angle	γ	$\tan^{-1}\left(\frac{m_x z_w}{d_1}\right)$	7.799179°
15	Normal Pressure Angle	α_n	$\tan^{-1}(\tan\alpha_x \cos\gamma)$	20.671494°
16	Lead	L	$\pi m_x z_w$	18.933146

* It should be determined by considering the size of tooth contact surface.

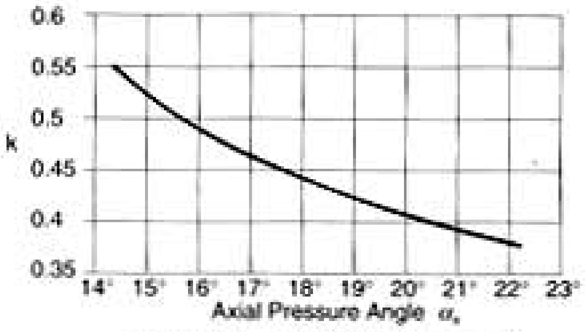


Fig. 9-10 The Value of Factor (*k*)

9.4 Self-Locking Of Worm Mesh

Self-locking is a unique characteristic of worm meshes that can be put to advantage. It is the feature that a worm cannot be driven by the worm gear. It is very useful in the design of some equipment, such as lifting, in that the drive can stop at any position without concern that it can slip in reverse. However, in some situations it can be detrimental if the system requires reverse sensitivity, such as a servo-mechanism.

Self-locking does not occur in all worm meshes, since it requires special conditions as outlined here. In this analysis, only the driving force acting upon the tooth surfaces is considered without any regard to losses due to bearing friction, lubricant agitation, etc. The governing conditions are as follows:

Let F_{u1} = tangential driving force of worm

Then, $F_{u1} = F_n (\cos\alpha_n \sin\gamma - \mu \cos\gamma)$ **(9-6)**

where:

α_n = normal pressure angle

γ = lead angle of worm

μ = coefficient of friction

F_n = normal driving force of worm

If $F_{u1} > 0$ then there is no self-locking effect at all. Therefore, $F_{u1} \le 0$ is the critical limit of self-locking.

Let α_n in **Equation (9-6)** be 20º, then the condition:

$F_{u1} \le 0$ will become:

$(\cos 20º \sin\gamma - \mu\cos\gamma) \le 0$

Figure 9-11 shows the critical limit of self-locking for lead angle γ and coefficient of friction μ. Practically, it is very hard to assess the exact value of coefficient of friction μ. Further, the bearing loss, lubricant agitation loss, etc. can add many side effects. Therefore, it is not easy to establish precise self-locking conditions. However, it is true that the smaller the lead angle γ, the more likely the self-locking condition will occur.

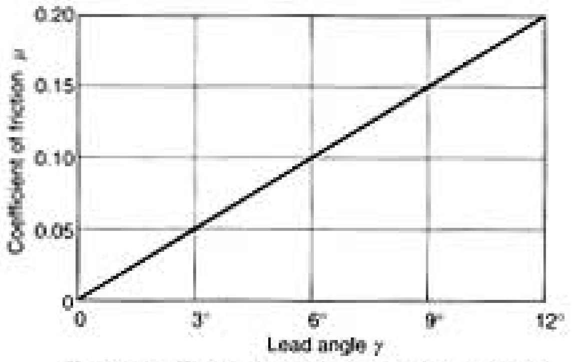


Fig. 9-11 The Critical Limit of Self-locking of Lead Angle γ and Coefficient of Friction μ